Math 1B03/1ZC3 - Tutorial 9

Mar. 14th/18th, 2014

Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
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Examples:

- **1.** Find a unit vector that has the same direction as $(-4, -3)$.

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- **1.** Find a unit vector that has the same direction as $(-4,-3)$.
- **Recall:** a vector of norm 1 is called a unit vector. i.e. if $||u|| = 1$, then u is a unit vector.

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- **2.** Let $u = (0,2,2,1)$ and $v = (1,1,1,1)$. Verify that the Cauchy-Schwartz inequality holds.

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- **Recall: Cauchy-Schwartz Inequality:** $|u \cdot v| \leq ||u||||v||$.

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- **Recall:** $\cos\theta = \frac{u \cdot v}{||u|| ||v||}$.

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- **Recall:** Two vectors u and v are **orthogonal** if $u \cdot v = 0$.
- **b)** Do u, v, and one of the unit vector you found form an orthogonal set?
- **Recall:** A nonempty set of vectors in $\mathbb{R}^n$ is called an orthogonal set if all pairs of distinct vectors in the set are orthogonal.

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- **Recall:** The **point normal equation** of a plane is: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$, where $P_0(x_0,y_0,z_0)$ is a specific point on the plane, $P=(x,y,z)$ is an arbitrary point on the plane, and $n=(a,b,c)$ is the normal vector to the plane.

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- **Recall:** If u and a are vectors in $\mathbb{R}^n$ such that $a \neq 0$, then we can write $u = w_1 + w_2$, where $w_1 = proj_a u = \frac{u \cdot a}{||a||^2} a$ (vector component of u along a; a.k.a. orthogonal projection of u along a),
  and $w_2 = u - w_1 = u - proj_a u$ (component of u orthogonal to a).

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  and $w_2 = u - w_1 = u - proj_a u$ (component of u orthogonal to a).
- **b)** Find the vector component of u orthogonal to a.

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- **7.** Find the distance between the point $(3,1,-2)$ and the plane $x+2y-2z=4$.

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- **Recall:)** In $\mathbb{R}^3$, the distance between a point $P_0(x_0,y_0,z_0)$ and a plane $ax+by+cz+d=0$ is: $\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$.

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- **8.** Consider two points $P(2,3,-2)$ and $Q(7,-4,1)$. Find the point on the line segment containing P and Q that is $\frac{3}{4}$ of the way from P to Q.

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- **Recall:** The vector with initial point $P_1(x_1,y_1,z_1)$ and terminal point $P_2(x_2,y_2,z_2)$ is given by the formula: $\overrightarrow{P_1P_2} = (x_2-x_1, y_2-y_1, z_2-z_1)$.

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- **d)** Find a line that is perpendicular to ℓ.

- **Recall:** Two lines are perpendicular if their dot product is zero.

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- **10.** Find a vector equation of the plane in $\mathbb{R}^4$ passing through the point $(2,-1,7,3)$ and parallel to both $(1,0,2,1)$ and $(3,2,4,5)$.

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- **Recall:** The equation of a plane passing through a point x_0 and parallel to v_1 and v_2, where v_1 and v_2 are not collinear, is $x = x_0 + v_1 t + v_2 s$.

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- **Recall:**

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix},$$

where i, j, and k are the standard unit vectors

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$