Math 1B03/1ZC3 - Tutorial 7

Feb. 28th/ Mar. 4th, 2014

Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .

Examples:

- **1.** Suppose the population of raccoons in the city in 2010 is 100 and the population of raccoons in the nearby forest is 300. Suppose we also know that 10% of the racoons in the forest move to the city, and 5% of the raccoons in the city move to the forest each year.

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- **c)** Does T have a steady-state vector? If so, what is it?
- **Recall:** If P is a regular transition matrix for a Markov chain, then $\exists!$ probability vector q such that $Pq = q$ (i.e. q is an eigenvector corresponding to $\lambda = 1$ and q's entries sum to 1). This vector is called the **steady-state vector**.

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- **e)** How many raccoons will be in the city after 20 years?
- **Recall:** We know $x_n = P^n x_0$, where x_0 is the initial state vector, x_n is the state vector at time n, and P is the transition matrix.

Examples:

- **2.** Express $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ as a real number.

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- **Recall:** The **argument** of z is multivalued, i.e. $\arg z = \theta + 2\pi k, k \in \mathbb{Z}$.
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- **d)** What is $\bar{z}$?
- **Recall:** If $z = a + bi$, then the **complex conjugate** of z is: $\bar{z} = a - bi$.

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- **4.** Express $(\sqrt{3} - i)^6$ in polar form.

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- **5.** Find the solutions to the equation $z^3 = -1$.

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- **Recall:** $z^{\frac{1}{n}} = \sqrt[n]{r}[\cos(\frac{\theta}{n} + \frac{2k\pi}{n}) + i\sin(\frac{\theta}{n} + \frac{2k\pi}{n})]$, $k = 0, 1, \ldots, n-1$.

Examples:

- **6. a)** Find the square roots of $2i$.

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- **b)** Express your two roots in rectangular coordinates.