Math 1B03/1ZC3 - Tutorial 4

Feb. 7th/ Feb. 11th, 2014

Tutorial Info:

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- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .

Examples:

- **1.** Consider

$$A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}.$$

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- So, since we're looking for vectors $\boldsymbol{x}$ such that $(A - \lambda I)\boldsymbol{x} = 0$ and we know that $\boldsymbol{x} \neq 0$ by definition, then by our equivalent statements about inverses that must mean that $\det(A - \lambda I) = 0$.

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- So, since we're looking for vectors $\boldsymbol{x}$ such that $(A - \lambda I)\boldsymbol{x} = 0$ and we know that $\boldsymbol{x} \neq 0$ by definition, then by our equivalent statements about inverses that must mean that $\det(A - \lambda I) = 0$.
- So, λ is an **eigenvalue** of $A \Leftrightarrow$ it satisfies the equation $\det(A - \lambda I) = 0$.

Examples:

- **b)** Find all eigenvectors of *A*.

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- **c)** Is A invertible?

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- **3.a)** Find the eigenvalues of

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- **b)** Find all eigenvectors corresponding to $\lambda = 2$.

Examples:

- **4.)** Consider

$$A = \begin{pmatrix} 5 & -3 \\ a & b \end{pmatrix}$$

and suppose

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is an eigenvector of A. What must the eigenvalue λ corresponding to $\boldsymbol{x}$ be?

Examples:

- **5.)** Find all eigenvalues and eigenvectors of A^{10}, if

$$A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}.$$