Math 1B03/1ZC3 - Tutorial 4

Jan. 31st/ Feb. 4th, 2014

Tutorial Info:

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- **Math Help Centre:** Wednesdays 2:30-5:30pm.
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Examples:

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- **Cofactor of entry $\mathbf{a_{ij}}$, $\mathbf{C_{ij}}$:** is kM_{ij}, where $k = 1$ or -1 in accordance with the pattern in the checkerboard array:

$$B = \begin{pmatrix} + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix},$$

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- **Recall:** You can find $\det(A)$ by multiplying the entries in any row or column by their corresponding cofactor and adding the resulting products.
- **Note:** We could have chosen a different row or column.

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- **2.** Consider

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- Doing the same operations on B's columns yield the same results.

Examples:

- **b)** Consider

$$B = \begin{pmatrix} t & 2t & 3t \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix},$$

for $t \in \mathbb{R}$. Find $\det(B)$.

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  (e) $\det(A^{-1}) = \frac{1}{\det(A)}$.

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$$A = \begin{pmatrix} 1 & x & 2 \\ 3 & 1 & -1 \\ -1 & 2 & 2 \end{pmatrix}.$$

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- **b)** When is A invertible?

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- **6.** Consider

$$A = \begin{pmatrix} 0 & 2 & 1 \\ -1 & -3 & 1 \\ -2 & -1 & -2 \end{pmatrix}.$$

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- **Recall:** If A is invertible, then $A^{-1} = \frac{1}{\det(A)} adj(A)$, where

$$adj(A) = \begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{pmatrix}^T.$$

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- **Cramer's Rule:** If $Ax=b$ is a system of n linear equations in n unknowns such that $\det(A)\neq 0$, then $Ax=b$ has a unique solution.
  This solutions is: $x_1=\frac{\det(A_1)}{\det(A)}$, $x_2=\frac{\det(A_2)}{\det(A)}$, ..., $x_n=\frac{\det(A_n)}{\det(A)}$, where A_j is the matrix obtained by replacing the entries in the jth column of A by the entries in the matrix b.