Math 1B03/1ZC3 - Tutorial 2

Jan. 21st/24th, 2014

Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
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then

$$AB = \begin{pmatrix} 10 & 2 & 0 & -4 \\ 7 & 2 & 1 & -3 \\ 9 & 6 & 7 & -5 \end{pmatrix},$$

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but BA is not defined.

- So no, it is not true in general that $AB = BA$.

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- What if A and B are both square (i.e. A and B are both $n \times n$ matrices)?
- Does $AB = BA$ for any possible A and B?
- Can you think of a counterexample?

Does the Commutative Law for Multiplication hold for Matrices?

- Is it ever possible to find an A and B such that $AB = BA$?

Zero Divisors?

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- Is this true for matrices? (i.e. if we have two matrices A and B such that $AB = 0$, is it true that we must have $A = 0$ or $B = 0$?)

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Recap: In general, it is not true that:

- $AB = BA$ (i.e. multiplicative commutativity fails)
- $AB = 0 \Rightarrow A = 0$ or $B = 0$ (i.e. $\exists$ non-zero zero divisors)
- $AB = AC \Rightarrow B = C$ (i.e. cancellation law fails)

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- **e.g.**

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- If A is a square $(n \times n)$ matrix such that $\exists$ a B such that $AB = I_{n \times n} = BA$, then A is said to be **invertible**, (a.k.a **nonsingular**), and B is called the inverse of A, $(B = A^{-1})$.

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- If A is a 2×2 matrix, then

$$A^{-1} = \frac{1}{ad - bc} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

b/c:

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- det(A)$= ad - bc$.
- A is nonsingular $\Leftrightarrow$ det(A)$\neq 0$.
- So, det(A)$= 0 \Leftrightarrow A$ is singular (i.e. A is not invertible).

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- **1.** Let

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- **c)** Is B invertible?.
- **d)** Find B^{-1}.
- **e)** Find $(AB)^{-1}$.

Examples:

- **2.** Let

$$A = \begin{pmatrix} 4 & x \\ x & 1 \end{pmatrix}.$$

For what values of x is A singular?

Examples:

- **3.** Solve for X: $A(X+B)=CA$ (where A is invertible).

Examples:

- **4.** Solve for X: $(2E+F)^T = G^{-1}X^T + F^T$.

Examples:

- **5.** Find the inverse of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 5 \\ 3 & 2 & 3 \end{pmatrix}$$

using row operations.

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- **b)** What sizes must F and W be in order for W to have a unique solution if E is $3 \times n$?