Math 1B03/1ZC3 - Tutorial 11

Mar. 28st/ Apr. 1st, 2014

Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .

Examples:

- **1.** Consider the following sets of vectors:

$$S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\},$$

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- **Recall:** If A is square, then $Ax = b$ is consistent for every $n \times 1$ matrix $b \iff \det(A) \neq 0$.

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- **Recall:** If a set of vectors $S = \{v_1, \ldots, v_r\}$ is such that the equation $\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_r v_r = \bar{0}$ has only the trivial solution (i.e. $\alpha_1 = \ldots = \alpha_r = 0$), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.

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- **b)** Find the vector $w \in \mathbb{R}^2$ whose coordinate vector relative to T is $[w]_T = (4,2)$.

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- **4.)** Which of the following are a basis for P_2 (where P_2 is the vector space of all polynomials of degree ≤ 2; i.e. $P_2 = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$.

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