

Math 1B03/1ZC3 - Tutorial 11



Mar. 28st/ Apr. 1st, 2014

Tutorial Info:

- **Website:** <http://ms.mcmaster.ca/~dedieula>.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .



Examples:

- 1. Consider the following sets of vectors:

$$S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\},$$

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- Recall:** The **span** of a set $S = \{w_1, \dots, w_r\}$, is the subspace formed by taking all possible linear combinations of the vectors in S . **i.e.**
 $\text{span}(S) = \{\alpha_1 w_1 + \dots + \alpha_r w_r \mid \alpha_1, \dots, \alpha_r \in \mathbb{R}\}.$



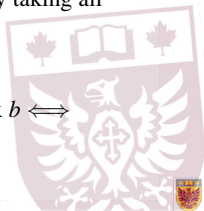
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- Recall:** If A is square, then $Ax = b$ is consistent for every $n \times 1$ matrix $b \iff \det(A) \neq 0$.



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- **Recall:** If a set of vectors $S = \{v_1, \dots, v_r\}$ is such that the equation $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_r v_r = \vec{0}$ has only the trivial solution (i.e. $\alpha_1 = \dots = \alpha_r = 0$), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.



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- b) Find the vector $w \in \mathbb{R}^2$ whose coordinate vector relative to T is $[w]_T = (4, 2)$.



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- 4.) Which of the following are a basis for P_2 (where P_2 is the vector space of all polynomials of degree ≤ 2 ; i.e. $P_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbb{R}\}$).



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