Math 1B03 - Tutorial #1

1. Let $A := \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 2 & 3 \end{pmatrix}$, $B := \begin{pmatrix} 3 & 0 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix}$.

ⓐ Can we multiply $A*B$?

- A has 3 rows and 2 columns: A_{32}
  B has 2 rows and 4 columns: B_{24}.

$_3A_2 * {}_2B_4$

2 2 same!

Yes, we can multiply $A*B$, & we will get a 3×4 matrix.

ⓑ Find AB.

$$-AB = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 0 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix} = \begin{pmatrix} (3*3+1*1) & (3*0+1*2) & (3*-1+1*3) & (3*-1+1*-1) \\ (2*3+1*1) & (2*0+1*2) & (2*-1+1*3) & (2*-1+1*-1) \\ (2*3+3*1) & (2*0+3*2) & (2*-1+3*3) & (2*-1+3*-1) \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 2 & 0 & -4 \\ 7 & 2 & 1 & -3 \\ 9 & 6 & 7 & -5 \end{pmatrix}.$$

ⓒ Can we multiply $B*A$?

- $_2B_4 * {}_3A_2$

4 3 Not same!

No, we can't multiply $B*A$, b/c # of columns in B ≠ # of rows in A (inner sizes don't match!).

2. What matrix A is the solution to

$$5A + \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} ?$$

$$\to 5A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$$

$$\to 5A = \begin{pmatrix} (1-1) & (-2-3) \\ (3-2) & (0-1) \end{pmatrix}$$

$$\to 5A = \begin{pmatrix} 0 & -5 \\ 1 & -1 \end{pmatrix} \to A = \frac{1}{5}\begin{pmatrix} 0 & -5 \\ 1 & -1 \end{pmatrix}$$

$$\to A = \begin{pmatrix} 0 & -\frac{5}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix} \to \boxed{A = \begin{pmatrix} 0 & -1 \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}}$$

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3. Consider the following system of equations:

$-x + z = 1$
$x - y + z = -1$
$-x + y - z = 1.$

ⓐ Find the augmented matrix for this system of equations.

$$\left(\begin{array}{ccc|c} -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{array}\right)$$

$\boxed{\text{IF a solution exists, then \#Free Variables = \#unknowns − \#leading 1's.}}$

ⓑ Put this matrix in reduced row echelon form.

<u>Recall</u>: <u>Elementary row operations:</u>

we can:

1. Multiply a row by a non-zero scalar.
2. Switch any two rows.
3. Add a multiple of another row to an existing row.

$$\left(\begin{array}{ccc:c} -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{array}\right) \begin{array}{l} \\ r_2 \leftarrow r_2 + r_1 \\ r_3 \leftarrow r_3 - r_1 \end{array} \quad \left(\begin{array}{ccc:c} -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \begin{array}{l} r_1 \leftarrow r_1 * -1 \\ r_3 \leftarrow r_3 + r_2 \end{array}$$

$$\left(\begin{array}{ccc:c} 1 & 0 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) r_2 \leftarrow r_2 * -1 \quad \left(\begin{array}{ccc:c} 1 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

ⓒ Find all solutions to the system of equations.

$x - z = -1$
$y - 2z = 0$
$z = t$

We can see that z will be a free variable, since there is no leading "1" in the "z" column. ($3-2=1$ Free variable).

$x = -1 + t$
$y = 2t$
$z = t$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} t.$$

infinitely many solutions!

<u>Check</u>

$$\begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} -1+t \\ 2t \\ t \end{pmatrix} = \begin{pmatrix} -1(-1+t) + 0 + t \\ -1+t \quad -2t + t \\ -1(-1+t) + 2t - t \end{pmatrix} = \begin{pmatrix} 1 - t + t \\ -1 \\ 1 - t + t \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \checkmark$$

3 3 3 3 1

4. Find an augmented matrix which corresponds to the following solution set: $x = 13 - s - t$

$y = t$

$z = s.$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 13 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

not necessarily unique.

5. Does the following homogeneous system have non-trivial solutions?

(At most 2 leading 1's. # Free variables $\geq 4-2=2$.)

$$x_1 + 2x_2 + x_3 - x_4 = 0$$
$$9x_1 + 999x_2 + 7x_3 + 23456x_4 = 0.$$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is the trivial solution.

Yes, there are non-trivial solutions b/c there are more unknowns than equations. (pg. 19)

6. Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) Find A^T.

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

(b) Find $Tr(A)$.

$$Tr(A) = 1 + 4 = 5.$$

7. (a) For what values of b will the system have infinitely many solutions?

$$2x - 6y = b$$
$$6x - 18y = 30$$

$$\left(\begin{array}{cc|c} 2 & -6 & b \\ 6 & -18 & 30 \end{array}\right) \; r_2 \leftarrow r_2 - 3r_1$$

$$\left(\begin{array}{cc|c} 2 & -6 & b \\ 0 & 0 & -3b+30 \end{array}\right)$$

To have infinitely many solutions we need a parameter. In fact, to even have a solution at all here we need $-3b + 30 = 0 \Leftrightarrow 3b = 30 \Leftrightarrow b = 10$.

Check

$$\left(\begin{array}{cc|c} 2 & -6 & 10 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} 2x - 6y = 10 \rightarrow 2x = 10 + 6t \rightarrow x = 5+3t \\ y = t \end{array}$$

So, if $b = 10$ we'll have infinitely many solutions.

i.e., $\begin{pmatrix} 2 & -6 \\ 6 & -18 \end{pmatrix} \begin{pmatrix} 5+3t \\ t \end{pmatrix} = \begin{pmatrix} 2(5+3t) - 6t \\ 6(5+3t) - 18t \end{pmatrix} = \begin{pmatrix} 10 + 6t - 6t \\ 30 + 18t - 18t \end{pmatrix} = \begin{pmatrix} 10 \\ 30 \end{pmatrix} \checkmark$

(b) For what values of b will the system have no solutions?

If $b \neq 10$, then we will have $-3b + 30 \neq 0$
$\Rightarrow 0x + 0y = -3b + 30 \neq 0$ ↯.

So, if $b \neq 10$, then this system has no solutions.