Math 1B03 Midterm 1

Monday 5 October 2009

1. Which of the following matrices are in reduced row echelon form?

(a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

(c) $(1\ 2\ 3\ 4)$

(d) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. Find a matrix A such that

$$\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} A = \begin{pmatrix} 2 & 4 & 6 \\ 7 & 14 & 21 \end{pmatrix}.$$

3. Calculate the determinant of the 5×5 matrix

$$B = \begin{pmatrix} 0 & 2 & 0 & 5 & 0 \\ 2 & 5 & 1 & 7 & -6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 4 & 0 & -1 & 2 \\ 1 & 0 & -1 & -1 & 9 \end{pmatrix}.$$

4. For the invertible matrix

$$C = \begin{pmatrix} 3 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix},$$

calculate each of the following.

(a) $\det C$

(b) $\operatorname{adj} C$

(c) C^{-1}

5. Consider the vectors

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \vec{y} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}.$$

Decide whether either of these vectors is an eigenvector for the matrix

$$\begin{pmatrix} 3 & -3 & 1 \\ -5 & 2 & 0 \\ 1 & 2 & -2 \end{pmatrix}$$

and, if so, determine the associated eigenvalue(s).

6. Let

$$D = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 2 & 4 \\ 2 & 2 & 4 \end{pmatrix}.$$

(a) Write down the augmented matrix $(D \mid I_3)$ and put it into reduced row echelon form.

(b) Is D invertible? If so, find D^{-1}.

7. Each of the following statements is either always true or sometimes false. If the statement is true, give a *brief* explanation of why. If the statement is false, give an *explicit* counterexample.

(a) If E and F are $n \times n$ matrices such that $EF = 0$, then either $E = 0$ or $F = 0$.

(b) If G is an $n \times n$ matrix such that $\det G = 0$ and $\vec{b}$ is a non-zero n-dimensional vector, then the equation $G\vec{x} = \vec{b}$ can never have a solution.

(c) A linear system with more variables than equations always has at least 1 solution.

(d) If H and K are $n \times n$ matrices such that HK is singular, then both H and K are singular.

(e) If L and M are $n \times n$ matrices such that $\operatorname{rank} L = \operatorname{rank} M$, then the reduced row echelon form of L is the same as the reduce row echelon form of M.

8. The inverse of the matrix

$$N = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

is the matrix

$$N^{-1} = \begin{pmatrix} -2 & -1 & 5 \\ 1 & 0 & -1 \\ 1 & 1 & -3 \end{pmatrix}.$$

Use this information to solve the linear system

$$N\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

9. Determine all possible values of h and k for which the linear system

$$\begin{aligned} -x_1 + 2x_2 &= 1 \\ x_1 - hx_2 &= k \end{aligned}$$

has:

(a) no solutions;

(b) exactly 1 solution;

(c) infinitely many solutions.