§14.3 cont'd.

3rd Order Partial Derivatives.

$f(x,y)$ has 8, 3rd order partial derivatives

$$\underbrace{f_{xxx} \quad f_{xxy} \quad f_{xyx} \quad f_{yxx}}$$

same by Clairaut's Th. provided the hypothesis are satisfied.

$$\overbrace{f_{yyy} \quad f_{yyx} \quad f_{yxy} \quad f_{xyy}}$$

Example. $f(x,y) = x^2 y^3$

$f_x = 2xy^3$ $\qquad$ $f_y = 3x^2y^2$

$f_{xx} = 2y^3$ $\qquad$ $f_{yy} = 6x^2y$

$f_{xy} = 6xy^2 = f_{yx}$

March 23, 2020 7:54 PM

$$f_{xxy} = 6y^2 = f_{xyx} = f_{yxx}$$

$$f_{yyx} = 12xy = f_{yxy} = f_{xyy}$$

A PARTIAL DIFFERENTIAL EQUATION is an equation involving a function of more than one variable and its partial derivatives.

Laplace's Eqn: $f_{xx} + f_{yy} = 0$. (2nd order PDE).

Sol'ns of PDE is all functions $f(x,y)$ that satisfy the PDE.

Some sol'ns of Laplace's Eq'n.

(1). $f(x,y) = x^2 - y^2$

$f_x = 2x$ $\qquad$ $f_y = -2y$

$f_{xx} = 2$ $\qquad$ $f_{yy} = -2$

$f_{xx} + f_{yy} = 2 + (-2) = 0$ ✓

March 23, 2020 8:08 PM

(ii) $f(x,y) = x^3 - 3xy^2$

$f_x = 3x^2 - 3y^2$ $\quad f_y = -6xy$

$f_{xx} = 6x$ $\quad f_{yy} = -6x$

$f_{xx} + f_{yy} = 6x + (-6x) = 0$ ✓

(iii) $f(x,y) = e^x \cos(y)$

$f_x = e^x \cos(y)$ $\quad f_y = -e^x \sin(y)$

$f_{xx} = e^x \cos(y)$ $\quad f_{yy} = -e^x \cos(y)$

$f_{xx} + f_{yy} = 0$ ✓

WAVE EQ'N $\quad f(x,t)$

x spatial

t time.

$$\frac{1}{c^2} f_{tt} = f_{xx}$$

A sol'n: $f(x,y) = \sin(x+ct)$

HEAT EQ'N $\quad f_t = k f_{xx}, \quad k > 0$