M1ZB3 Lecture 30 Part 1 (C02) Dr. Wolkowicz
March 24

March 17, 2020 10:00 AM

§14.3 Partial Derivatives

Defn. The PARTIAL DERIVATIVES of $f(x,y)$ at (a,b) in the domain of f are:

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

wrt to x

$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

wrt y

IDEA:

1. To calculate f_x, regard y as a constant and differentiate wrt x.

i.. $g(x) = f(x,b)$

$$g'(a) = \lim_{h\to 0} \frac{g(a+h) - g(a)}{h}.$$

$$= f_x(a,b).$$

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2. To calculate f_y, regard x as a constant and differentiate wrt y.

i.e. $k(y) = f(a, y)$

$$k'(b) = \lim_{h \to 0} \frac{k(b+h) - k(b)}{h}$$

$$= f_y(a, b).$$

Alternate NOTATION: $z = f(x, y)$

$$f_x = \frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = D_1 f = D_x f$$

wrt 1st variable

$$f_y = \frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = D_2 f = D_y f$$

symbol "∂" is called "del".

Example $f(x,y) = \sin(x^3y^2)$

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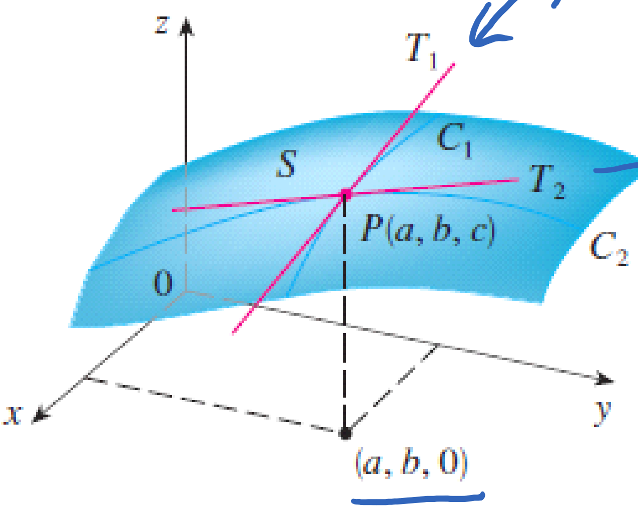
$$\frac{\partial f}{\partial x} = \cos(x^3y^2)\,\frac{\partial}{\partial x}(x^3y^2)$$
$$= \cos(x^3y^2)\,(3x^2y^2)$$

$$\frac{\partial f}{\partial y} = \cos(x^3y^2)\,\frac{\partial}{\partial y}(x^3y^2)$$
$$= \cos(x^3y^2)\,(2yx^3)$$

GEOMETRIC Interpretation.

T_1 has slope $f_x(a,b)$



T_2 has slope $f_y(a,b)$.

$c = f(a,b)$

FIGURE 1
The partial derivatives of f at (a, b) are the slopes of the tangents to C_1 and C_2.

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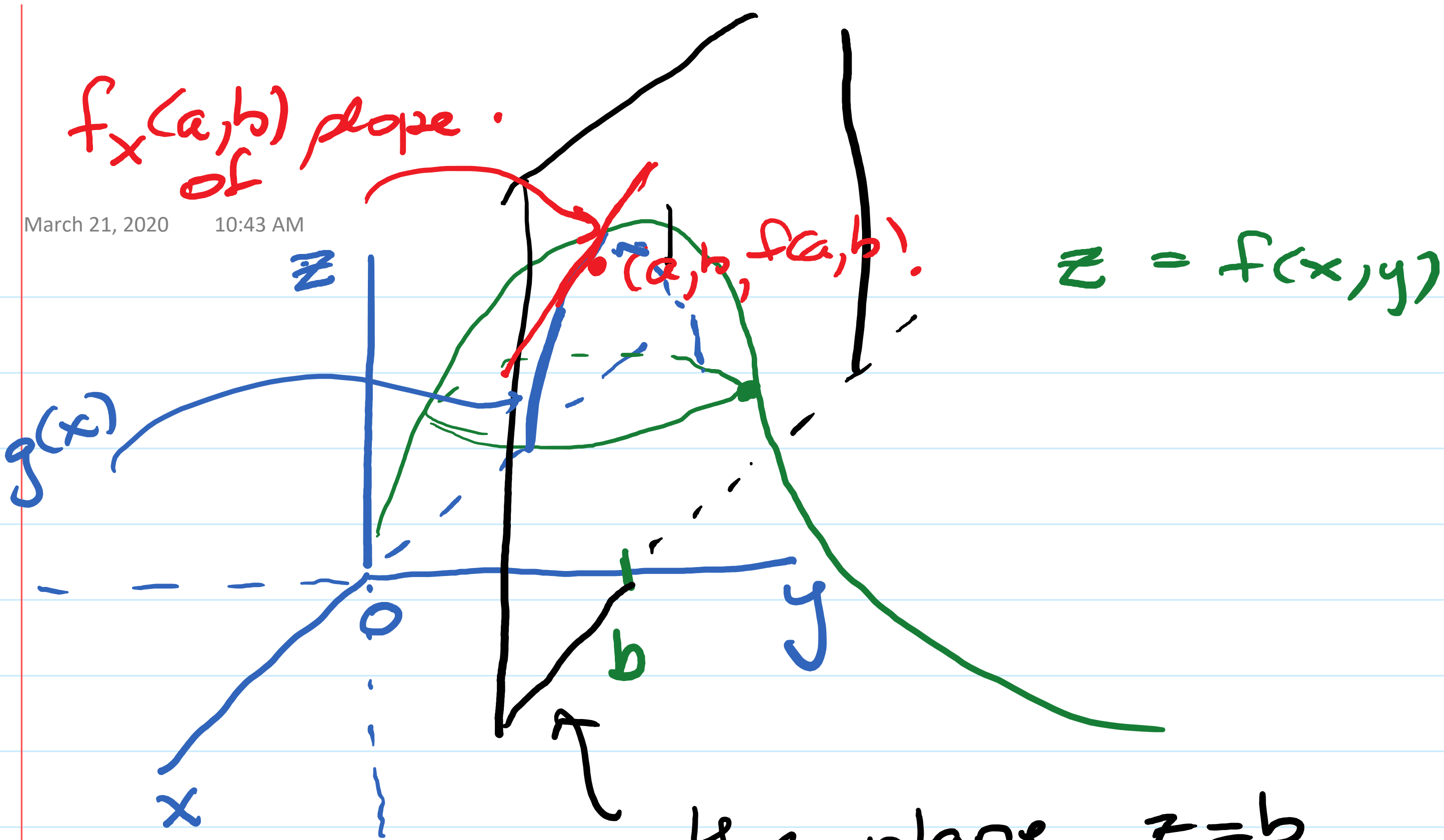

the plane $z=b$ parallel to the xz-coordinate plane.

$f_x(a,b)$

$f_x(a,b) = g'(a) =$ slope of the tangent to $g(x) = f(x,b)$ at $x=a$.

...

Similarly, Take a slice parallel to the yz-coordinate plane

$f_y(a,b) = k'(b)$

$=$ slope of the tangent to $k(y) = f(a,y)$ at $y=b$.