§14.2 CONTINUITY

Def'n: $f(x,y)$ is continuous at $(a,b) \in D$ (the domain of f) if:

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$$

If $f(x,y)$ is continuous it means it is continuous at every $(a,b) \in D$.

Examples functions of 2 variables.

1. Polynomial functions.

$$f(x,y) = \sum_{i=0}^{N} \sum_{j=0}^{M} a_{ij} x^i y^j$$

e.g. $$f(x,y) = \sum_{i=0}^{1} \sum_{j=0}^{2} a_{ij} x^i y^j$$

$$= a_{00} + a_{01} y + a_{02} y^2 + a_{10} x + a_{11} xy + a_{12} x y^2$$

$$f(x,y) = 3x + 2xy + 4xy^2$$

some of the $a_{ij} = 0$.

2. Rational functions.

$f(x,y) = \dfrac{g(x,y)}{h(x,y)}$, where $g(x,y)$ and $h(x,y)$ are polynomial functions. $h(x,y) \neq 0$.

Domain D
$\{(x,y) \in \mathbb{R}^2 \mid h(x,y) \neq 0\}$

then $f(x,y)$ is continuous on D.

3. $f(x,y) = \begin{cases} 0 & \text{if } (x,y) = (0,0) \\ \dfrac{3x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \end{cases}$

RECALL last lecture

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} = 0.$$

If $(a,b) \neq (0,0)$

$$\lim_{(x,y)\to(a,b)} \frac{3x^2y}{x^2+y^2} = \frac{3a^2b}{a^2+b^2} = f(a,b).$$

4. Composition of continuous functions is continuous

If $h = g \circ f$

i.e. If $h(x,y) = g(f(x,y))$

and g and f are continuous then $h(x,y)$ is continuous

Example. $h(x,y) = \ln(1+x+y)$

Where is $h(x,y)$ continuous?

$h(x,y) = g(f(x,y))$

where $f(x,y) = 1+x+y$

and $g(t) = \ln(t)$

$f(x,y)$ is continuous for all $(x,y) \in \mathbb{R}^2$

$g(t)$ is continuous for $t>0$.

$\therefore$ require $\underbrace{1+x+y}_{t} > 0$.

$h(x,y)$ is continuous provided $1+x+y > 0$.