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§4.1 Functions of OF SEVERAL VARIABLES

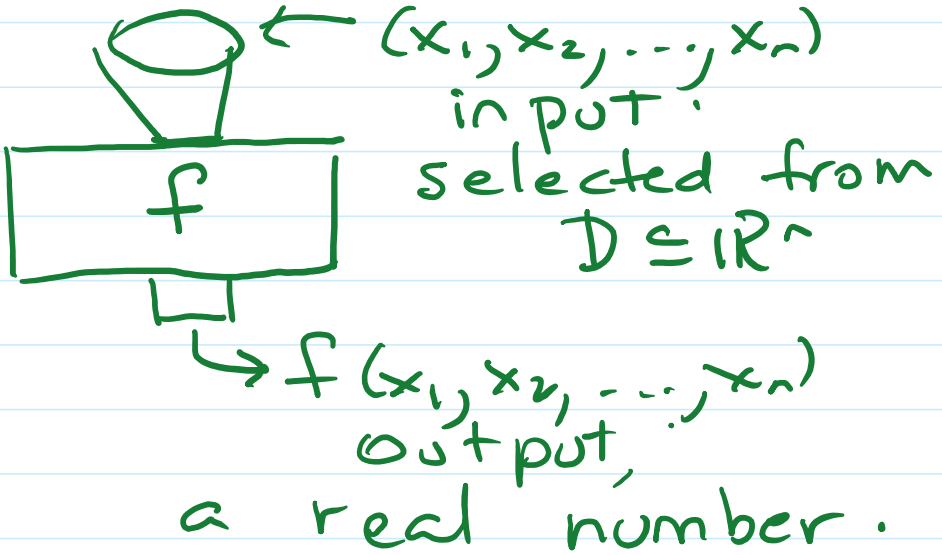
Let n be a positive integer.

Def'n. A function of n-varichles is a rule that assigns to each n-tuple in a given set $D \subseteq \mathbb{R}^n$ a real number $f(x_1, x_2, \ldots, x_n)$.

$D \subseteq \mathbb{R}^n$, n-tuples $(x_1, x_2, x_2, \ldots, x_n)$

$f: D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$

D is called the DOMAIN of f.



The largest set that makes sense is called the "natural domain".

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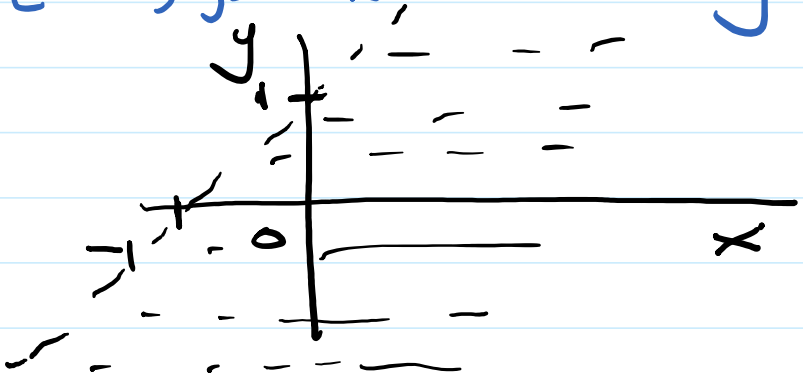
Examples. (two variables. $n=2$)

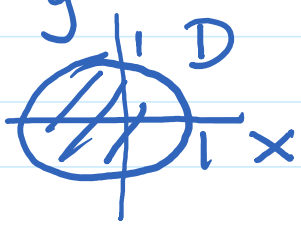
① $f(x,y) = \ln(1+x-y)$

$D = \{(x,y) \in \mathbb{R}^2 : 1+x-y > 0\}$



② $f(x,y) = (1-x^2-y^2)^{1/2}$

$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$.



$(n>2)$

③ $f(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \cdots + x_n = \sum_{i=1}^{n} x_i$

$D = \mathbb{R}^n$.

④ $f(x_1, x_2, \ldots, x_n) = \sum_{i<j} x_i x_j$

e.g. $n = 3$

$$f(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + x_2x_3$$

Def'n The RANGE of function $f(x_1, x_2, \ldots, x_n)$ with domain D is the set of all possible values of $f(x_1, x_2, \ldots, x_n)$ as $(x_1, x_2, \ldots, x_n)$ ranges over D.

Range of $f(x_1, x_2, \ldots, x_n)$

$$= \{f(x_1, x_2, \ldots, x_n) \in \mathbb{R} : (x_1, x_2, \ldots, x_n) \in D\}.$$

Examples.

① range of $f(x,y) = \cos(x+y)$ is $[-1, 1] = \{z \in \mathbb{R} : -1 \leq z \leq 1\}$.

Domain of ω is $\mathbb{R}^2$

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② range of $f(x,y) = e^{x^2-y^2}$

Domain of f is $\mathbb{R}^2$

Range of f is $(0,\infty)$

$= \{z \in \mathbb{R} : z > 0\}$.

i.e. all positive real numbers.

Defn The graph of $f(x,y)$ with DOMAIN D is the set

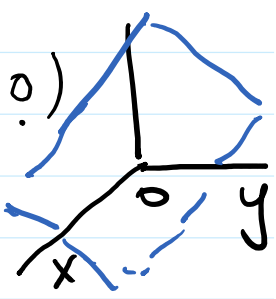
$$\{(x,y,f(x,y)) \in \mathbb{R}^3 : (x,y) \in D\} \subseteq \mathbb{R}^3.$$

Examples.

① $f(x,y) = ax + by$, $a, b \in \mathbb{R}$ (fixed)

$D \subseteq \mathbb{R}^2$

Graph is a PLANE in $\mathbb{R}^3$ through the origin $(0,0,0)$



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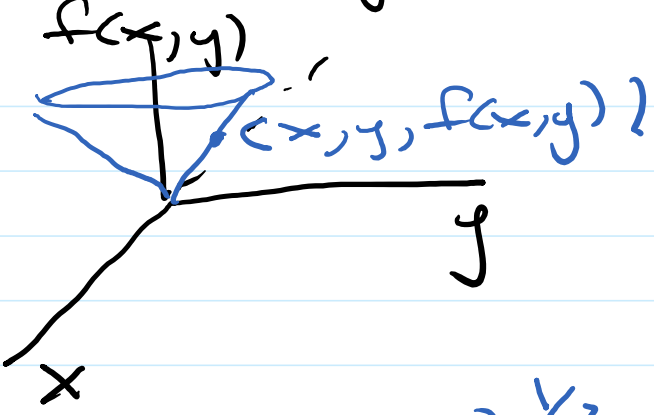
(2) $f(x,y) = (x^2+y^2)^{1/2}$

$D \subseteq \mathbb{R}^2$



(3) $f(x,y) = (1-x^2-y^2)^{1/2}$

$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$

Graph is the Northern hemisphere of a sphere, with radius 1 and centred at the origin. (0,0,0).

Def'n. The LEVEL CURVES of a function $f(x,y)$ with domain $D \subseteq \mathbb{R}^2$ are the curves

$\{(x,y) \in D : f(x,y) = c\}$.

(here c are constants.)

Examples.

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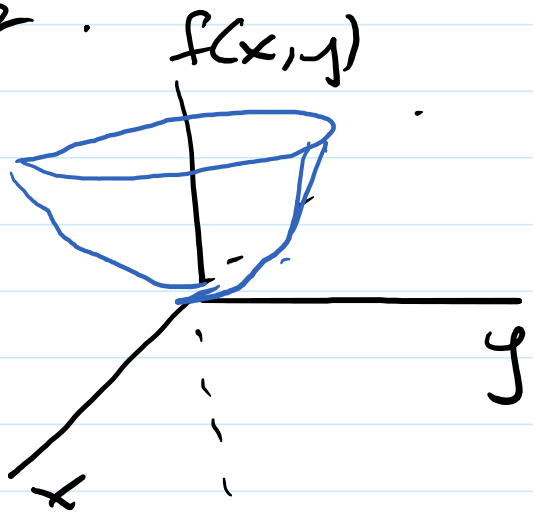
$f(x,y) = x^2 + y^2$. paraboloid.



$f(x,y) = c$.

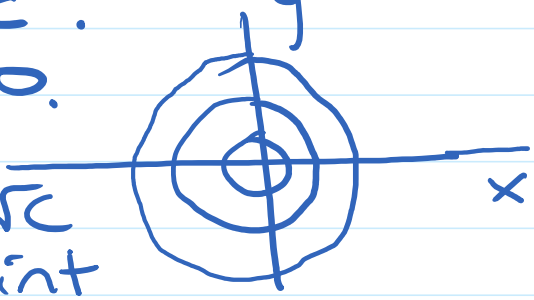
Cut the graph with a horizontal plane.

$z = c$.

to obtain the circle

$$x^2 + y^2 = c$$

LEVEL CURVES are circles centred at $(x,y) = (0,0)$. with radius $\sqrt{c}$. provided $c \geq 0$.



$c > 0$: circle of radius $\sqrt{c}$

$c = 0$: we get the point $(0,0)$

$c < 0$: empty set.

Topographic Maps.

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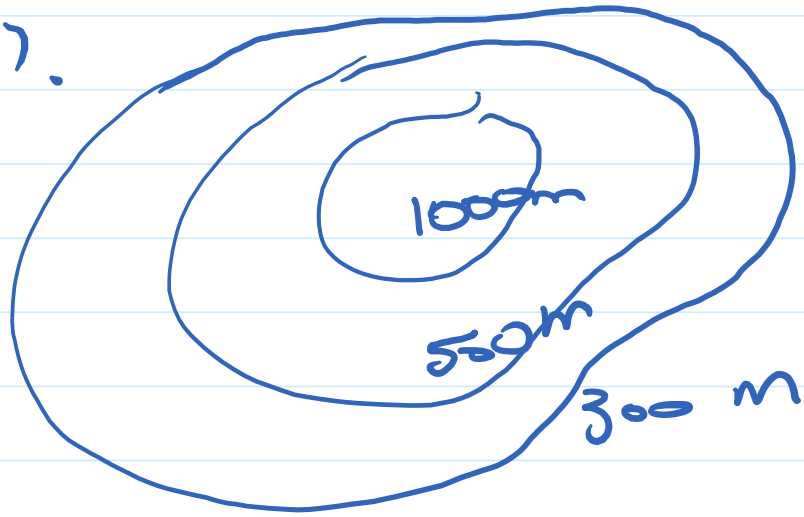
$f(x,y)$ represent elevation above sea level at locations with coordinates (x,y).



Isotherms on weather maps.

See the textbook for nice maps and pictures in § 4.1.