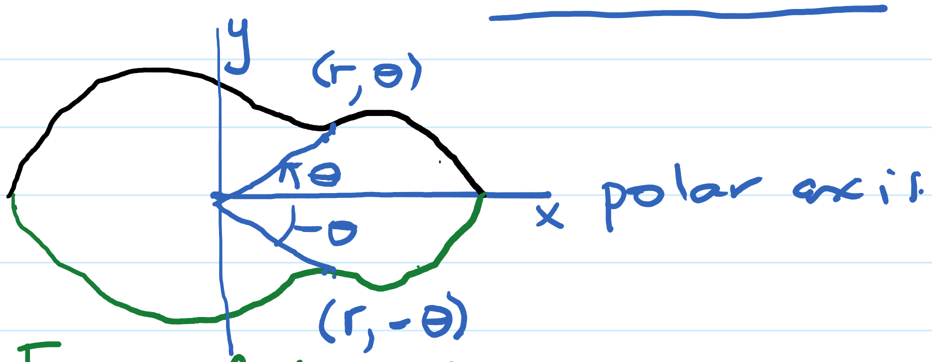
Classes and Monday's test CANCELLED as of end of today
- updates will be posted.

§ 10.3 Polar Coordinates cont'd.

Polar Curve: $r = f(\theta)$

Symmetry.

① If $f(-\theta) = f(\theta)$
curve is symmetric about the POLAR axis



Example: $r = \cos(\theta)$
$f(\theta) = \cos(\theta)$
$f(-\theta) = \cos(-\theta) = \cos(\theta)$

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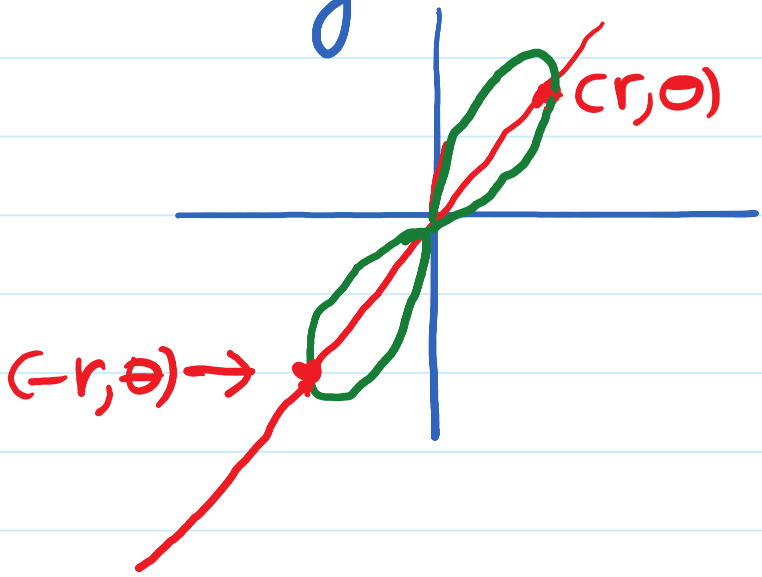
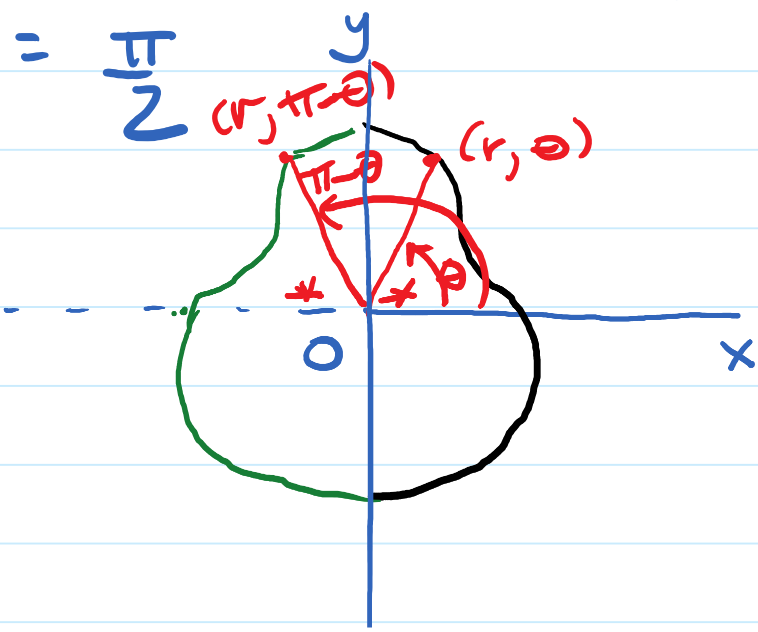
② If $f(\theta) = f(\theta + \pi)$
curve is unchanged
if r is replaced by $-r$
i.e. if rotate curve
by 180° (i.e by π rad)



③ If $f(\theta) = f(\pi - \theta)$.
curve is symmetric
about the vertical line
$\theta = \frac{\pi}{2}$



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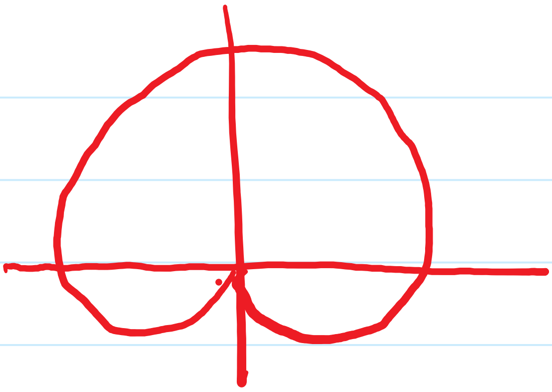
Cardioid

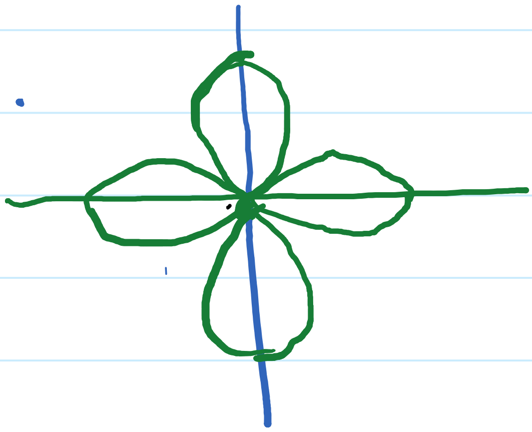
$r = 1 + \sin\theta$



Symmetric about the vertical axis.

4-leaved-rose.

$r = \cos(2\theta)$



- symmetric about the polar axis since $\cos(2\theta) = \cos(-2\theta)$

- symmetric about the vertical axis.

$$\cos(2(\pi-\theta)) = \cos(2\pi - 2\theta) = \cos(-2\theta) = \cos(2\theta).$$

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Tangents to Polar Curves.

$r = f(\theta)$

$x = r\cos\theta = f(\theta)\cos(\theta)$

$y = r\sin\theta = f(\theta)\sin(\theta)$

Slope of the tangent line:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

BEWARE r is a function of θ $r = f(\theta)$.

NOTE: at $r=0$ this simplifies to

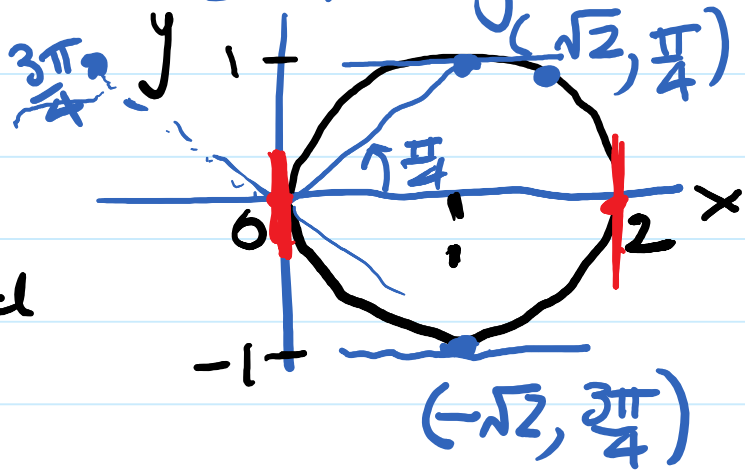
$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

provided $\frac{dr}{d\theta} \neq 0$

Example: $r = 2\cos\theta$

We showed this is: $(x-1)^2 + y^2 = 1$

Find points where there are vertical and horizontal tangents?



$$x = r(\theta)\cos(\theta) \qquad r(\theta) \quad 2\cos(\theta)$$

$$y = r(\theta)\sin(\theta) \qquad \frac{dr}{d\theta} = -2\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r(\theta)\cos\theta}{\frac{dr}{d\theta}\cos\theta - r(\theta)\sin\theta}.$$

$$= \frac{-\cancel{2}\sin\theta\sin\theta + \cancel{2}\cos\theta\cos\theta}{-\cancel{2}\sin\theta\cos\theta - \cancel{2}\cos\theta\sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{-2\sin\theta\cos\theta}$$

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horizontal tangent

(usually $\frac{dy}{d\theta} = 0$, $\frac{dx}{d\theta} \neq 0$

OR just look at numerator $= 0$
denominator $\neq 0$

numerator : $\cos^2\theta - \sin^2\theta = 0$.

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

$r = 2\cos\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$.

$\theta = \frac{3\pi}{4}$. gives $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

$\theta = \frac{5\pi}{4}$ give $\left(-\sqrt{2}, \frac{5\pi}{4}\right)$

$\theta = \frac{7\pi}{4}$ give $\left(\sqrt{2}, \frac{7\pi}{4}\right)$

$$= \left(-\sqrt{2}, \frac{3\pi}{4}\right) = \left(\sqrt{2}, -\frac{\pi}{4}\right)$$

Note denominator
$-2\sin\theta\cos\theta \neq 0$.

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Vertical tangents.

when $\frac{dy}{dt} \neq 0$, $\frac{dx}{dt} = 0$

OR. denominator $= 0$

numerator $\neq 0$.

denominator : $-2\sin\theta\cos\theta$

$r = 2\cos\theta$

$\sin\theta = 0$, $\theta = 0, \pi, 2\pi$

$(2, 0)$. $(-2, \pi)$ $(2, 2\pi)$

$\cos\theta = 0$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$(0, \frac{\pi}{2})$. $(0, \frac{3\pi}{2})$

NOTE: the numerator $\neq 0$ at any of these angles.

§4.1 Functions of Several Variables.