Test #2 Review Sessions

Seating #1
Day: Thurs. Mar. 12
Room: PGCLL B138
Time: 3:30-5:20pm
(15:30-17:20)

Seating #2
Day: Fri. Mar. 13
Room: CNH 104
Time: 3:30-5:20pm
(15:30-17:20)

§ 10.2 cont'd.

Surface AREA.

$x = f(t), \quad y = g(t)$

$c \le t \le d.$

f, g diff'ble

traverse curve once as t increases from c to d.

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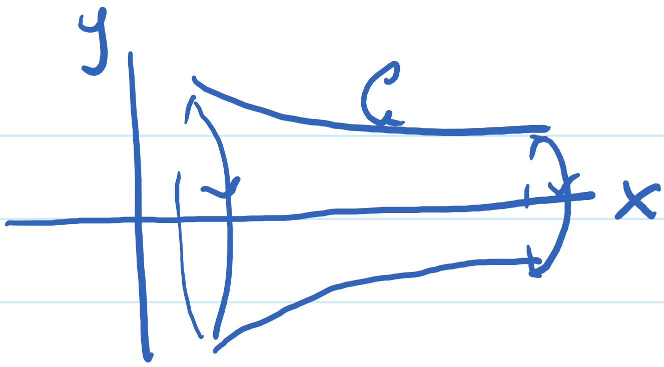
Rotate about x-axis



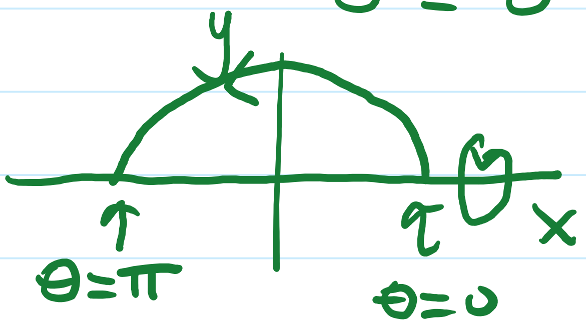
rotate C about x xis

$$\text{Surface Area} = \int_c^d 2\pi \underbrace{g(t)}_{y} \underbrace{\sqrt{(f'(t))^2 + (g'(t))^2}\, dt}_{ds}$$

Example.

$x = r\cos\theta = f(\theta)$, $y = r\sin\theta = g(\theta)$

$0 \le \theta \le \pi$



semi-circle. with radius r.

Find Surface Area of SPHERE.

$$\text{Surf Area} = \int_{\theta=0}^{2\pi} 2\pi \underbrace{(r\sin\theta)}_{y} \sqrt{(-r\sin\theta)^2 + (r\cos\theta)^2}\, d\theta$$

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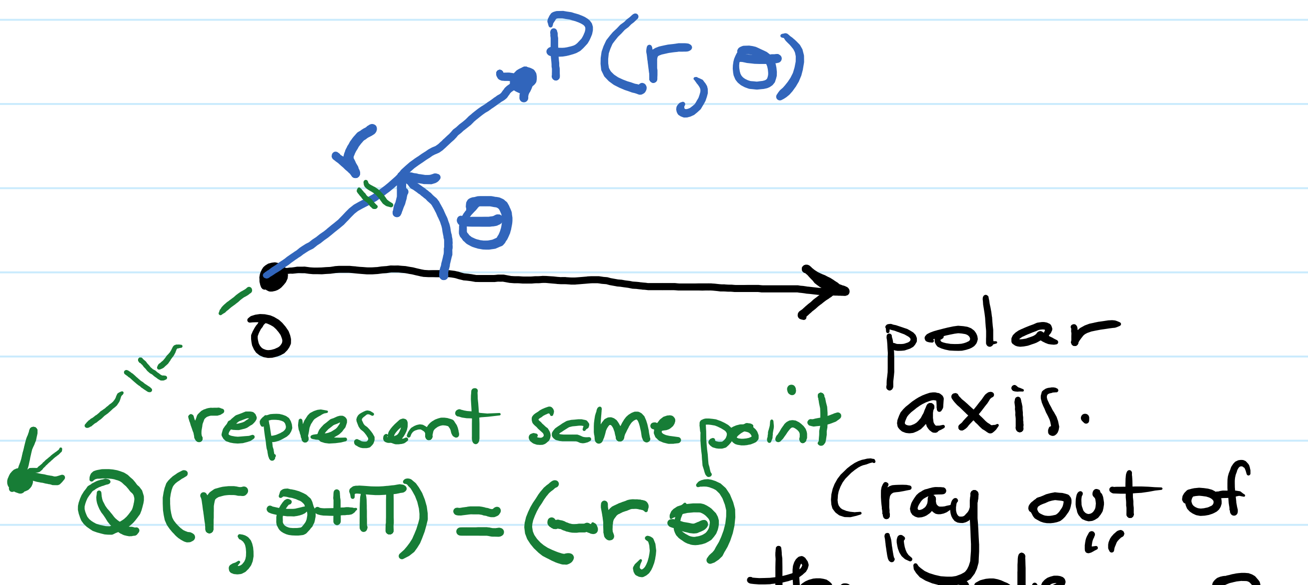
$$= \int_0^{\pi} 2\pi r^2 \sin\theta \, d\theta$$

$$= 2\pi r^2 \left(-\cos\theta \Big|_0^{\pi} \right)$$

$$= -2\pi r^2 (-1-1)$$

$$= \underline{4\pi r^2}$$ surface area of a SPHERE.

§10.3. Polar Coordinates



Measure angles in the counter clockwise direction.

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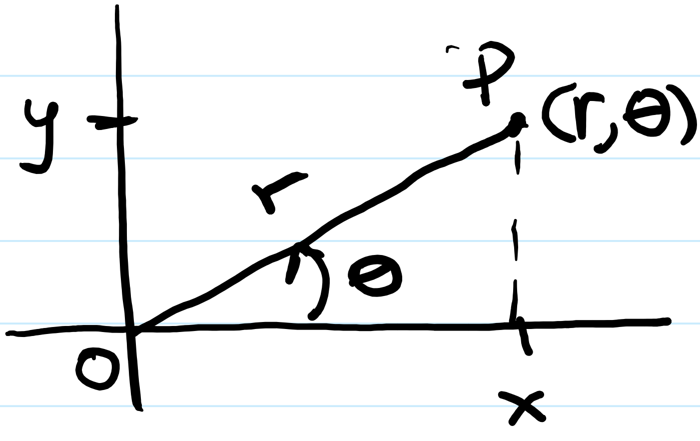
Relationship between Cartesian coordinates and Polar coordinates.

In cartesian coordinates P has coordinates (x, y)



$$\begin{cases} x = r\cos\theta \qquad y = r\sin\theta \\ x^2 + y^2 = r^2 \\ \tan\theta = \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{y}{x} \end{cases}$$

Polar axis = positive x-axis.

Polar curves.

Polar equation: $r = f(\theta)$

The graph is the curve of points with polar coordinates $(f(\theta), \theta)$

$\uparrow$
r

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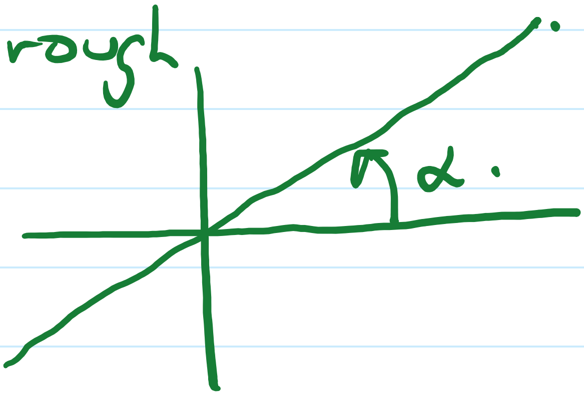
Example:

1. $r = c$ (constant)
   - a circle of radius $|c|$ centred at the pole, O.

2. $\theta = \alpha$ (constant)
   a line through the pole



3. $r = 2\cos\theta$ What curve?
   Convert to Cartesian Coordinates.

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \leftarrow \text{time 2}$$

$$\begin{aligned} 2x &= 2r\cos\theta \\ &= r(2\cos\theta) \\ &= r(r \\ &= r^2 \end{aligned}$$

$$2x = r^2 = x^2 + y^2$$

$$x^2 + y^2 - 2x = 0 \quad \leftarrow \text{(Add \& Subtract 1 from LHS)}$$

$$x^2 - 2x + 1 + y^2 - 1 = 0$$

$$(x-1)^2 + y^2 = 1$$

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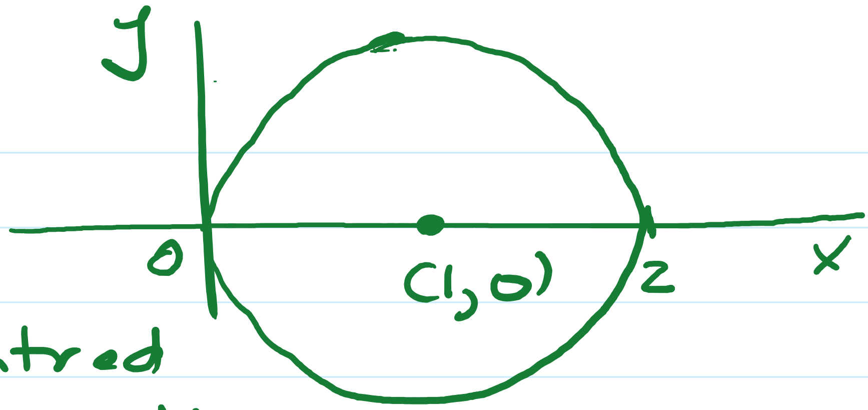
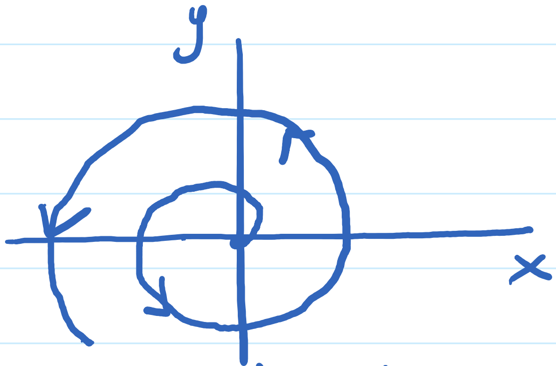


Circle, centred at (1,0) with radius 1.

4. $r = \theta$

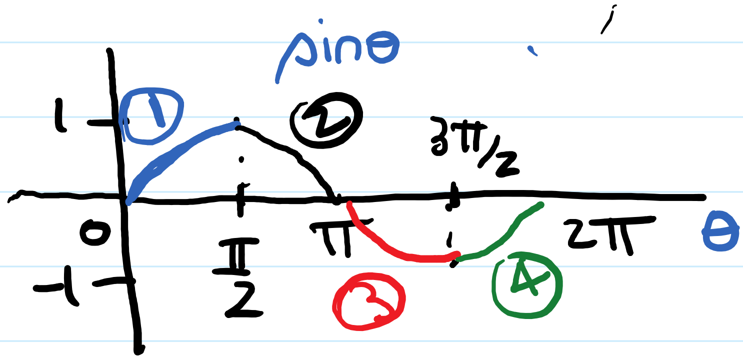


spiral.

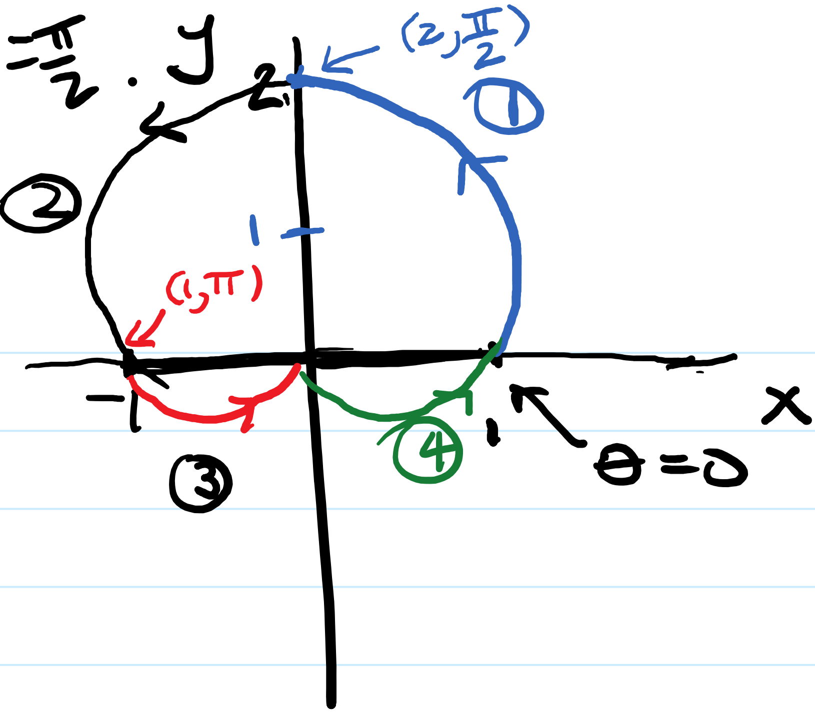
winding in counterclockwise direction moving further from the pol, as θ increases.

5. Cardoid. (heart)

$r = 1 + \sin\theta$

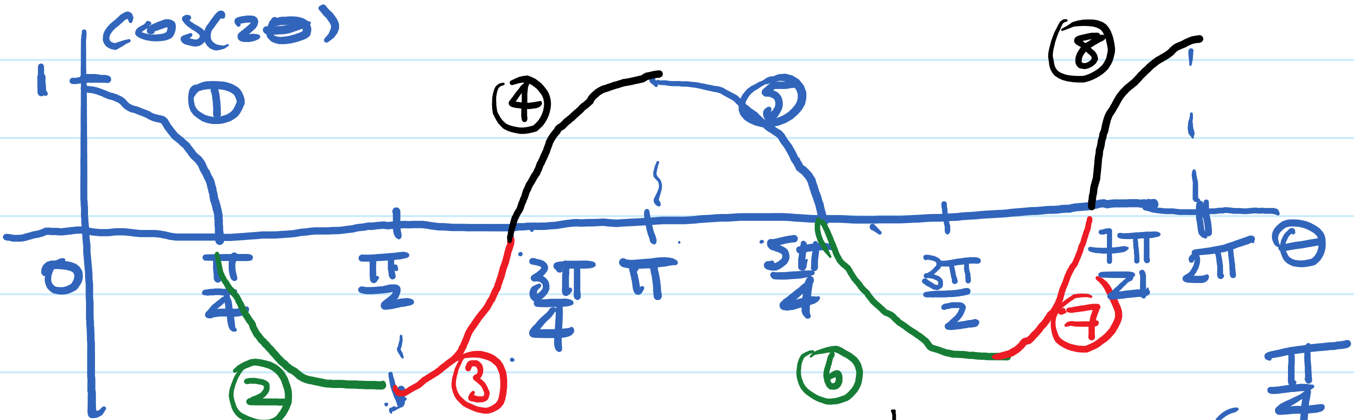


As θ goes from 0 to $\frac{\pi}{2}$
r increase from 1 to 2.



θ goes from $\frac{\pi}{2}$ to π
r decreases from 2 to 1.

Example. 4-leaved rose.
$r = \cos(2\theta)$.



$\theta: 0 \to \frac{\pi}{4}$
r decreases from 1 to 0

$\theta: \frac{\pi}{4} \to \frac{\pi}{2}$
from 0 to -1