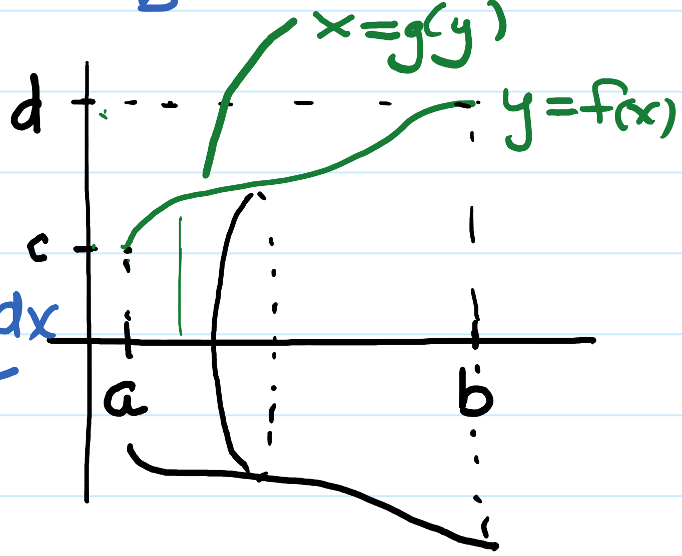
§ 8.2 cont'd.

ROTATE $y=f(x)$ $\boxed{\text{about } x\text{-axis}}$
from a to b



Surface Area of REVOLUTION

(**) $$S = \int_a^b \underbrace{2\pi f(x)}_{\text{circumference}} \underbrace{\sqrt{1+(f'(x))^2}\,dx}_{\text{arc length}}$$

In Leibnitz notation: $y=f(x)$

$$S = \int_a^b 2\pi y \underbrace{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} dx}_{ds}$$

(*) $$= \int_a^b 2\pi y\, ds \quad \text{arc length.}$$

If curve is given as
$x=g(y)$, $c \le y \le d$.
$(f(a)=c,\ f(b)=d)$

If you STILL want to rotate about the x-axis:

$$S = \int_c^d \underbrace{2\pi y}_{\text{Circumference}} \underbrace{\left(1+\left(\frac{dx}{dy}\right)^2\right)^{1/2} dy}_{\substack{ds \\ \text{arc length.}}}$$

To ROTATE $\boxed{\text{about the } y\text{-axis}}$

interchange the roles of x and y in the previous formulas for rotation about x-axis.

$y = f(x) \qquad a \le x \le b.$

$$(*) \quad S = \int_a^b 2\pi x \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx$$

OR $\quad x = g(y) \qquad c \le y \le d$

$$S = \int_c^d 2\pi\, g(y) \left(1+(g'(y))^2\right)^{1/2} dy \quad (**)$$

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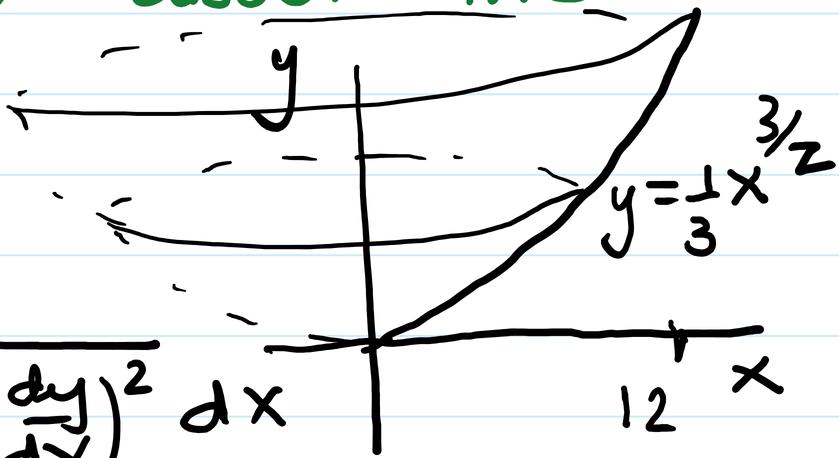
Example.

Find the surface area if

$$y = \frac{1}{3}x^{3/2}, \qquad 0 \le x \le 12$$

is rotated about the y-axis.



$$S = \int_0^{12} 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = \frac{1}{3}x^{3/2} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{3}\right)\left(\frac{3}{2}\right)x^{\frac{1}{2}} = \frac{1}{2}x^{1/2}$$

$$S = \int_0^{12} 2\pi x \sqrt{1 + \frac{1}{4}x}\, dx$$

$$= \int_0^{12} \frac{\cancel{2}\pi x}{\cancel{2}} \sqrt{4 + x}\, dx$$

Integrate by parts

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$u = x \qquad v = \frac{2}{3}(4+x)^{3/2}$

$du = dx \qquad dv = \sqrt{4+x}\,dx$

$= \pi \left(\frac{2}{3} x (4+x)^{3/2} \Big|_0^{12} - \int_0^{12} \frac{2}{3}(4+x)^{3/2}\,dx \right.$

$= \pi \left(\frac{24}{3}(16)^{3/2} - \left[\frac{2}{3} \left(\frac{2}{5}(4+x)^{5/2} \Big|_0^{12} \right] \right) \right.$

$= \pi \left((8)(4)^3 - \frac{4}{15}\left\{ 16^{5/2} - 4^{5/2} \right\} \right.$

$= \pi \left(512 - \frac{4}{15}\left\{ 4^5 - 2^5 \right\} \right. = 3712/15$

§ 8.3 Applications in Physics.

Hydrostatic Pressure.
(force due to water pressure).

<u>SI units</u>

g = gravitational constant
9.8 m/sec^2

ρ = fluid density for water
$= 1000\ kg/m^3 = 1\ g/cm^3$

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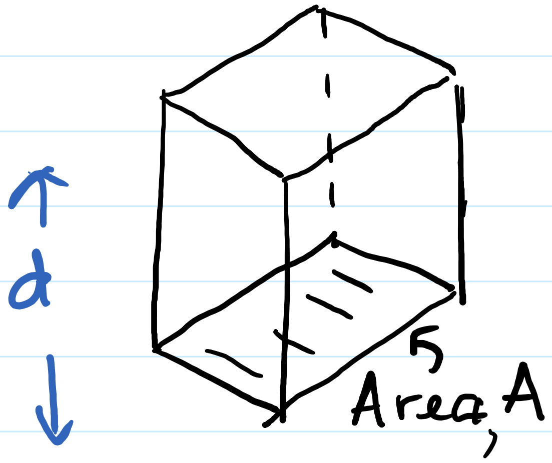

Volume = d A

mass = (density)(volume)
$= \rho\,(dA)$ (kg)

d depth.

Newton's Law

Force = F = mg = $(\rho d A) g$

units Newtons, N
$\frac{kg\ m}{sec^2}$.

Pressure = P = Force per unit Area

$= F/A = \rho d g$

unit Pascals, Pa
$kg/m\,sec^2$.

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Hydrostatic Force

- force exerted by a liquid at rest against a vertical plate or a wall or a dam.

$F = PA$ (pressure)(area).

NOTE: US Customary units. lbs pounds is a force.

$P = \rho g d = \delta d$
where $\delta = \rho g$ weight density

Weight density of water is
$62.5 \text{ lb}/\text{ft}^3$.

NOTE: At any point in a fluid, the pressure is the same in every direction.

Similar Triangles.

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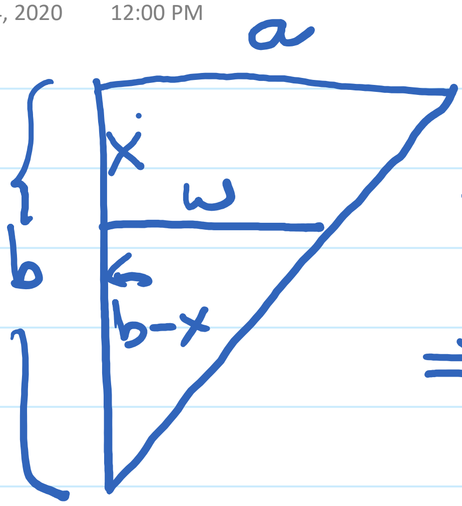
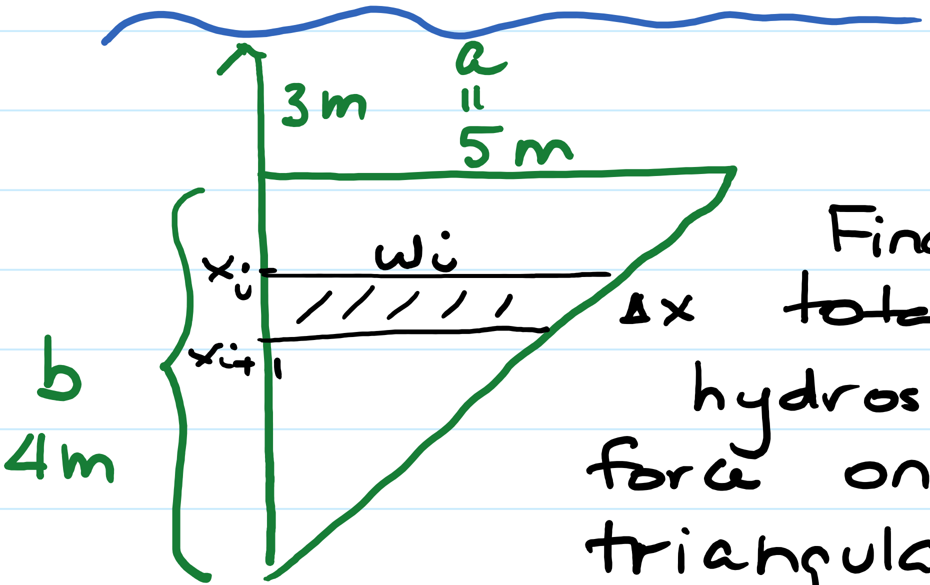


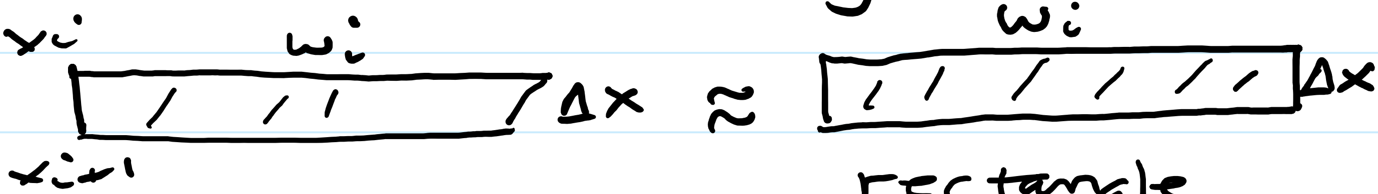
$$\frac{w}{b-x} = \frac{a}{b}$$

$$\Rightarrow w = \frac{a}{b}(b-x)$$

Vertical triangular plate.



Find the total hydrostatic force on the triangular plate.



rectangle approx.

By similar triangles:

$$w_i = \frac{5}{4}(4 - x_i)$$

$$\text{area} \approx w_i \Delta x = \underbrace{\frac{5}{4}(4-x_i)\Delta x}_{a}$$

Hydrostatic Force

$$\approx (\rho g d) A$$

$$= \rho g (3+x_i) \overbrace{\frac{5}{4}(4-x_i)\Delta x}$$

(Pressure)(Area).

Total force obtained by adding the force on each strip. and letting n, number of strips $\to \infty$.

$$\text{Total Force} = \lim_{n\to\infty} \sum_{i=1}^{n} \rho g (3+x_i) \frac{5}{4}(4-x_i)\Delta x$$

limit of a Reimann sum.

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$$\text{Force} = \int_0^4 \rho g \underbrace{(3+x)}_{\substack{\text{depth}\\ m}} \underbrace{\frac{5}{4}(4-x)\,dx}_{\text{Area } m^2}$$

$$\frac{kg\, m}{sec^2}, \quad N.$$

$$= \int_0^4 \rho g \frac{5}{4}(-x^2+x+12)\,dx$$

$\vdots$ HW

$$= \rho g \frac{130}{3}$$

$$= 1000 \frac{kg}{m^3} \quad 9.8 \frac{m}{s^2} \left(\frac{130}{3}\right) m^3$$

$$\simeq 424{,}666.\dot{6} \text{ N}, \text{ i.e. } \frac{kg\, m}{sec^2}$$

$$\approx 425 \text{ kN}$$

kilo Newton

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HW Example 2
pg. 559–560.

Hydrostatic force
on a circular cylinder.