Test 1 Information

Date: Mon. Feb. 10, 2020

Time: 7:00-8:30pm & 9:00-10:30pm

Duration: 1.5hrs (90 min)

Location: find where on childsmath

Request an alternate test (if conflict)

Format: ~19 Multiple Choice

Topics: Anything covered in class up to & including the section on radius and intervals of convergence.

This includes: 8th ed. of Textbook 7.5, 7.8, App. E, 11.1-11.6, 11.8

Look over all lecture notes, sections of the textbook, current assignments, and go over Sample Tests for test #1.

NO CALCULATORS

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Test 1 Reviews Session

Seating #1

Date: Thurs. Feb. 6

Time: 3:30-5:20pm

Room: PGCLL B138

Seating #2

Date: Fri. Feb. 7

Time:3:30-5:20pm

Room: CNH104

TODAY

§11.9 Power Series as FUNCTIONS

§11.9.

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When is $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ meaningful?

Example. Geometric Series.

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ if } |x|<1 \text{ meaningful}$$

if $|x| \geq 1$, series diverges NOT meaningful.

Example: Write $\frac{1}{1+x^2}$ as a p.s.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \frac{1}{1-u} \quad \text{if } u = -x^2$$

$$= \sum_{n=0}^{\infty} u^n, \quad |u|<1$$

$$= \sum_{n=0}^{\infty} (-x^2)^n, \quad |-x^2|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n}, \quad |x|<1$$

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Example: Write $\frac{x^4}{1+x^2}$ as a p.s.?

$$x^4\left(\frac{1}{1+x^2}\right) = x^4 \sum_{n=0}^{\infty} (-1)^n x^{2n}, \quad |x|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n+4}, \quad |x|<1.$$

Polynomials.

$$p(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n$$

n finite.

Can differentiate and integrate term-by-term.

$$p'(x) = c_1 + 2c_2 x + \cdots + n c_n x^{n-1}$$

$$\int p(x)\,dx = K + c_0 x + c_1 \frac{x^2}{2} + \cdots + c_n \frac{x^{n+1}}{n+1}.$$

ROC Radius of Convergence; R

IOC Interval of " .

FACT

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If a p.s. has ROC, $R>0$ OR $R=\infty$ then if

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

then it is differentiable and $\therefore$ continuous and integrable. and one can differentiate and integrate term by term. in its open I.O.C.

i) $$f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$$

$$= c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \dots +$$

ii) $$\int f(x)\,dx = K + \sum_{n=0}^{\infty} \frac{c_n}{n+1} (x-a)^{n+1}$$

K arb constant.

$$f(x) = K + c_0(x-a) + \frac{c_1}{2}(x-a)^2 + \frac{c_2}{3}(x-a)^3 + \dots$$

NOTE The ROC of (i) and (ii) is the same as the original p.s., but the I.O.C. could be different at the end points.

Example.

Find the p.s. for $\frac{1}{(1-x)^2}$, $|x|<1$.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x|<1.$$

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^n = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}(1-x)^{-1}$$

$$= (-1)(1-x)^{-2}(-1) = \frac{1}{(1-x)^2}$$

$$\therefore \frac{1}{(1-x)^2} = \frac{d}{dx}\sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=0}^{\infty} \frac{d}{dx} x^n \quad \text{(Can diff. term by term)}$$

$$= \sum_{n=0}^{\infty} n x^{n-1}$$

$$= \sum_{n=1}^{\infty} n x^{n-1} \quad \text{(Since first term} = 0\text{)}$$

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Check endpoints

I.O.C is $(-1, 1)$.

Example: Find p.s. for $\ln(1-x)$ if $|x|<1$.

$$\int \frac{1}{1-x}\,dx = -\ln(1-x), \quad |x|<1.$$

$$-\ln(1-x) = \int \frac{1}{1-x}\,dx$$

$$= \int \sum_{n=0}^{\infty} x^n \,dx$$

$$= \sum_{n=0}^{\infty} \int x^n \,dx \quad \text{(Integrate term by term)}$$

$$= \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} + K$$

to be determined.

$$-\ln(1-x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} + K$$

Find K: let $x=0$.

$-\ln(1-0) = 0$

$\therefore$ RHS must be 0. $\Rightarrow K=0$.

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$$\therefore \; -\ln(1-x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

$$\therefore \; \ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}.$$

$|x|<1$.

OR.

$m = n+1$, $n = m-1$

$$\ln(1-x) = -\sum_{m=1}^{\infty} \frac{x^m}{m}$$

$|x|<1$.

At the endpoints

$\uparrow$ ROC.

$$x=1 \rightarrow -\sum_{m=1}^{\infty} \frac{1}{m}$$

Diverges.

$$x=-1 \rightarrow -\sum_{m=1}^{\infty} \frac{(-1)^m}{m}$$

Converges by (AST)

$\therefore R=1$

IOC $= [-1, 1)$.

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HW Example 7

pg. 756. of text book.

$$\tan^{-1}(x) = \int \frac{1}{1+x^2}\,dx.$$

Example. $\int e^{-x^2}\,dx$ non-elementary

FACT

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad R=\infty$$

Let $u = -x^2$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

$$\int e^{-x^2}\,dx = \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}\,dx$$

$$= C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)\,n!}, \quad R=\infty.$$

Important $\int$ in probability theory.