Test 1 Information

Date: Mon. Feb. 10, 2020

Time: 7:00-8:30pm & 9:00-10:30pm

Duration: 1.5hrs (90 min)

Location: find where on childsmath

Request an alternate test (if conflict)

Format: ~19 Multiple Choice

Topics: Anything covered in class up to & including the section on radius and intervals of convergence.

This includes: 8th ed. of Textbook 7.5, 7.8, App. E, 11.1-11.6, 11.8

Look over all lecture notes, sections of the textbook, current assignments, and go over Sample Tests for test #1.

NO CALCULATORS

Test 1 Reviews Session

<u>Seating #1</u>

Date: Thurs. Feb. 6

Time: 3:30-5:20pm

Room: PGCLL B138

<u>Seating #2</u>

Date: Fri. Feb. 7

Time:3:30-5:20pm

Room: CNH104

§11.8 cont'd.

February 3, 2020 3:23 PM

Let R denote Radius of Convergence.

Example. Find R for $\sum_{n=0}^{\infty} n!\,x^n$.

Use RATIO TEST:

$$\lim_{n\to\infty} \left| \frac{(n+1)!\,x^{n+1}}{n!\;x^n} \right|$$

$$= \lim_{n\to\infty} |n+1|\,|x| = \infty, \quad x \neq 0.$$

$\therefore R = 0$, since series ONLY converges at $x = 0$.

useless series.

Example. $S = \sum_{n=0} \frac{x^n}{n!}$. Find R.

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \Big/ \frac{x^n}{n!} \right| \quad \text{(Ratio Test)}$$

$$= \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \frac{x^{n+1}}{x^n} \right| = \lim_{n\to\infty} \left| \frac{1}{n+1} \right| |x|$$

$= 0$ for all x.

$\therefore R = \infty$, i.e. series converges all x, i.e. $x \in (-\infty, \infty)$, i.e., $-\infty < x < \infty$.

February 4, 2020 11:37 AM

Remark: We will see that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x \in \mathbb{R}.$$

<u>Example</u>. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$. Find R & interval of convergence

Series is centred at $\underline{x=1}$. $(c_0 = 0)$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)\,2^{n+1}} \Big/ \frac{(x-1)^n}{n\,2^n} \right|$$

$$= \lim_{n\to\infty} \left| \left(\frac{2^n}{2^{n+1}}\right)\left(\frac{n}{n+1}\right) \frac{(x-1)^{n+1}}{(x-1)^n} \right|$$

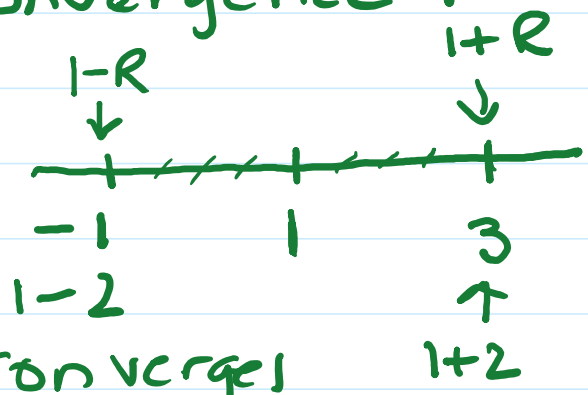
$$= \lim_{n\to\infty} \left| \frac{1}{2} \frac{n}{n+1} (x-1) \right| = \frac{1}{2}|x-1| < 1 \text{ for convergence}$$

$$\frac{1}{2}|x-1| < 1 \Rightarrow |x-1| < 2 = R.$$

February 4, 2020 11:45 AM

Interval of Convergence.

$R = 2$:



$\therefore$ the series converges at least for $-1 < x < 3$

i.e., $(-1, 3)$.

MUST TEST the END POINTS separately, and DO NOT use the RATIO or Root test on the END POINTS.

if $x = 3$, series is $\sum_{n=1}^{\infty} \frac{2^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

Diverges. (harmonic series)

if $x = -1$, series is $\sum_{n=1}^{\infty} \frac{(-2)^n}{n 2^n}$

$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, converges by AST (alternating harmonic series)

$\therefore$ INTERVAL of convergence is $[-1, 3)$ OR $-1 \le x < 3$

February 4, 2020 11:58 AM

Example. Find R and the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}} . \quad (*)$$

$$= \sum_{n=1}^{\infty} \frac{\left(2(x-\frac{1}{2})\right)^n}{5^n\sqrt{n}}$$

$$= \sum_{n=1}^{\infty} \frac{2^n (x-\frac{1}{2})^n}{5^n\sqrt{n}}$$

Series is centred at $x = \frac{1}{2}$.

Ratio Test on $(*)$

$$\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}} \Big/ \frac{(2x-1)^n}{5^n\sqrt{n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\sqrt{n}}{\sqrt{n+1}} \left(\frac{1}{5} \right| |2x-1| \right.$$

for convergence by RATIO TEST.

$$= \frac{1}{5} |2x-1| < 1 \Rightarrow |2x-1| < 5$$

$$\Rightarrow -5 < 2x-1 < 5$$

February 4, 2020 12:05 PM

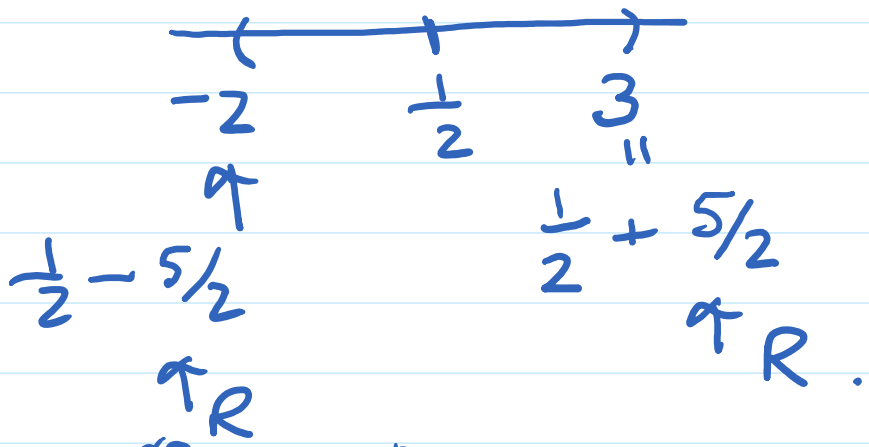
$$\left.\begin{aligned} -5 &< 2x-1 < 5 \\ \Rightarrow -4 &< 2x < 6 \\ \Rightarrow -2 &< x < 3 \end{aligned}\right\}$$

$R = \frac{1}{2}$ length of interval of convergence.

$R = (3-(-2))/2 = 5/2$

$\therefore$ Interval of convergence contains $(-2, 3)$

BUT MUST CHECK END POINTS.



At $x = -2$, $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n}{5^n \sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$

converges by AST.

$x = 3$, $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{5^n \sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Divergent p-series $(p = 1/2 < 1)$.

$\therefore R = 5/2$ and Interval of convergence is $\underline{[-2, 3)}$.

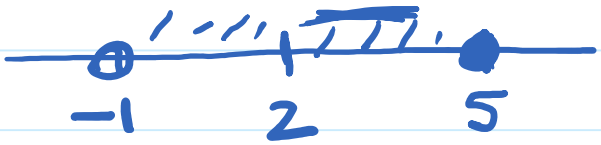
February 4, 2020 12:11 PM

Example.

Assume the series $\sum_{n=0}^{\infty} c_n (x-2)^n$ converges at $\underline{x=5}$.

1. Does it converge at $x=0$?

Centred at $x=2$.



$\therefore R \geq 3$.

2. Does series converge at $x=-1$?
   - need more information.

$x=-1$ <u>might</u> be an end point of the interval of convergence.

3. Does series converge at $x=-2$?
   - don't know.

the radius of convergence could be bigger than 3.