HELP

MONDAY-THURSDAY			HELP Centre HH/104	2:30-8:30
FRIDAY			HELP Centre HH/104	2:30-6:30
MONDAY	TA	Uyen Le	HH/104 (Help Centre)	2:30-5:30
TUESDAY	Instructor	McLean	BSB/B124	10:00-11:30
	Instructor	Childs	HH/213	1:30-2:20
	Instructor	Yang	HH/417	3:30-5:30
WEDNESDAY	Instructor	McLean	BSB/124	10:00-11:30
	Instructor	Wolkowicz	HH/318	11:00-12:00
	TA	Lee van Brussell	HH/104 (Help Centre)	2:30-5:30
	Instructor	Lane	HH/407	4:00-6:00
THURSDAY	Instructor	Childs	HH/213	1:30-2:20
FRIDAY	Instructor	McLean	BSB/B124	10:00-11:30
	Instructor	Childs	HH/213	1:00-2:20
	Instructor	Wolkowicz	HH/318	2:00-3:00

Last lecture:

Def'n Derivative of f at a

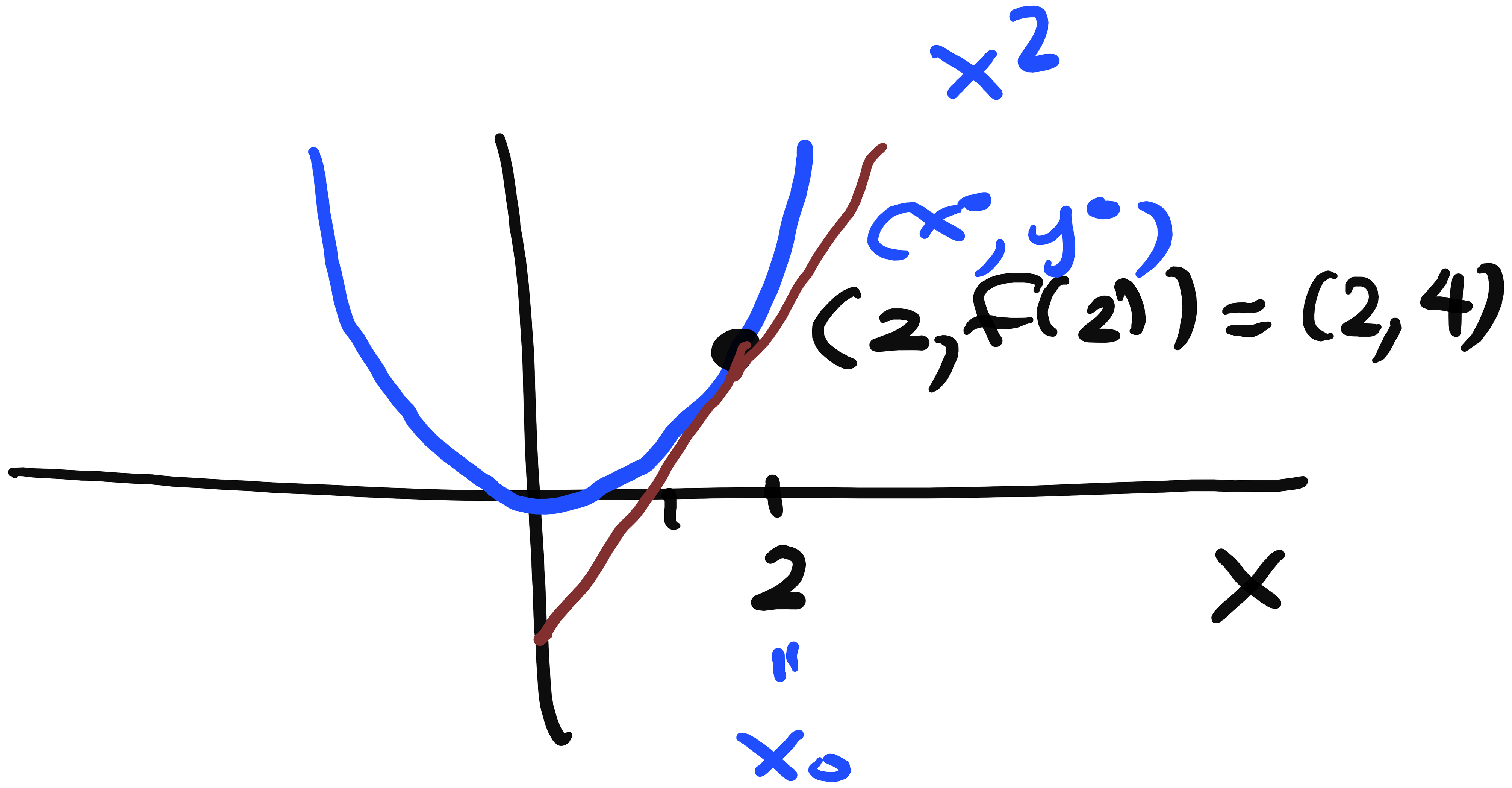
$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

OR letting $x = a+h \Rightarrow h = x-a$
and $h \to 0$ iff $x \to a$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

this is the slope of the TANGENT LINE to f at a.

Example: Find an equation of the tangent line to $y=x^2$ at point $(2,4)$.



Sol'n Point slope formula of a line:

$$y - y_0 = m(x-x_0)$$

m ↑ slope

(x_0, y_0) point

To find the slope at $a=2$ of $f(x)=x^2$.

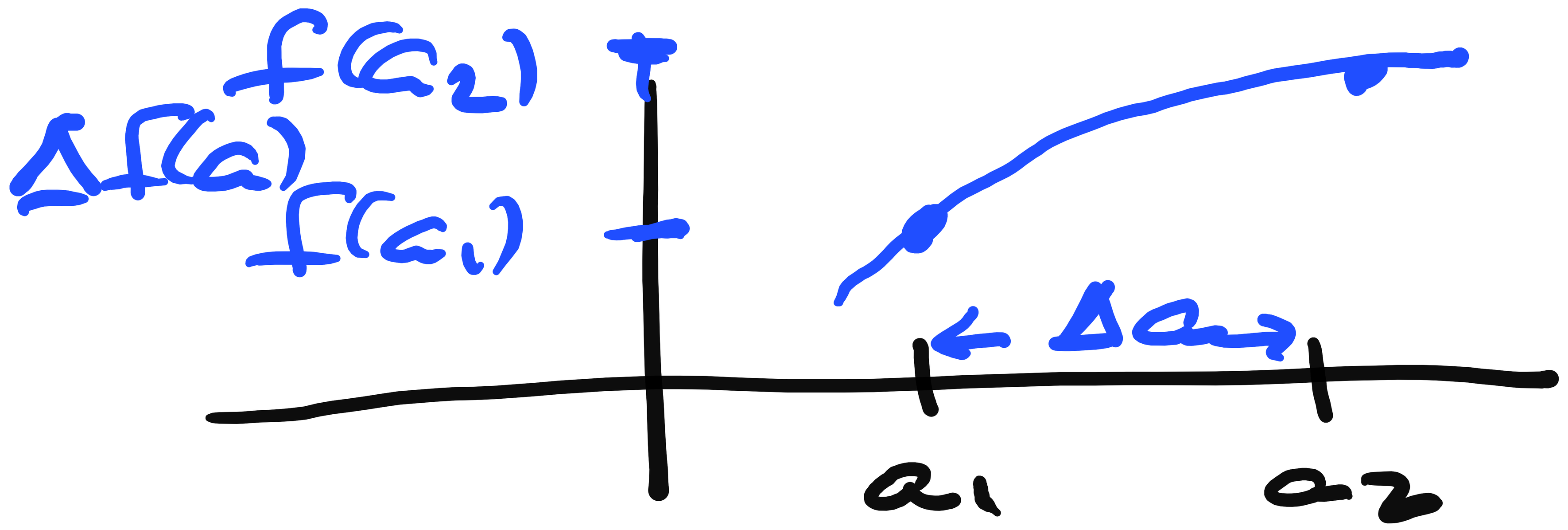
$$\text{slope } m = \lim_{x \to 2} \frac{f(x) - f(\overset{a}{2})}{x - \underset{a}{2}}$$

$$= \lim_{x \to 2} \frac{x^2 - 2^2}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

The tangent line is:

$$y - \underset{y_0}{4} = \underset{m}{4}(x - \underset{x_0}{2}), \quad \Rightarrow \quad y = 4x - 4.$$

Interpretation of the derivative of f at a as "an instantaneous rate of change of f at a". ($f'(a)$).



$\Delta a = a_2 - a_1$ increment of a

$\Delta f(a) = f(a_2) - f(a_1)$

average rate of change of f wrt x

on $[a_1, a_2]$

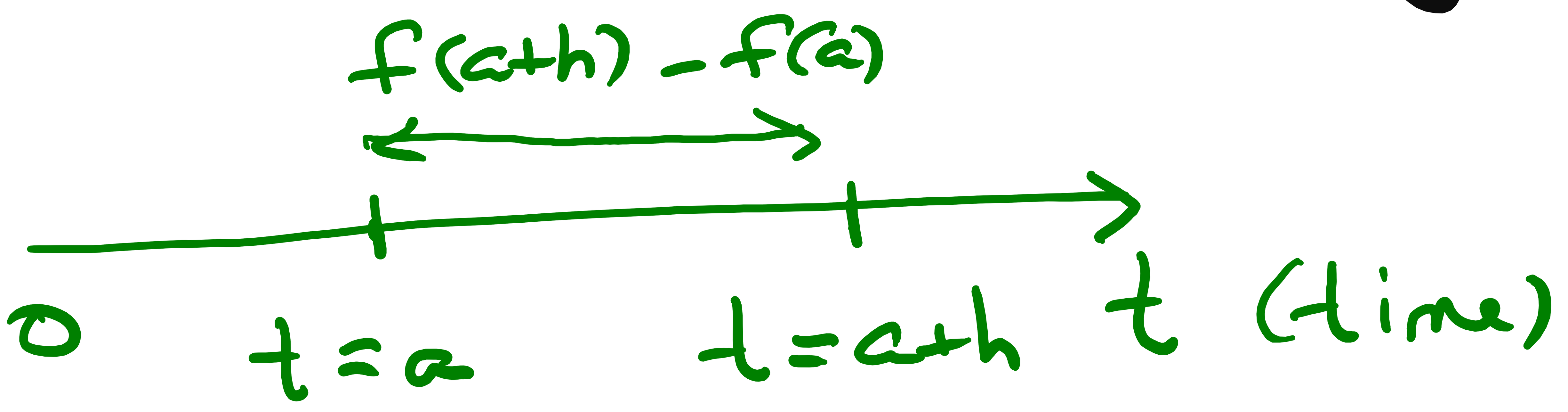
= difference quotient.

$$= \frac{\Delta f(a)}{\Delta a} = \frac{f(a_2) - f(a_1)}{a_2 - a_1}$$

instantaneous rate of change

$$\lim_{a_2 \to a_1} \frac{\Delta f(a)}{\Delta a} = \lim_{a_2 \to a_1} \frac{f(a_2) - f(a_1)}{a_2 - a_1} = f'(a_1)$$

Another interpretation of $f'(a)$ is of the "instantaneous velocity of $f(t)$ at a."



$f(a+h) - f(a)$

= displacement

or change of position

from time $t = a$ to time $t = a+h$.

average velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{f(a+h) - f(a)}{h}$

h

$a+h - a$

slope of SECANT LINE.

let $h \to 0$ to obtain

instantaneous velocity of f at a

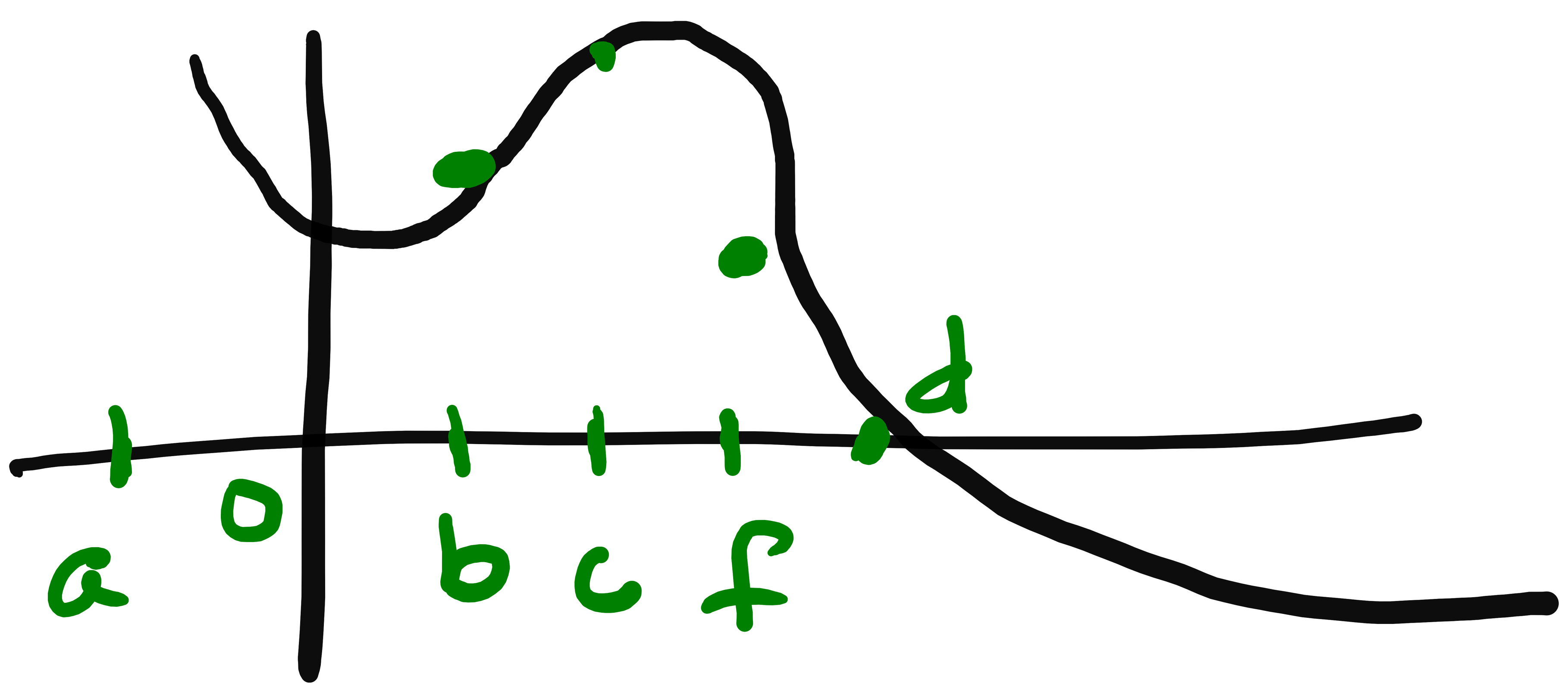
$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a).$$

§ 2.8 The Derivative as a Function.

Define the function $f'(x)$ as

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NOTE: The fcn (function) $f'(x)$ has a domain & range.



The function $f(x)$ is:

strictly decreasing on $[a, 0)$

$\therefore$ $f'(x) < 0$ $x \in [a, 0)$

$f'(0) = 0$

strictly increasing $(0, b)$

$f'(x) > 0$ $x \in (0, c)$

$f'(c) = 0$

strictly decreasing $(c, d]$

$f'(x) < 0 \quad (c, d]$

What about the function $f'(x)$?

$$f''(x) = f'(f'(x))$$ the 2^{nd} derivative.

$f''(x) > 0 \quad [a, b)$, $f''(b) = 0$ inflection point

$f''(x) < 0 \quad (b, f)$, $f''(f) = 0$ " "

$f''(x) > 0 \quad x \in (f, d]$

<u>Notation</u> $\quad y = f(x)$

$$f'(x) = \frac{dy}{dx} = y' = \frac{df}{dx} = \frac{d}{dx} f(x) = D f(x) = D_x f(x)$$

2nd derivative.

Liebniz notation

$$(f'(x))' = f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2 y}{dx^2}$$

3^{rd} derivative

$$\left(f''(x)\right)' = f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

$\vdots$

n^{th} derivative

$$f^{(n)}(x) = \frac{d^ny}{dx^n}$$

Find $f'(x)$ directly from the def'n

for $f(x) = \sqrt{8-x}$.

Sol'n

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{8-(x+h)} - \sqrt{8-x}}{h}\right) \cdot \left(\frac{\sqrt{8-(x+h)} + \sqrt{8-x}}{\sqrt{8-(x+h)} + \sqrt{8-x}}\right)$$

$\uparrow = 1$

$$= \lim_{h\to 0} \frac{8-(x+h)-(8-x)}{h\left(\sqrt{8-(x+h)}+\sqrt{8-x}\right)}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\left(\sqrt{8-(x+h)}+\sqrt{8-x}\right)}$$

$$= \frac{-1}{2\sqrt{8-x}}.$$

Defn A function f is differentiable at a if $f'(a)$ exists.

It is differentiable on a <u>OPEN</u> interval (i.e. (a,b) (or (a,∞) or $(-\infty,a)$ or $(-\infty,\infty)$) if it is differentiable at every point in the interval.

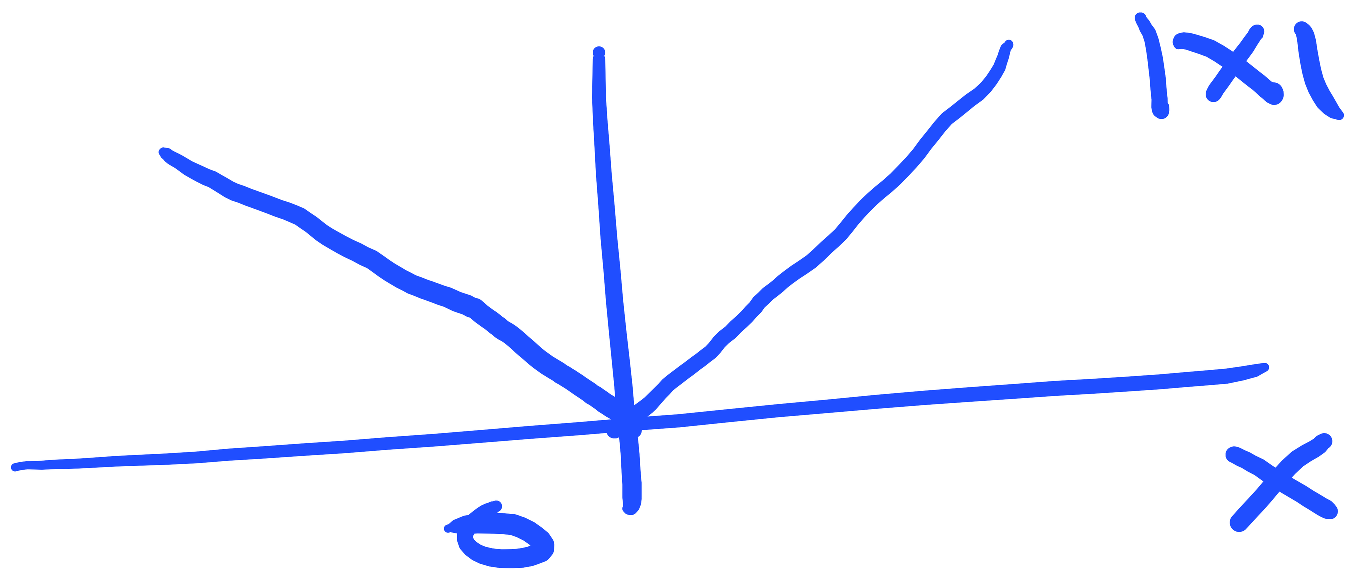
thm. If f is differentiable at a
then f is continuous at a.

BEWARE: f continuous at a

diff'ble

$\not\Rightarrow$ f is differentiable at a.

e.g. $f(x) = |x|$



continuous at $x=0$

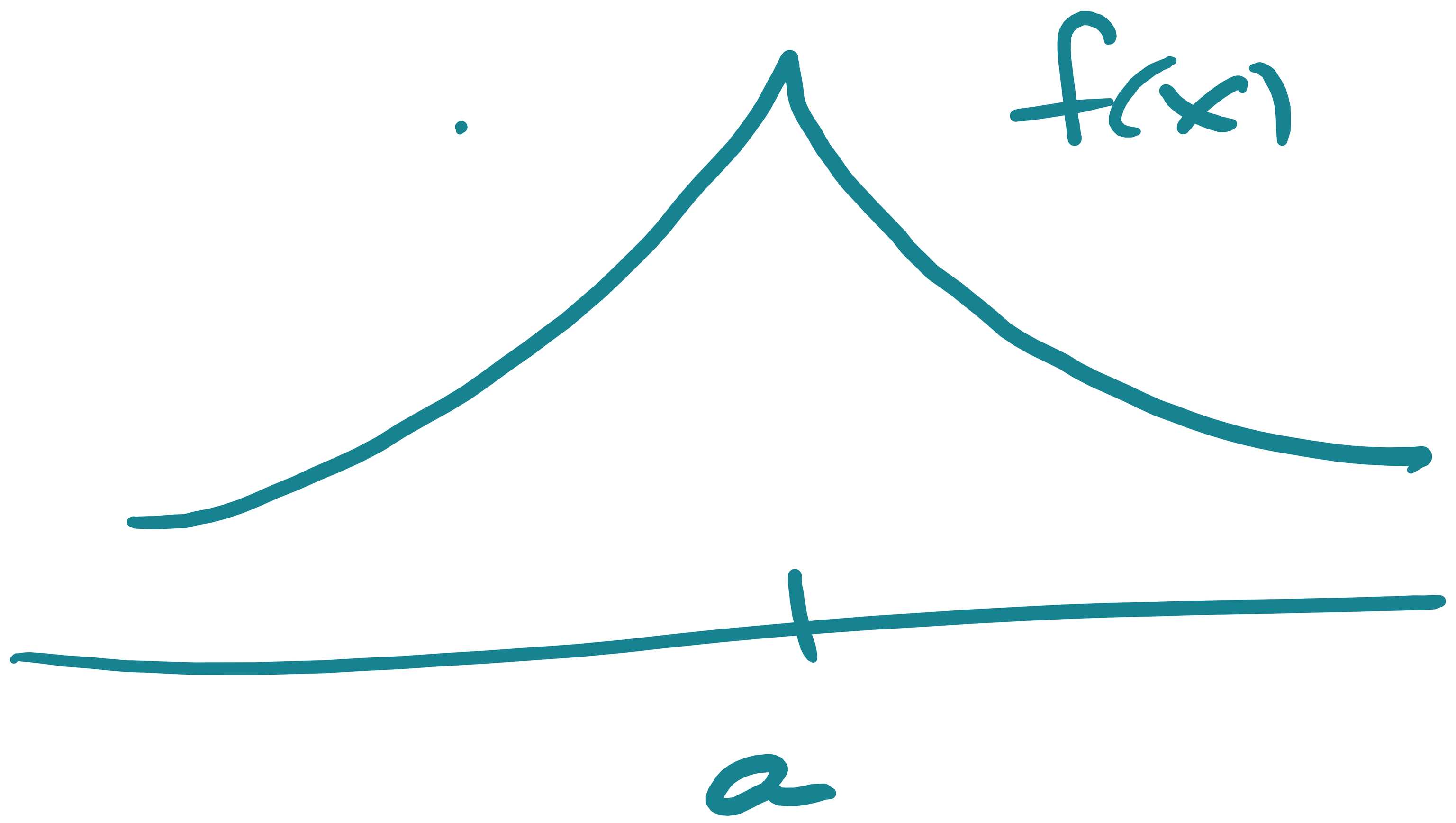
NOT diff'ble at $x=0$.

Example: $f(x) = \sqrt{8-x}$ domain $(-\infty, 8]$

$f'(x) = -\dfrac{1}{2\sqrt{8-x}}$ domain $(-\infty, 8)$

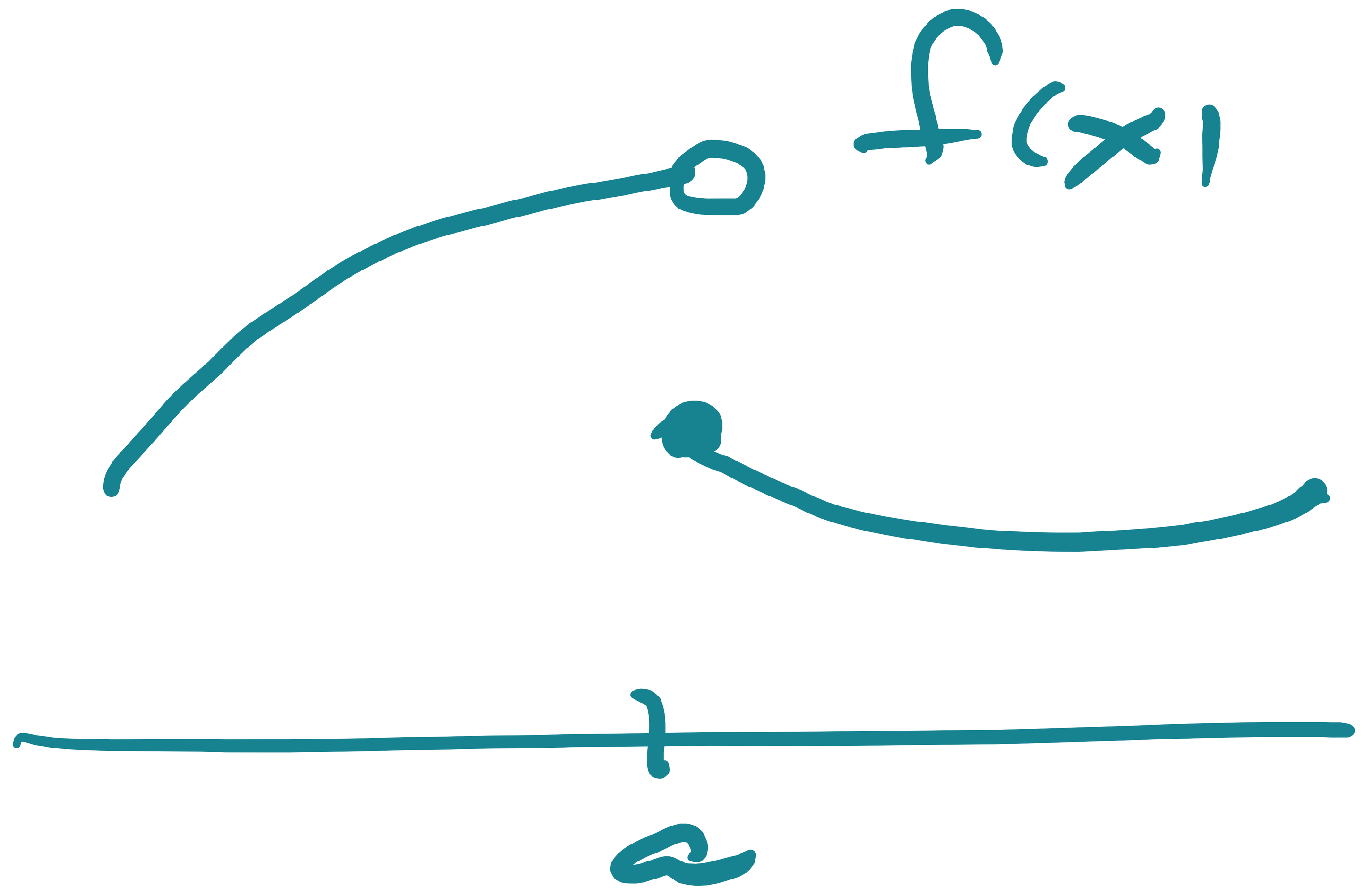
Domain of $f'(x)$ can be smaller than or equal to the domain of $f(x)$.

How can functions fail to be diff'ble at a:



corner

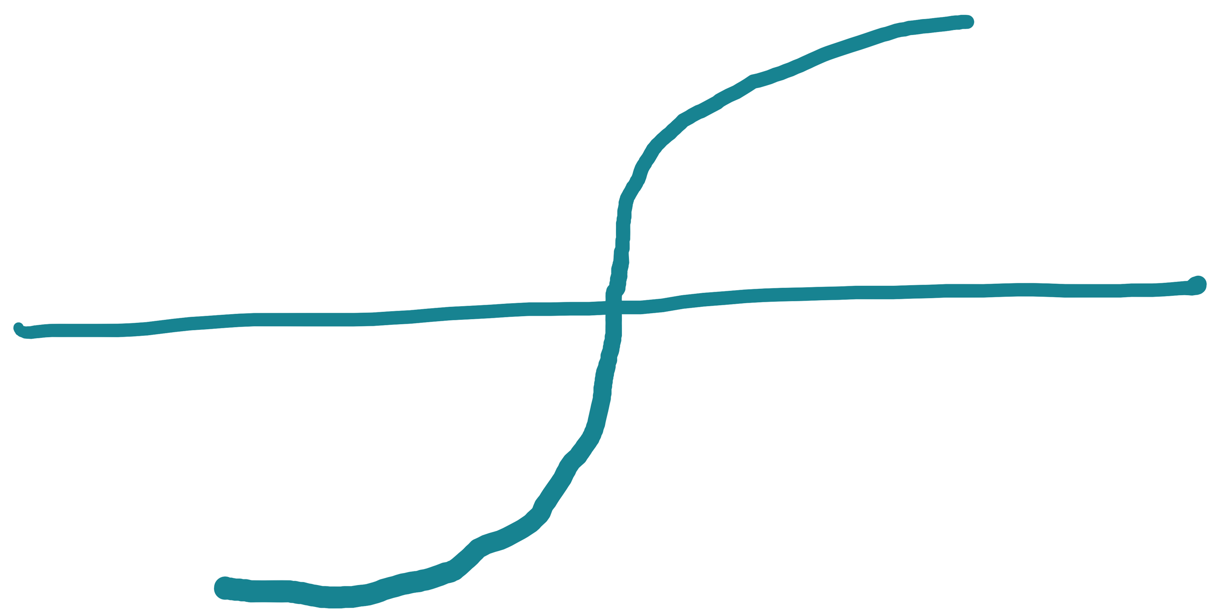
$f'(a)$ not. diff'ble at a



discontinuity of f

$f'(a)$ not diff'ble at a

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\sqrt{-x} & x < 0 \end{cases}$$



vertical tangent at a.

$\lim_{x \to a} f'(a) = \infty$.

§3.1 Derivatives of Polynomials & Exponentials.

From the def'n:

1. $\frac{d}{dx} c = 0$ if c is constant.

2. $\frac{d}{dx} x^n = n x^{n-1}$ any $n \in \mathbb{R}$, power rule

3. $\frac{d}{dx}\left(c f(x)\right) = c \frac{df(x)}{dx} = c \frac{d}{dx} f(x)$

   Constant multiple rule

4. $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{df(x)}{dx} \pm \frac{dg(x)}{dx}$.

   sum / difference rule.