Math 1A03 Calculus 1   Section C01   Dr. Wolkowicz

Lecture 5   m1A03
Sept. 11, 2019.

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Course website: www.childsmath.ca/childsa/forms/main_login.php

My lecture notes will be posted after classes. (Volunteer Notetakers wanted)

There are links to my notes from my website, as well as from the course website -> Course Information (on the left see Dr. Wolkowicz's Notes)

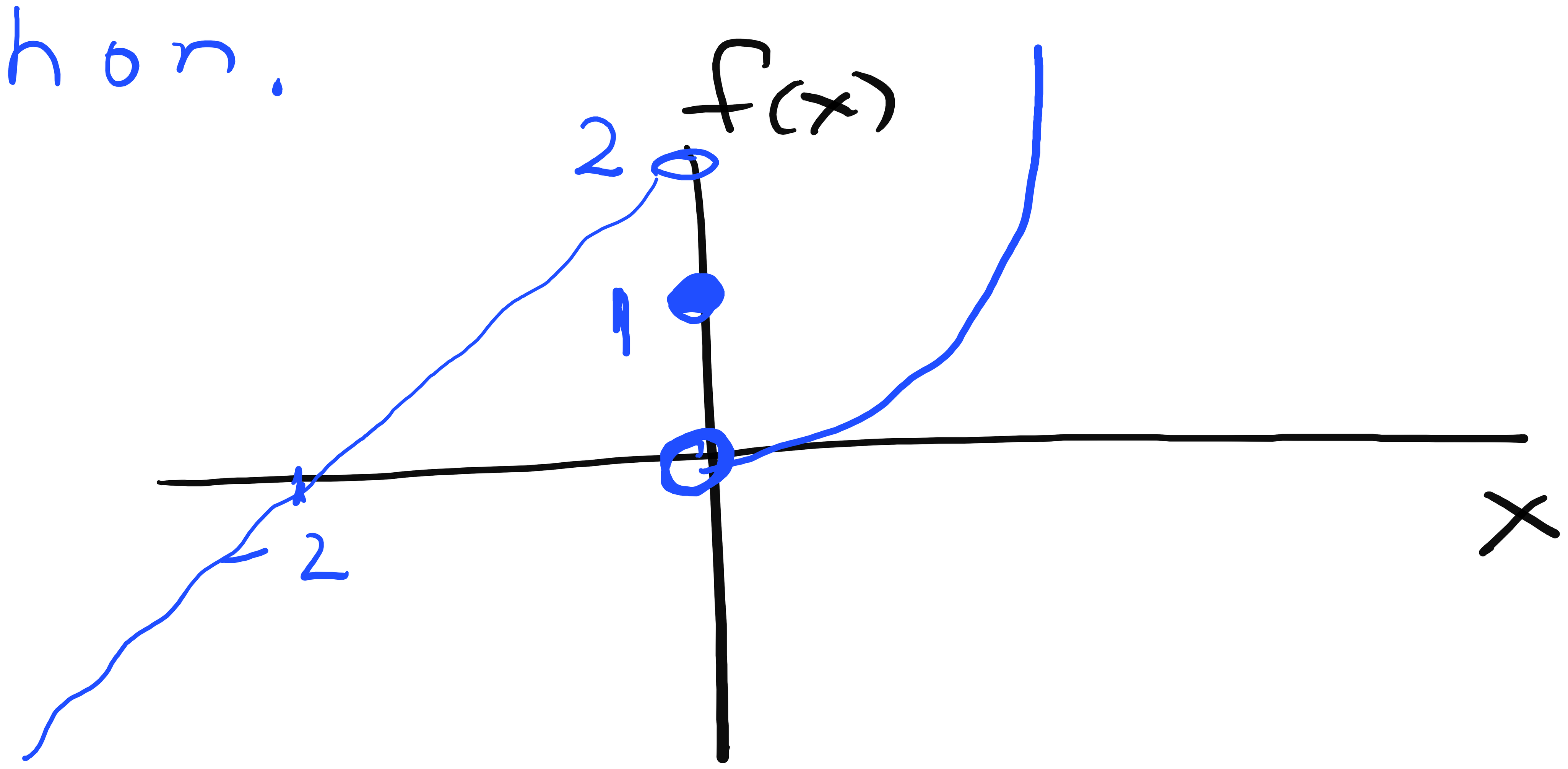
There is also lots of important information, including links to Announcements, Important Dates, Lecture Schedule, MSAF FAQ, Suggested Problems etc.

TUTORIALS Start week of Monday, Sept. 9.
Help Centre HH/104 starts Wednesday, Sept. 11.

§ 2.1 – 2.3 (from High School). read.

2.4 omitted.

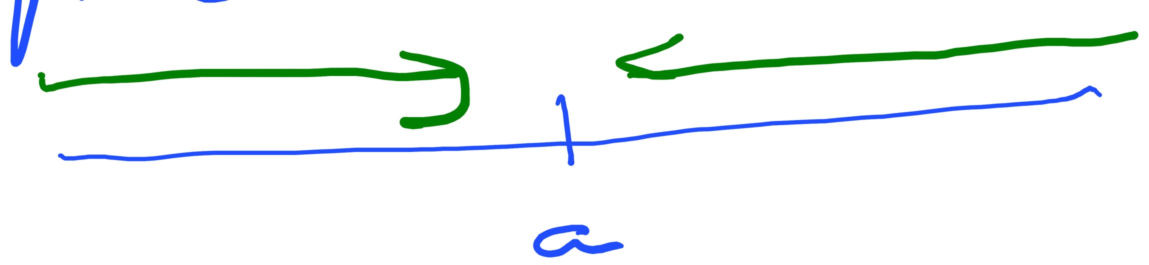
§ 2.5. Piecewise defined function.



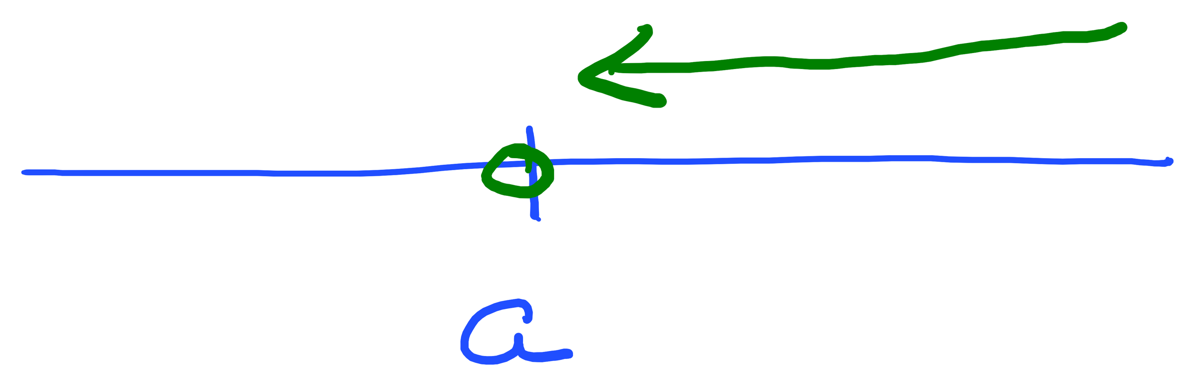
Ex. 1

$$f(x) = \begin{cases} x^2 & x > 0 \\ 1 & x = 0 \\ x+2 & x < 0 \end{cases}$$

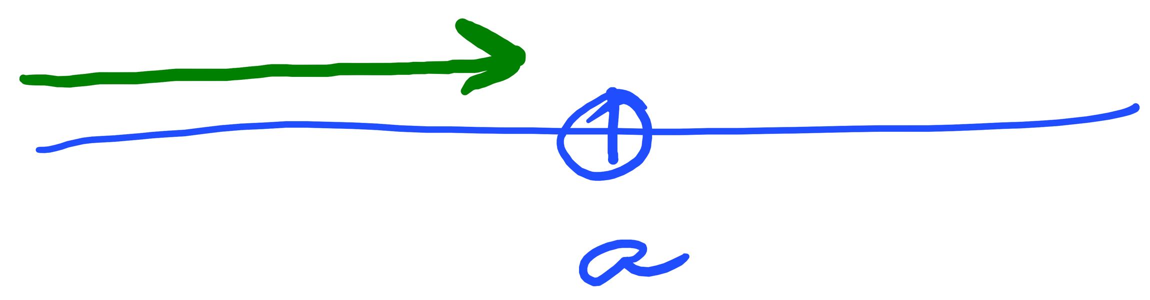
$\lim\limits_{x\to a} f(x) = L$ "limit of $f(x)$ as x approaches a equals L."



$\lim\limits_{x\to a^+} f(x) = L$ limit of $f(x)$ as x approaches a from the right



$\lim\limits_{x\to a^-} f(x) = L$ limit of $f(x)$ as x approaches a from the left



In our example.

$\lim\limits_{x\to 0^-} f(x) = 2$; $\lim\limits_{x\to 0^+} f(x) = 0$ $\lim\limits_{x\to 0} f(x)$ DNE does not exist.

Def'n A function f is CONTINUOUS at a if $\lim_{x \to a} f(x) = f(a)$.

This means

(1) $f(a)$ is defined. (i.e., a is in the domain of f)

(2) $\lim_{x \to a} f(x)$ exists (finite)

(3) $\lim_{x \to a} f(x) = f(a)$, the value of f at a.

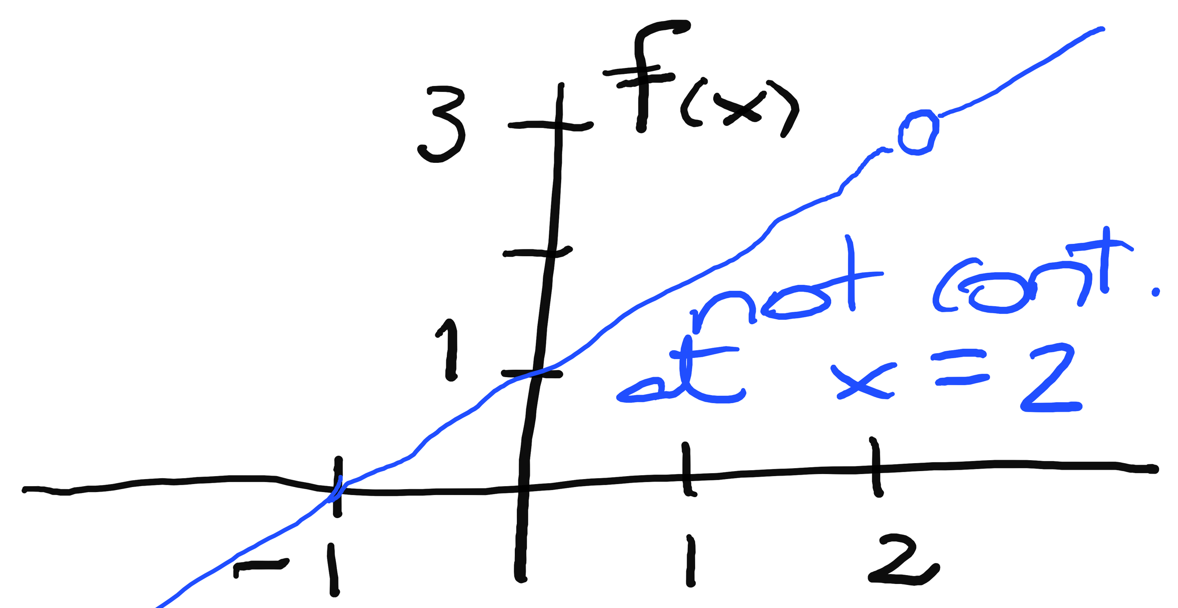
The above example of $f(x)$ is continuous at all points in $\mathbb{R}$ except at $x = 0$.

At $x=0$ there is a JUMP discontinuity.

i.e. the value of the function jumps to a finite value.

Example 2 $f(x) = \dfrac{x^2 - x - 2}{x-2}$

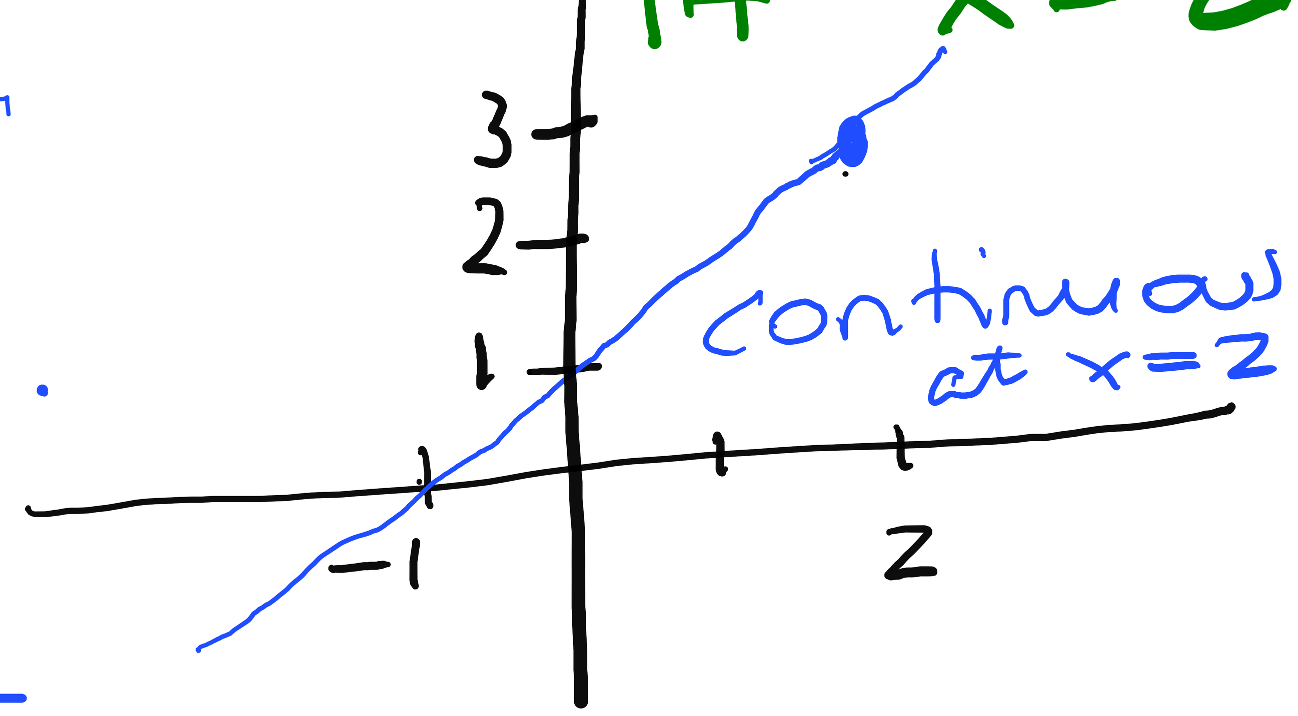
If f continuous at $x = 2$?



Sol'n: No. since f is not defined at $x=2$.

Modified Example: 3 $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x-2} & \text{if } x \neq 2 \\ 3. & \text{if } x = 2. \end{cases}$

If f continuous at $x=2$.



Sol'n. f is defined at $x=2$ ✓.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^2-x-2}{x-2}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = 2+1 = 3,$$

$\therefore$ the limit exists. (finite) ✓

$f(2) = 3 = \lim_{x\to 2} f(x)$ ✓ $\therefore$ this function IS continuous at $x=2$.

$x=2$ is called a REMOVEABLE DISCONTINUITY of Example 2.

i.e. we can redefine the function at one point and make it continuous.

Example. 4 $\quad f(x) = \dfrac{\sin x}{x}$

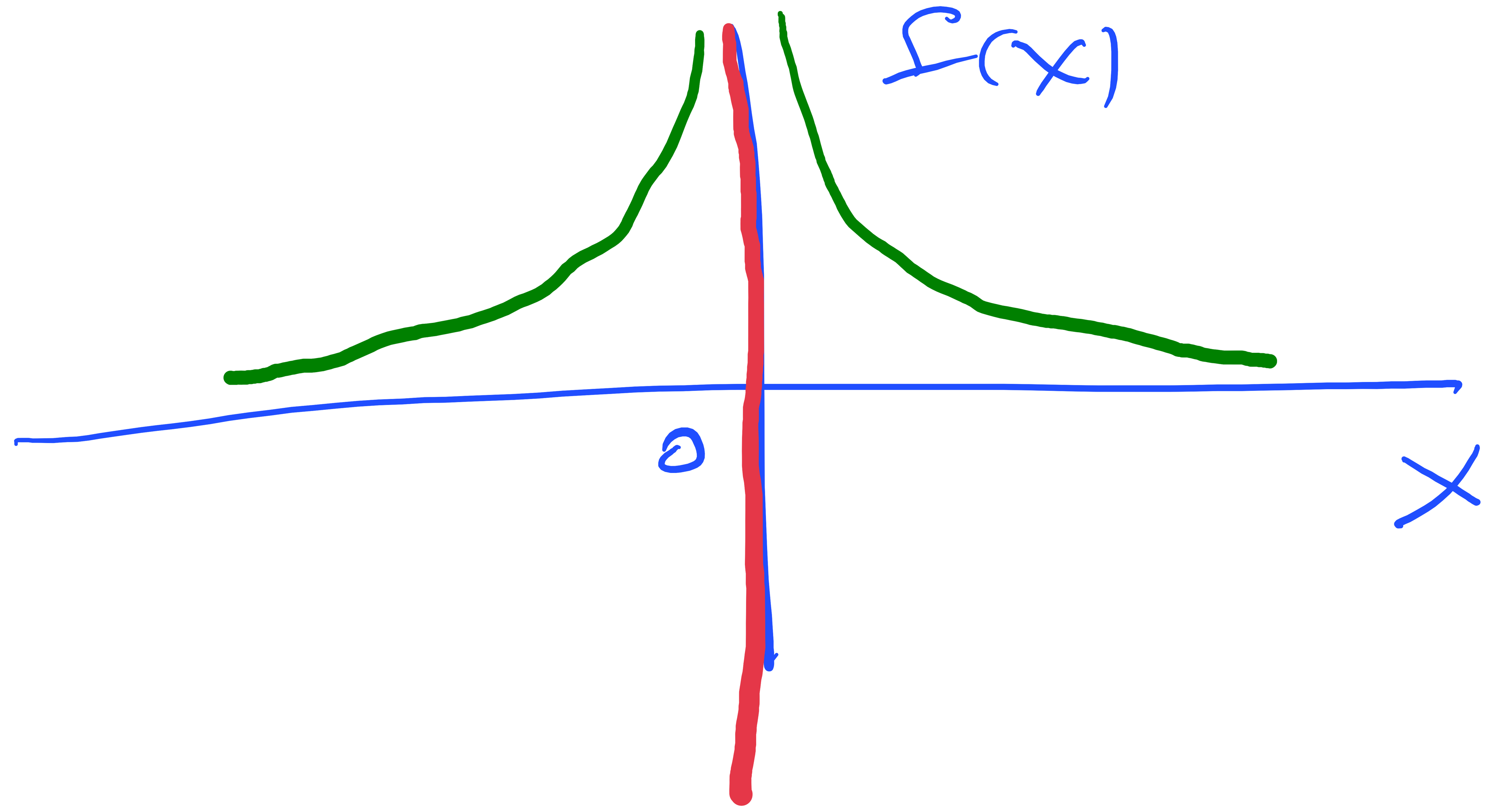
has a removable discontinuity at $x=0$.

since $f(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1. = f(0).$$

l'Hospital's Rule.

Example $f(x) = \frac{1}{x^2}$.



$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = +\infty$

$f(x)$ is NOT continuous at $x=0$.

since the limit DNE (it is not finite)

The vertical line at $x=0$ is called

a vertical asymptote.

This is called an INFINITE DISCONTINUITY.

(it is NOT removeable).

NOTE: f is continuous if it is continuous from the left and continuous from the right.

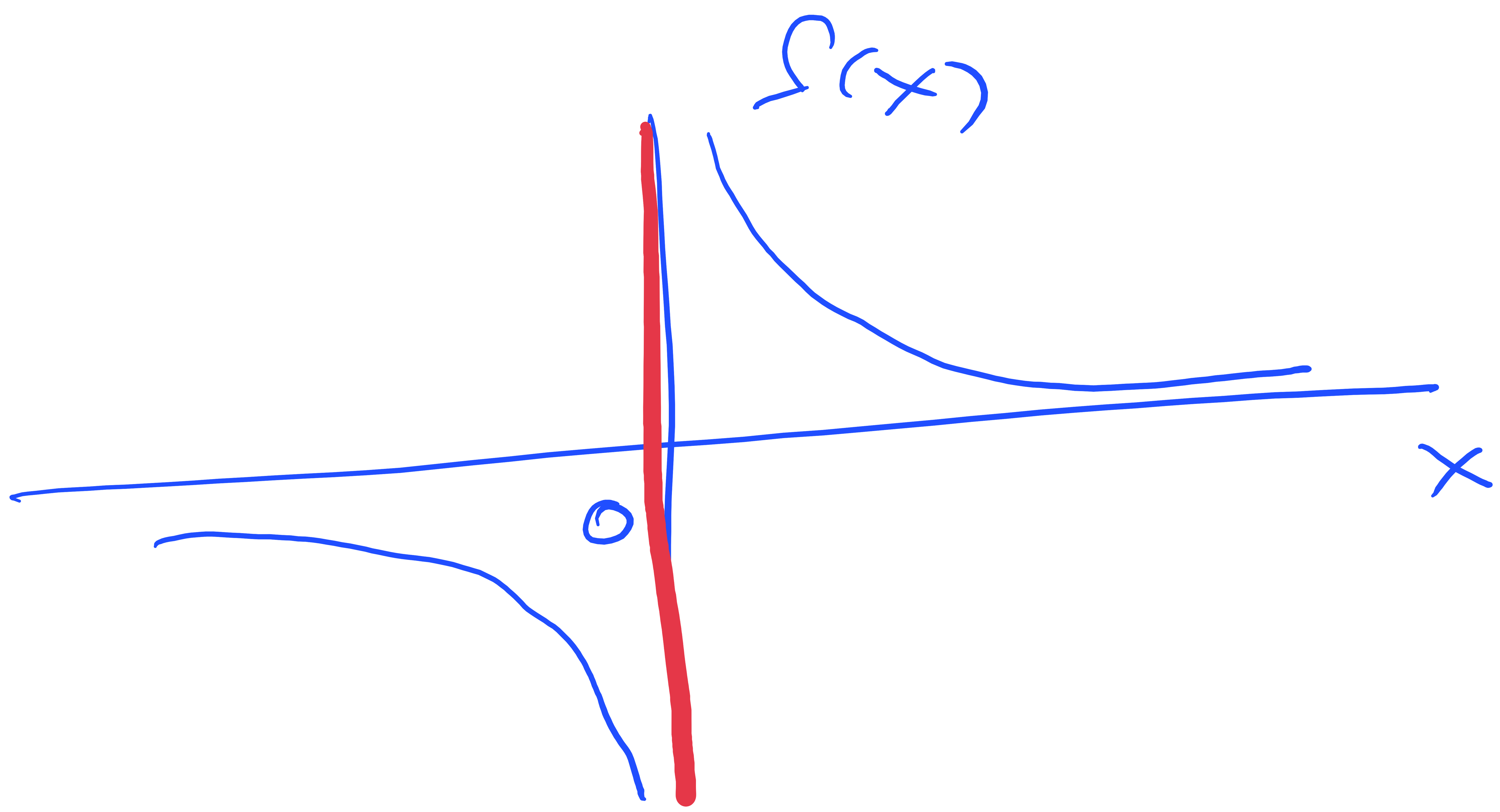
Def'n. f is continuous from the right at a if $\lim_{x \to a^+} f(x) = f(a)$

" " " the left at a if $\lim_{x \to a^-} f(x) = f(a)$.

Example $f(x) = \frac{1}{x}$

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$.

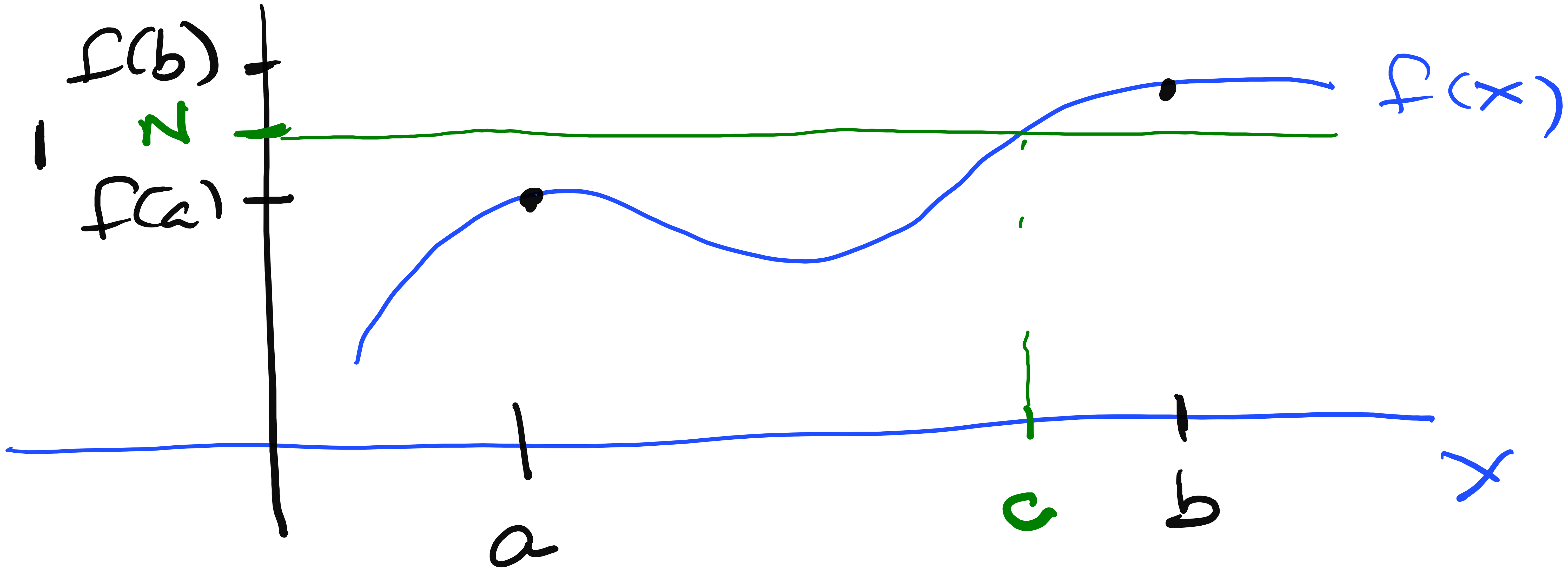


Def'n. A function is continuous on its domain if it is continuous at every point in its domain.

Intermediate Value Thm (IVT)

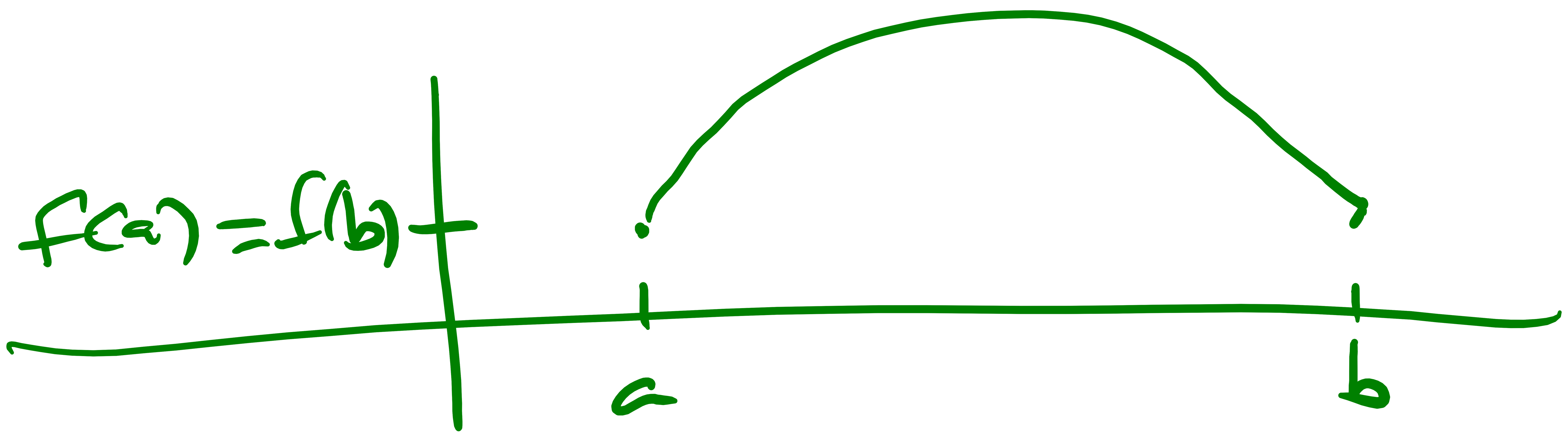
Assume that f is a continuous function on the closed interval $[a, b]$. Let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$.

Then there exists a number $c \in (a, b)$ such that $f(c) = N$.



$f(c) = N.$

there could be more than one value for c



NO value in (a,b)
so that $z = f(a)$.