Math 1A03 Calculus 1 Section C01 Dr. Wolkowicz

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Course website: www.childsmath.ca/childsa/forms/main_login.php

My lecture notes will be posted after classes.

There are links to my notes from my website, as well as from the course website -> Course Information (on the left see Dr. Wolkowicz's Notes)

There is also lots of important information, including links to Announcements, Important Dates, Lecture Schedule, MSAF FAQ, Suggested Problems etc.

TUTORIALS start week of Monday, Sept. 9.
Help Centre HH/104 starts Wednesday, Sept. 11.

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STUDENT AFFAIRS

Textbook: Calculus, Early Transcendentals, 8th Edition, James Stewart, Brooks/Cole

To prepare: Besides the **Pre-Calc Review** & **Calculus Warm-Up** on childsmath
see
page xxiii: To the Student
pages xxvi-xxx: Diagnostic tests

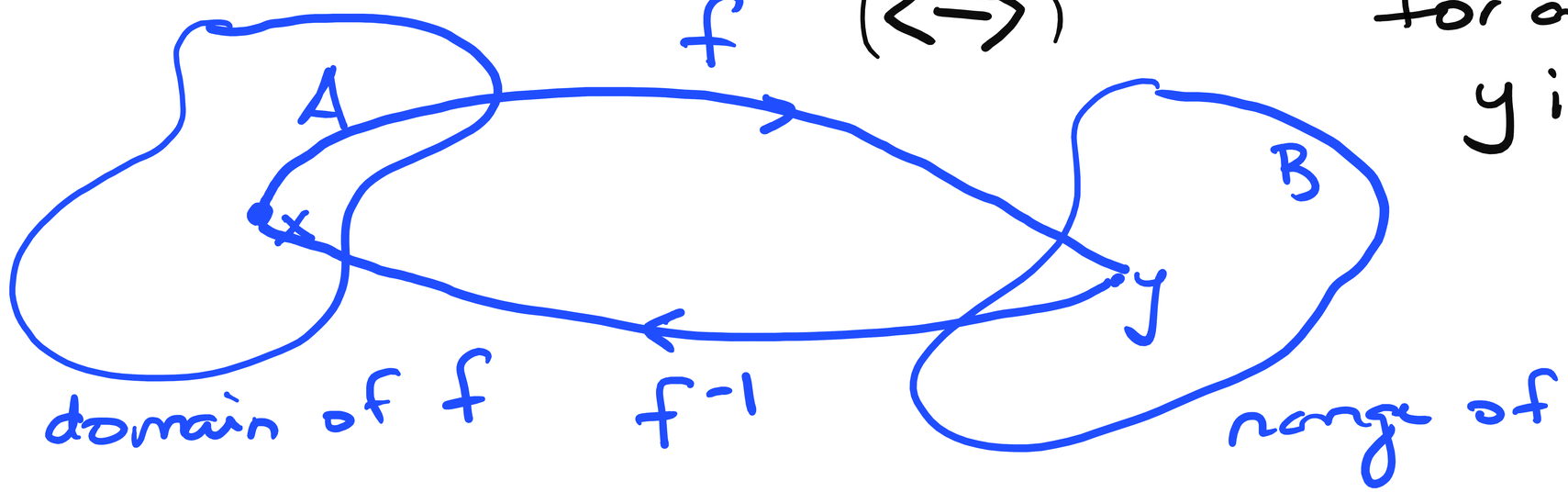
Motivation:
page 1 - 8 A Preview of Calculus
pages 9-54 Sections 1.1-1.4 Background that you are expected to know from High School
pages 77-104 Sections 2.1 - 2.3
We begin with a review of Trigonometry (see Appendix D pages A24-A33.)
& Section 1.5 Inverse Functions and Logarithms

Assignment 1 due Sept. 13

Def'n Let f be a one-to-one function with domain A and range B. Then the inverse function, f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \text{if and only if} \quad f(x) = y \quad (\Leftrightarrow)$$

for any y in B.



In general $f(f^{-1}(x)) = x = f^{-1}(f(x)) = x$.

Example: $f(2) = 6$ then $f^{-1}(6) = 2$

$$f^{-1}(f(2)) = f^{-1}(6) = 2$$

BEWARE: $f^{-1}(x) \neq \frac{1}{f(x)} = (f(x))^{-1}$

Finding inverses.

$f(x) = \sqrt{1+x}$, find $f^{-1}(x)$.

Solution: Let $y = \sqrt{1+x}$

swap the roles of x & y

and solve for y.

swap the dep & indep variables.

(y) (x)

Swap: $x = \sqrt{1+y}$

Solve for y: $x^2 = 1+y$

$y = x^2 - 1$

$\therefore$ $f^{-1}(x) = x^2 - 1$. (not one to one)

Domain is $B = [0, \infty)$

Check. $f^{-1}f(x) = f^{-1}(\sqrt{1+x}) = 1 + x - 1 = x$ ✓

domain of $f(x) = \sqrt{1+x}$ is one-to-one on its domain.

is $x \in [-1, \infty) = A$.

range of $f(x)$ is $[0, \infty) = B$

BEWARE

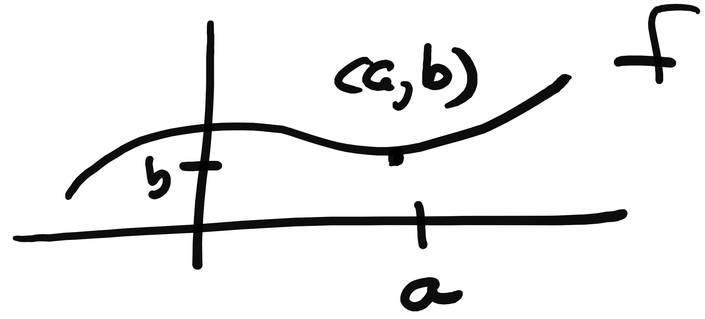
domain of $f^{-1}(x)$ is $B = [0, \infty)$

range of $f^{-1}(x)$ is $A = [-1, \infty)$.

Graphs of inverses

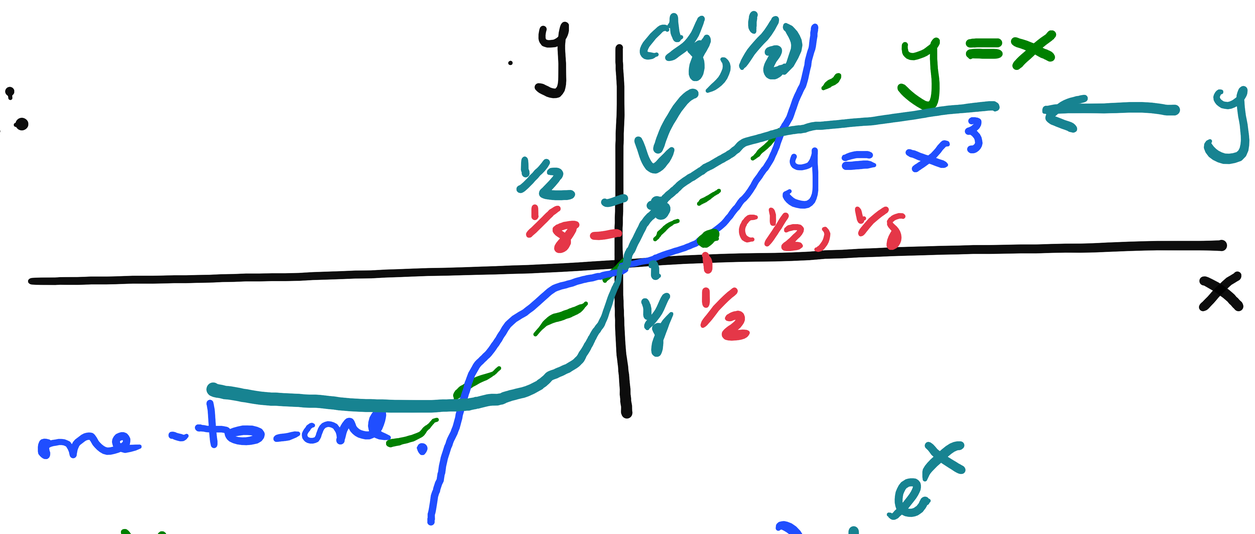
Points on the graph of f are (a, b)

Points on the graph of f^{-1} are $\therefore (b, a)$



$\therefore$ The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

Example:

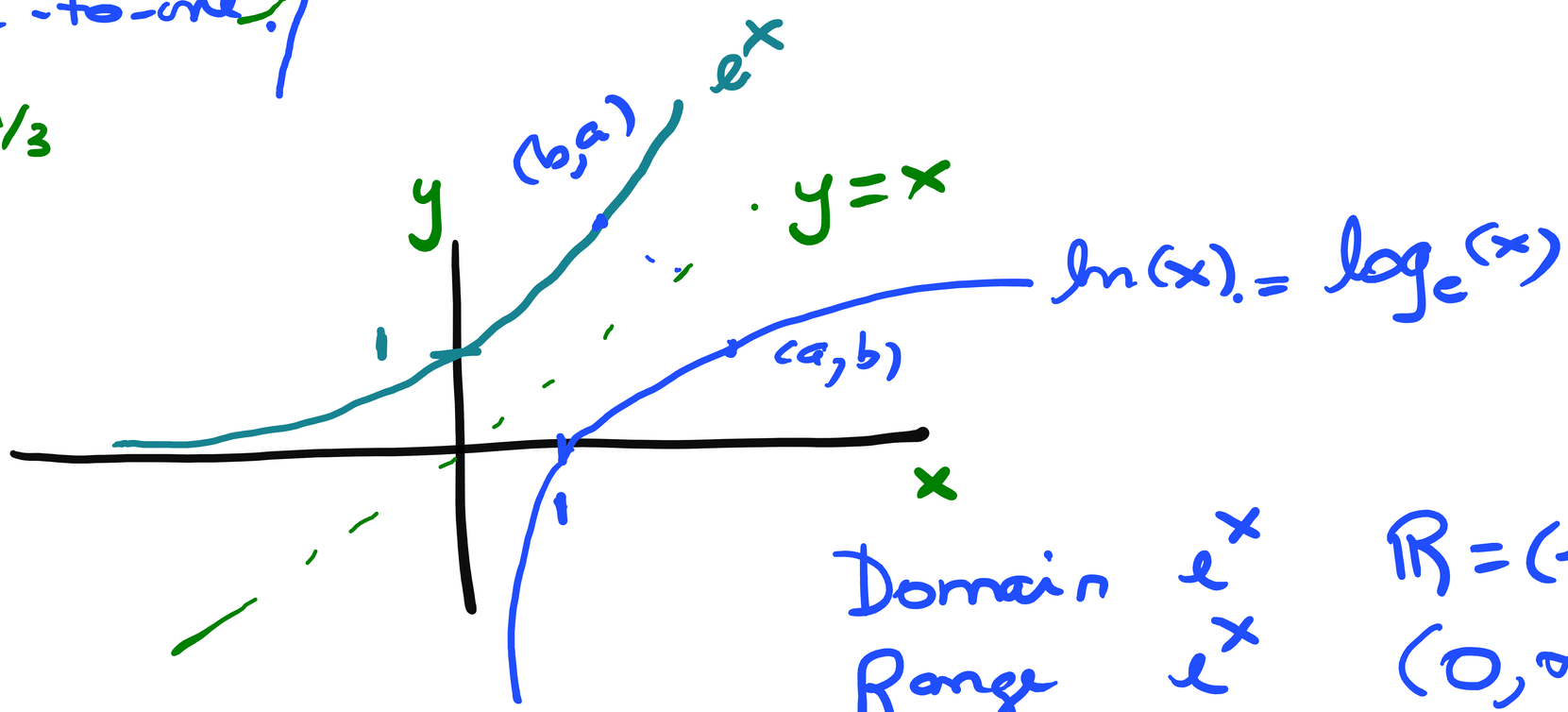


$y = x^{1/3} = f^{-1}(x)$.

$y = x^3$ is one-to-one.

$f^{-1}(x) = x^{1/3}$

Example



$\ln(x) = \log_e(x)$

$\ln(x)$ Natural logarithm.

Domain e^x $\mathbb{R} = (-\infty, \infty)$

Range e^x $(0, \infty)$

Domain $\ln(x)$ $(0, \infty)$

Range $\ln(x)$ $\mathbb{R} = (-\infty, \infty)$

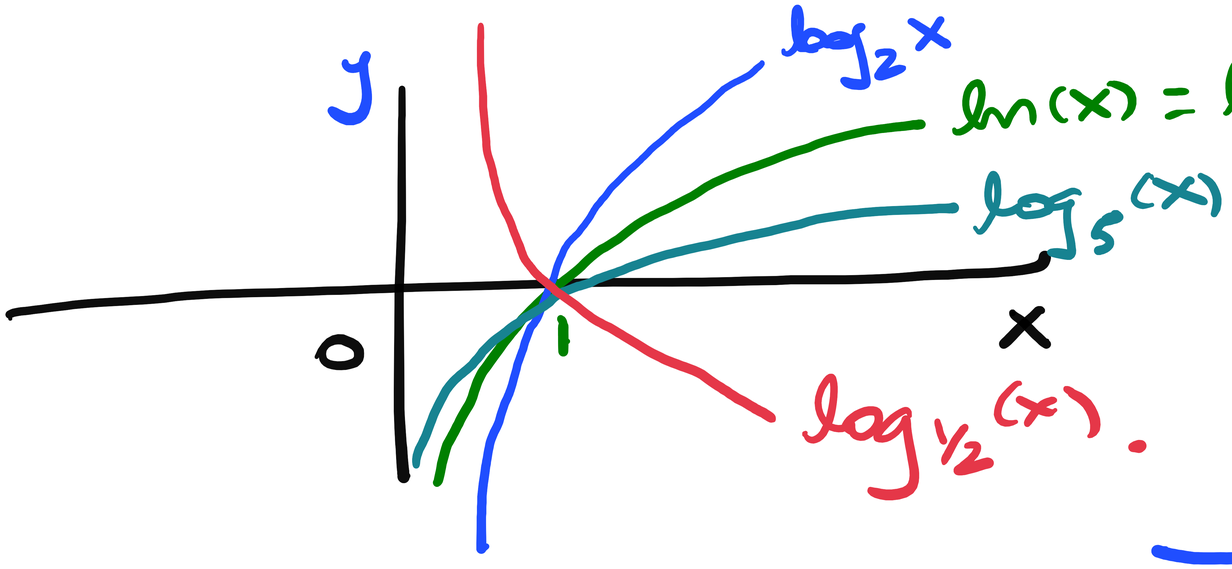
What does $x \in \mathbb{R}$ mean?

It means $x \in (-\infty, \infty)$

Logs can have different bases.

$b > 0$, $b \neq 1$. $\log_b(x)$ is defined.

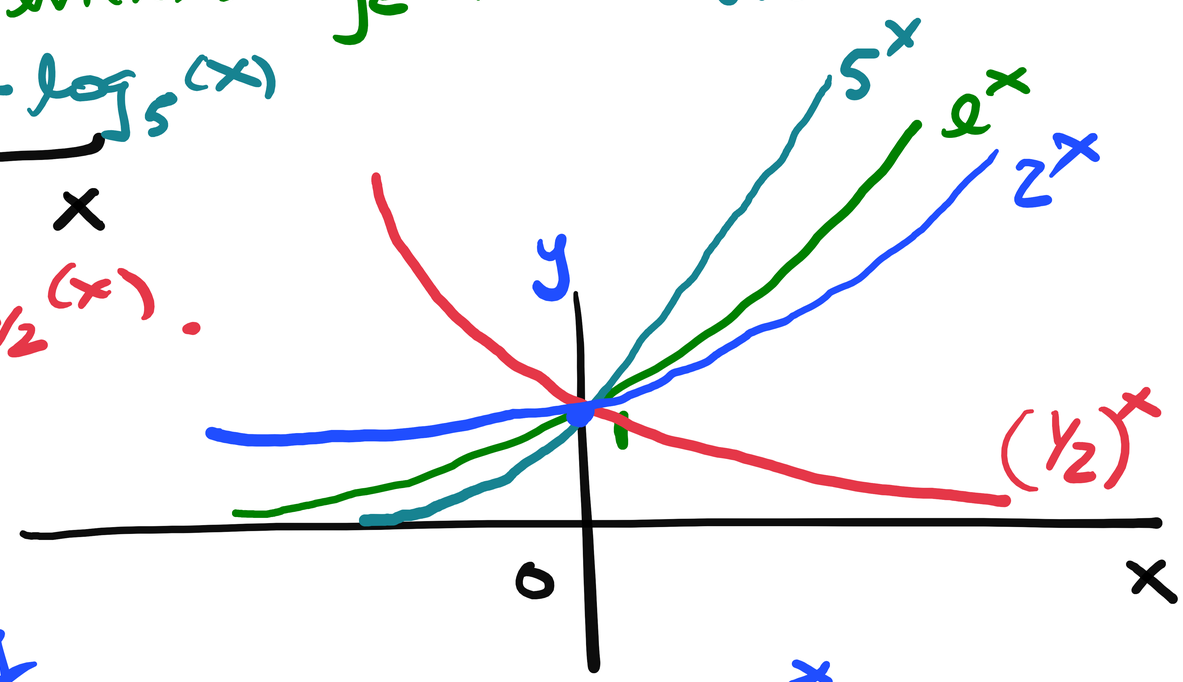
and is one-to-one.



$e \approx 2,1782818$

$b > 1$ $\log_b(x)$ increasing

$0 < b < 1$ decreasing.



b^x

$b > 1$ b^x increasing

$0 < b < 1$ b^x decreasing.

Examples:

$f(x) = 2^x$

$f(3) = 2^3 = 8$

$f(-1) = 2^{-1} = 1/2$

$f^{-1}(x) = \log_2(x)$

$f^{-1}(8) = 3$

$f^{-1}(1/2) = -1$.

$$f^{-1}(f(x)) = x \qquad f(f^{-1}(x)) = x.$$

$$\log_b(b^x) = x \qquad b^{\log_b(x)} = x.$$

$$\log_b b = 1.$$

$$\ln(x) = \log_e(x) \qquad e^{\ln x} = x$$

In math + physics. usually $\log x$ mean $\ln x$.

But in engineering. often $\log x$ means $\log_{10} x$.

Laws of logarithms.

1. $\log_b(xy) = \log_b x + \log_b y$.
2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3. $\log_b(x^r) = r\log_b(x)$, $r \in \mathbb{R}$

Example: $\log_2(125) - \log_2(5)$

$= \log_2\left(\frac{125}{5}\right)$ by Rule 2.

$$= \log_2(25)$$

$$= \log_2(5^2)$$

$$= 2\log_2(5). \quad \text{by Rule (3).}$$

Example. Expand $\ln\left(\frac{3^x(x^2-4)}{x^8(x^2+4)}\right)$

$$= \ln(3^x(x^2-4)) - \ln(x^8(x^2+4)) \quad \text{Rule 2}$$

$$= \ln(3^x) + \ln(x^2-4) - [\ln x^8 + \ln(x^2+4)] \quad \text{Rule 1.}$$

$$= x\ln 3 + \ln((x-2)(x+2)) - 8\ln x - \ln(x^2+4)$$

Rule 3

Rule 3

$$= x\ln 3 + \ln(x-2) + \ln(x+2) - 8\ln x - \ln(x^2+4)$$

↑ Cannot expand.