Review

Important Def'ns.

1) Derivative:

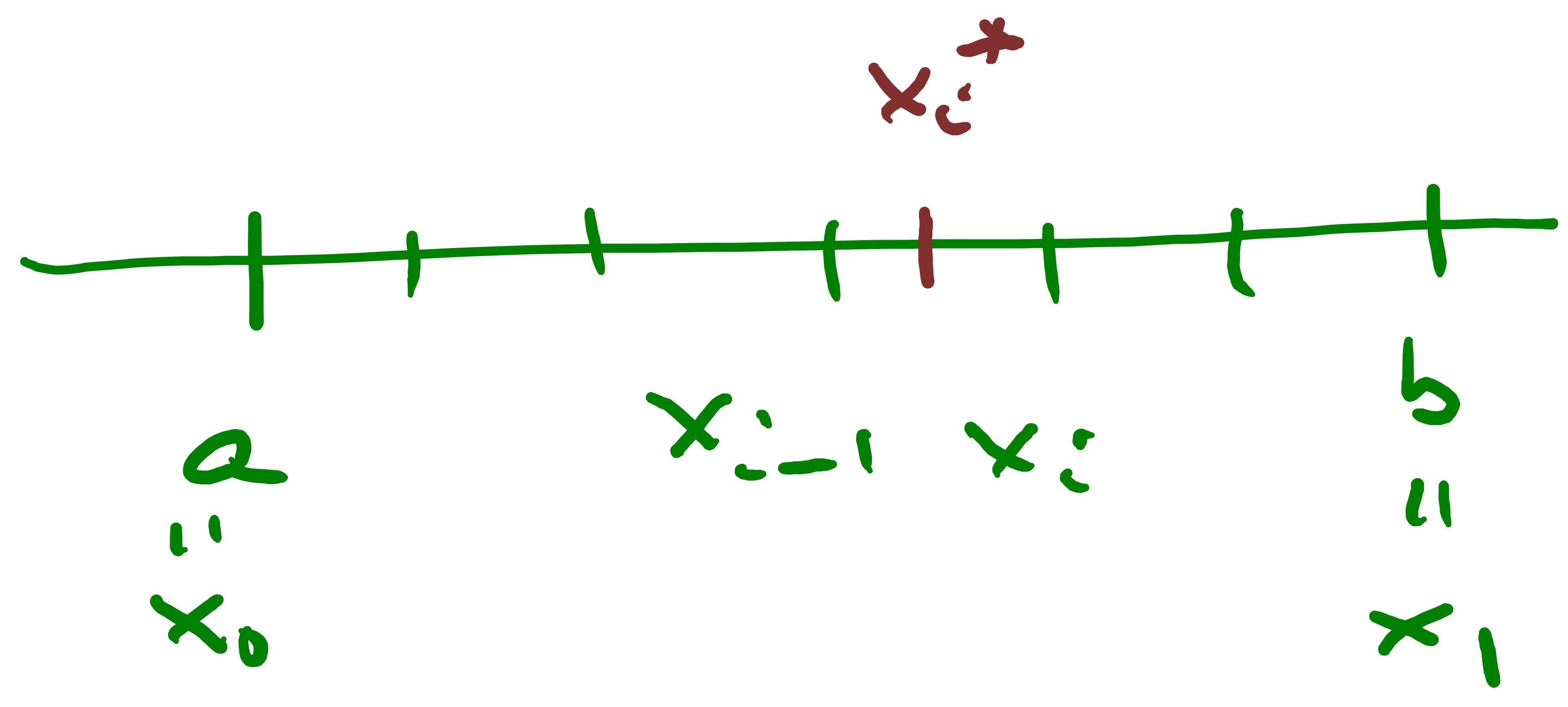
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

OR

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

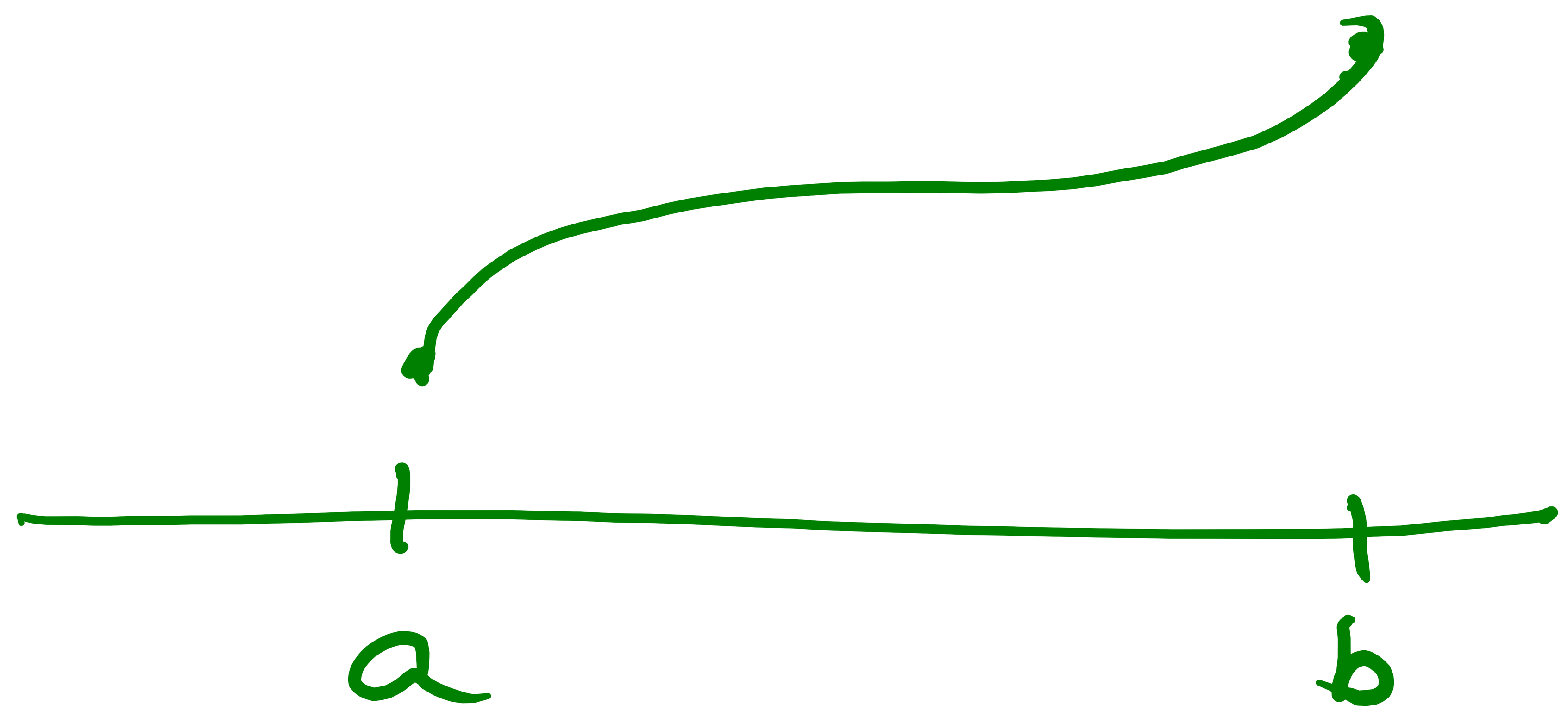
2). Integral.

$$\int_a^b f(x)\,dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i$$

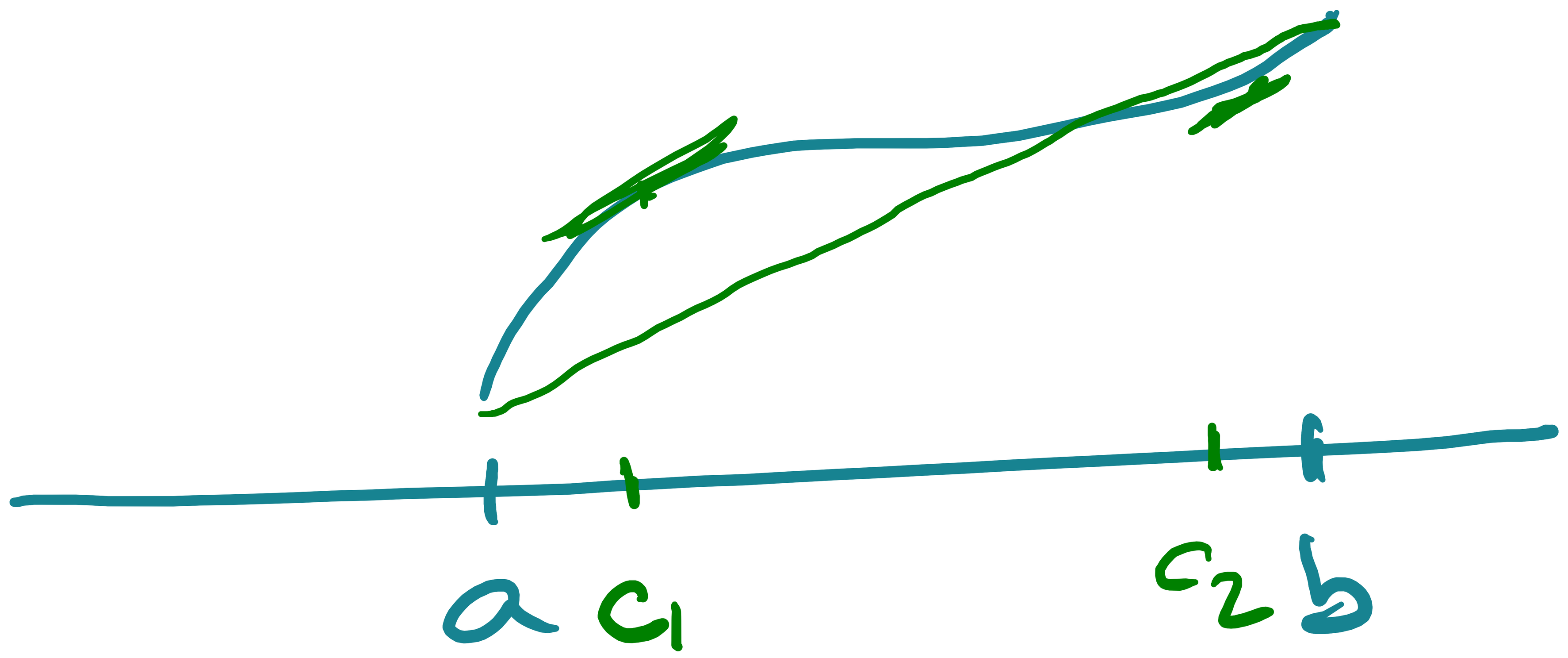


Important Theorems.

Intermediate value thm.



Mean Value Thm.



f cont $[a,b]$.

f diff'ble (a, b).

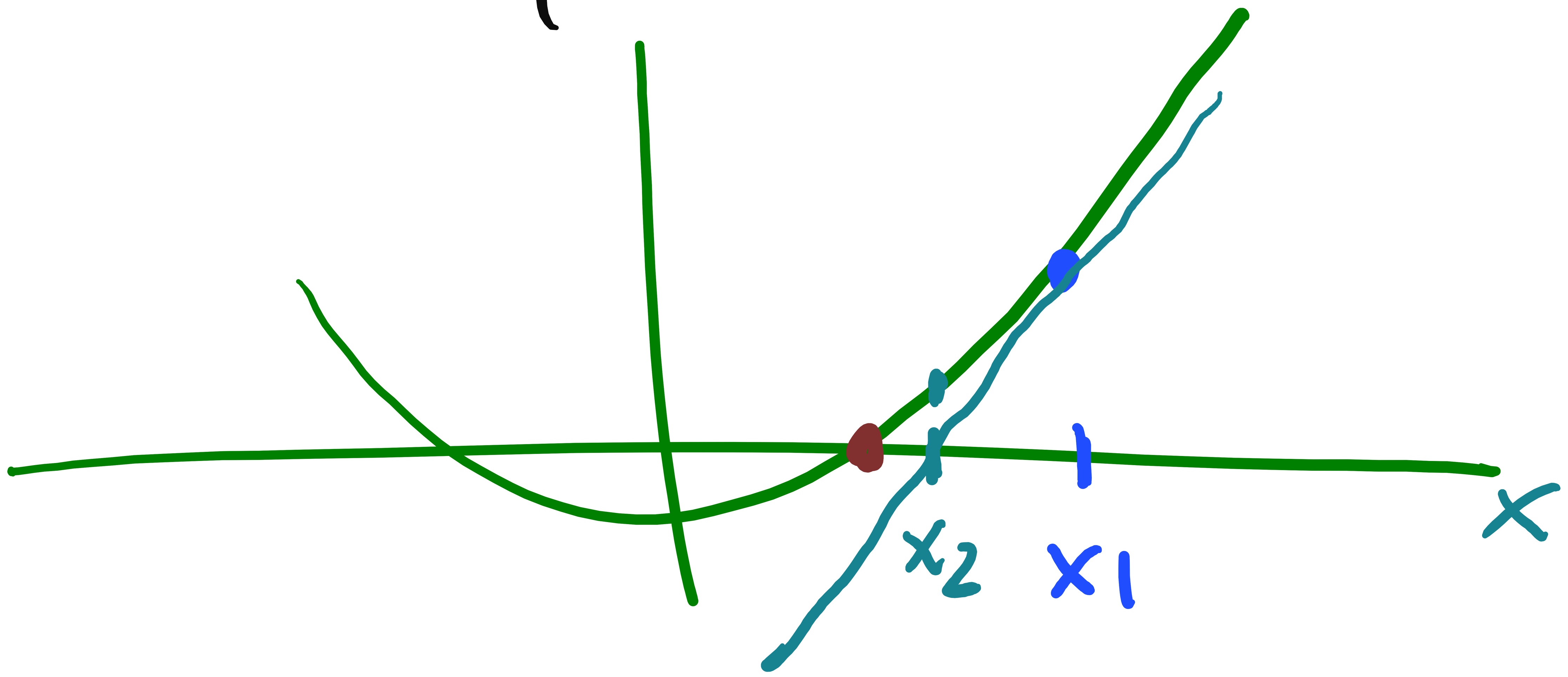
there exists $c \in (a,b)$ with $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Fundamental Thm of Calc

PI

PII.

Newton's Method. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$



Work. $= \int_a^b F(x)\,dx$

↑ force wrt to distance.

Arc Length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx.$

Avg Value $f(x)$ on $[a, b]$ $= \frac{1}{b-a} \int_a^b f(x)\,dx.$

Area btw Curves $= \int_a^b |f(x)-g(x)|\,dx$

Volume of Revolution.

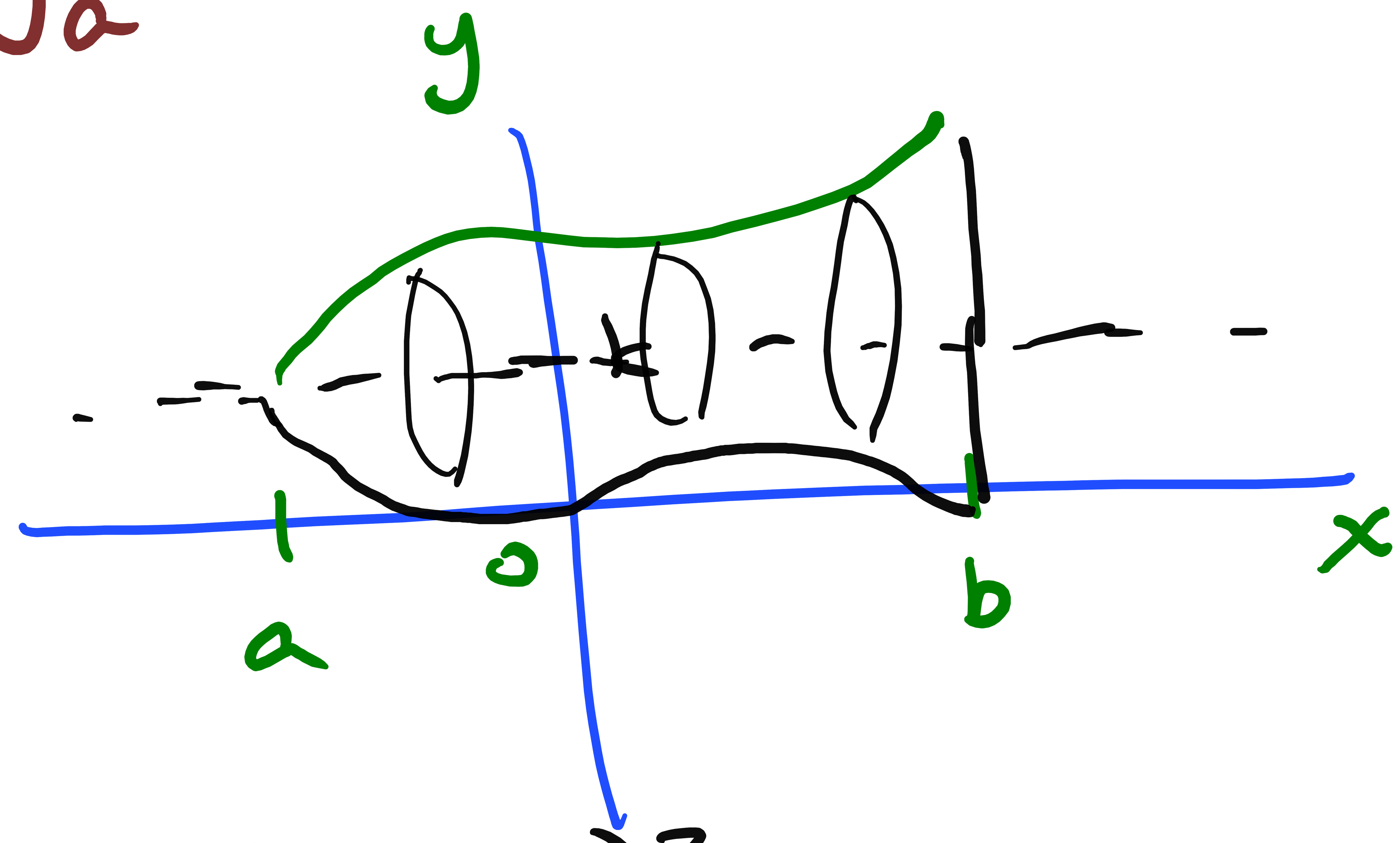
→ about x-axis. $= \int_a^b \pi (f(x))^2\,dx$

$y = f(x)$

→ about y axis. $= \int_a^b \pi (g(y))^2\,dy.$

$x = g(y)$

→ about a horizontal line $y = k$.



$y = f(x)$

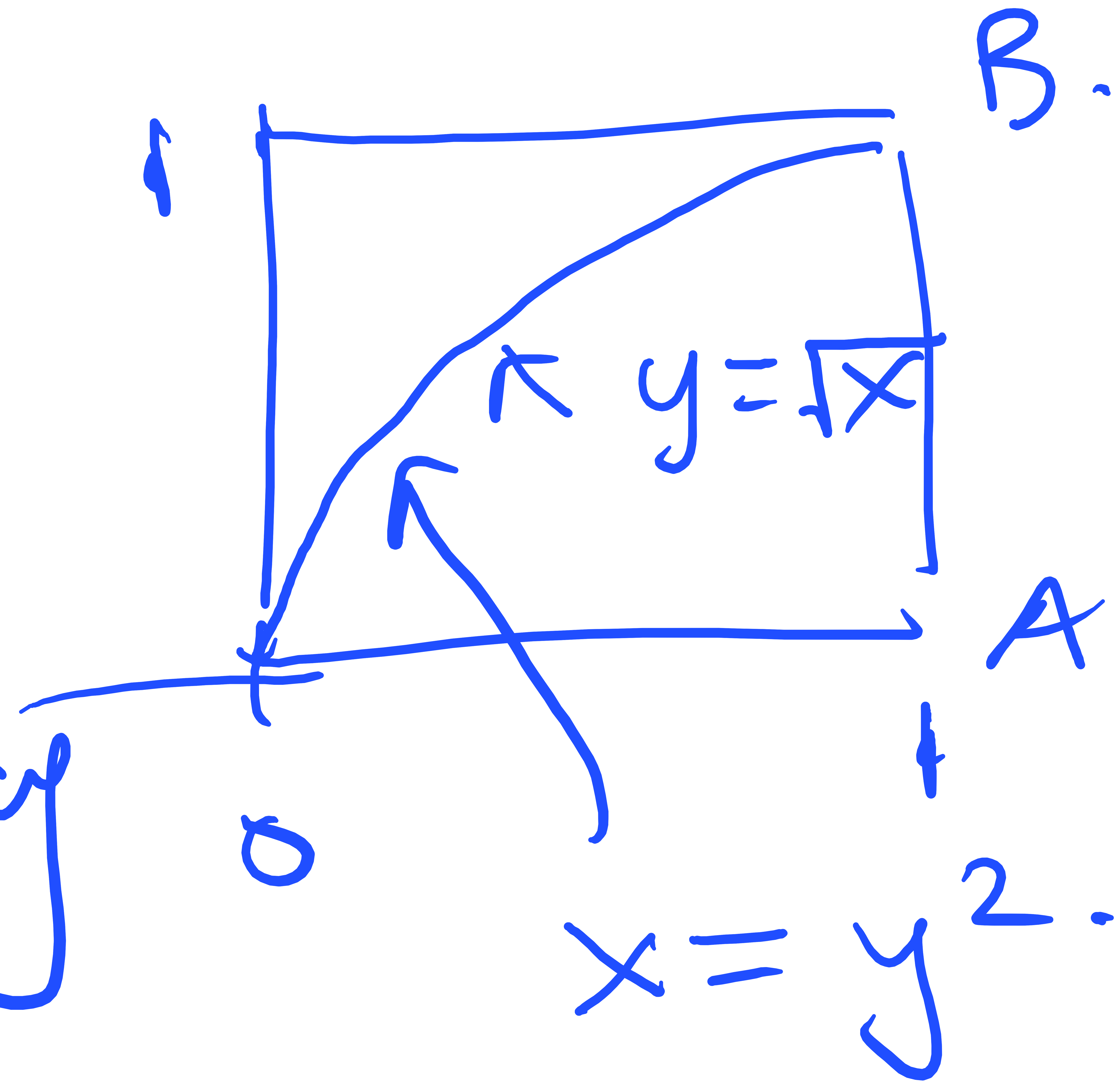
$$= \int_a^b \pi \big(\underbrace{f(x)-k}_{\text{radius.}}\big)^2\,dx.$$

Limits: L'Hospital's Rule.

5 Sample

Area

$$= \pi \int_0^1 (1)^2 - (y^2-1)^2 \, dy$$



16. $I = \int \frac{\sqrt{x^2-4}}{x^6} \, dx$

$\tan^2\theta + 1 = \sec^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

$x = 2\sec\theta$

$dx = 2\sec\theta \cdot \tan\theta \, d\theta$

$$I = \int \frac{\sqrt{4\sec^2\theta - 1}}{2^6 \sec^6\theta}\, 2\sec\theta\tan\theta\, d\theta .$$

$$= \int \frac{2\tan\theta}{2^6\sec^6\theta}\, 2\sec\theta\tan\theta\, d\theta .$$

$$= \frac{1}{2^4}\int \frac{\tan^2\theta}{\sec^5\theta}\, d\theta$$

$$= \frac{1}{2^4}\int \frac{\sec^2\theta - 1}{\sec^5\theta}\, d\theta$$

$$= \frac{1}{2^4}\int \frac{1}{\sec^3\theta} - \frac{1}{\sec^5\theta}\, d\theta$$

$$= \frac{1}{2^4}\int \cos^3\theta - \cos^5\theta\, d\theta = \frac{1}{16}\int \cos^3\theta(1-\cos^2\theta)\,d\theta$$

$$= \frac{1}{16}\int \cos^3\theta\sin^2\theta\, d\theta$$

$$= \frac{1}{2^4} \cos\theta \left(\cos^2\theta - \cos^4\theta\right) d\theta .$$

$$= \frac{1}{2^4} \int \left((1 - \sin^2\theta) - (1 - \sin^2\theta)^2 \right) \cos\theta \, d\theta$$

$$u = \sin\theta$$

$$du = \cos\theta \, d\theta$$

$$= \frac{1}{2^4} \int (1 - u^2) - (1 - u^2)^2 \, du$$

$$= \frac{1}{2^4} \int 1 - u^2 - (1 - 2u^2 + u^4) \, du$$

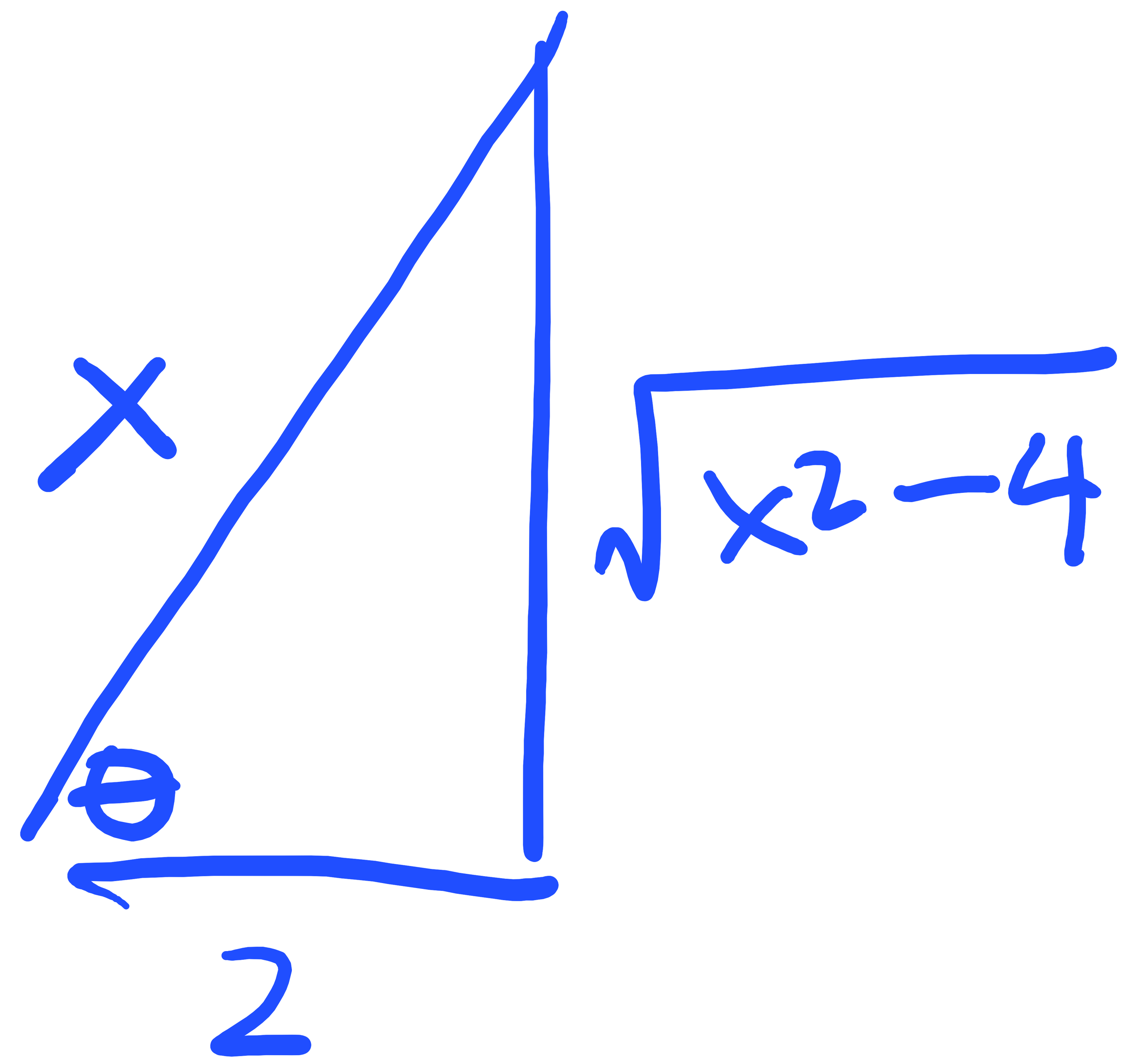
$$= \frac{1}{16} \int u^2 - u^4 \, du$$

$$= \frac{1}{16}\left(\frac{u^3}{3} - \frac{u^5}{5} \right) + C$$

$$= \frac{1}{16}\left(\frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5} \right) + C$$

$$\frac{x}{2} = \sec\theta$$

$$\frac{2}{x} = \cos\theta$$



$$\sin\theta = \frac{\sqrt{x^2-4}}{x}$$

$$= \frac{1}{16}\left(\left(\frac{\sqrt{x^2-4}}{x}\right)^3 \Big/ 3 - \left(\frac{\sqrt{x^2-4}}{x}\right)^5 \Big/ 5 \right) + C.$$

#21. $I = \int \ln(x^2+3)\, dx$

Integration by parts:

$u = \ln(x^2+3) \qquad v = x$

$du = \frac{1}{x^2+3} 2x\, dx \qquad dv = 1\, dx$

$I = x\ln(x^2+3) - \int \frac{x}{x^2+3} \cdot 2x\, dx$

$= x\ln(x^2+3) - \int \frac{2x^2}{x^2+3}\, dx.$

$$\begin{array}{r|l} & 2 \\ \hline x^2+3 & 2x^2 \\ & 2x^2+6 \\ \hline & -6 \end{array}$$

$$I = x\ln(x^2+3) - \int 2 - \frac{6}{x^2+3}\,dx.$$

$$= x\ln(x^2+3) - 2x + 6\int \frac{1}{x^2+3}\,dx.$$

$$= x\ln(x^2+3)$$
$$-2x$$
$$+\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C.$$

$a = \sqrt{3}$. $\int \frac{1}{x^2+a^2}\,dx$
$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

\# 27. Partial Fraction Decomp.

$$\frac{x^3-2x^2+x+1}{x^4+5x^2+4} = \frac{x^3-2x^2+x+1}{(x^2+1)(x^2+4)}$$

$$= \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$$

$$(Ax+B)(x^2+4) + (Cx+D)(x^2+1)$$

$$= x^3 - 2x^2 + x + 1.$$

Coeff:

$$\begin{array}{llll}
x^3: & A + C = 1 & & (1) \\
x^2: & B + D = -2 & & (2) \\
x: & 4A + C = 1 & & (3) \\
1: & 4B + D = 1 & & (4)
\end{array}$$

$③ - ①$ $\quad 3A = 0$ $\quad \left[\begin{array}{l} A = 0 \\ C = 1 \end{array}\right]$

$④ - ②$ $\quad 3B = 3 \quad B = 1$

$4 + D = 1 \Rightarrow D = -3.$

$$I = \int \frac{Ax + B}{x^2+1} + \frac{Cx + D}{x^2+4} \, dx$$

$$= \int \frac{1}{x^2+1} + \frac{x}{x^2+4} - \frac{3}{x^2+4} \, dx.$$

$$= \tan^{-1}(x) + \frac{1}{2}\ln(x^2+4) - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c.$$

$$\int \frac{x}{x^2+4}\,dx = \int \frac{1}{2u}\,du = I$$

$$u = x^2+4$$

$$du = 2x\,dx \qquad \frac{1}{2}du = x\,dx$$

$$I = \frac{1}{2}\ln|u| + c.$$

$$= \frac{1}{2}\ln|x^2+4| + c.$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

Good luck on Saturday.