Review Sessions for the Exam (organized through the Math Help Centre) will be on the following days and times:
- Thursday December 5th, 10:30am-12:20pm in LRW/B1007
- Thursday December 5th, 12:30pm-2:20pm in LRW/B1007

Extra office hour on Wednesday Dec. 4. Besides 11:00am-12:00noon, as well, 2:00pm-4:00pm.
My offie: HH/318

Exam information on childsmath for office hours of the other professors after classes end, once they decide.

§7.4 Partial Fraction Decomp. cont'd.

Case 4 $Q(x)$ contains repeated irreducible quadratic factor.

i.e factor of the form $(ax^2+bx+c)^r$ where $b^2-4ac<0$ and $r>1$. (integer)

Include r terms of the form:

$$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_rx+B_r}{(ax^2+bx+c)^r}$$

Integrate by completing the square and making a $\tan\theta$ substitution.

Example: $\dfrac{x^2-5x}{x^3(x^2+4)^2}$

$$= \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \frac{A_5x+B_5}{x^2+4} + \frac{A_6x+B_6}{(x^2+4)^2}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$$

OR.

$$\frac{x(x-5)}{x^3(x^2+4)^2} = \frac{(x-5)}{x^2(x^2+4)^2}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2}$$

Example. $I = \int \frac{1}{(x^2+2x+2)^2}\,dx$

$b^2 - 4ac$
$= 4 - 4(1)(2)$
$= -4 < 0$.
$\therefore$ irreducible quadratic.

Complete the square.

$$= \int \frac{1}{((x+1)^2+1)^2}\,dx$$

$$\frac{Ax+B}{((x+1)^2+1)} + \frac{Cx+D}{((x+1)^2+1)^2} =$$

Would get: $A=0, B=0, C=0, D=1$

Didn't need to do this since already in its partial fraction decomposition.

$$I = \int \frac{1}{((x+1)^2+1)^2}\,dx,$$

substitute

$u = x+1$
$du = dx$.

$$I = \int \frac{1}{(u^2+1)^2}\,du.$$

substitute a 2^{nd} time:

let $u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$= \int \frac{\sec^2\theta\, d\theta}{(\underbrace{\tan^2\theta+1}_{\sec^2\theta})^2} = \int \frac{\sec^2\theta}{(\sec^2\theta)^2}\,d\theta$$

$$= \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\, d\theta$$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$$= \int \frac{1}{2}(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C.$$

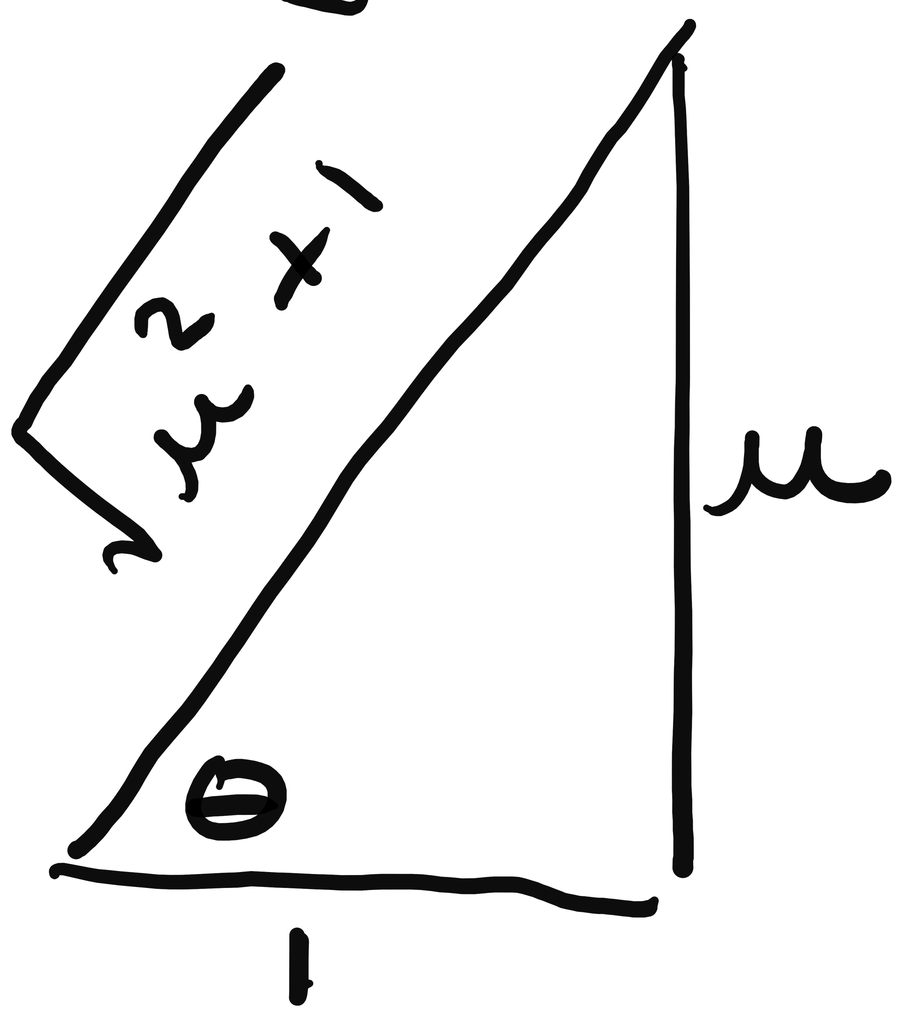
Go Back to ORIGINAL VARIABLE X.

$u = \tan\theta \Rightarrow \theta = \tan^{-1}(u)$

$$= \frac{1}{2}\left(\tan^{-1}(u) + \frac{\cancel{2}}{\cancel{2}}\sin\theta\cos\theta\right) + C$$

using $\left(\sin 2\theta = 2\sin\theta\cos\theta\right)$

$$= \frac{1}{2}\left[\tan^{-1}(\mu) + \left(\overset{\sin\theta}{\overset{\shortparallel}{\frac{\mu}{\sqrt{\mu^2+1}}}}\right)\left(\overset{\cos\theta}{\overset{\shortparallel}{\frac{1}{\sqrt{\mu^2+1}}}}\right)\right] + C.$$



$$\frac{\mu}{1} = \tan\theta$$

$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}$$

$$\cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$

But $\mu = x+1$

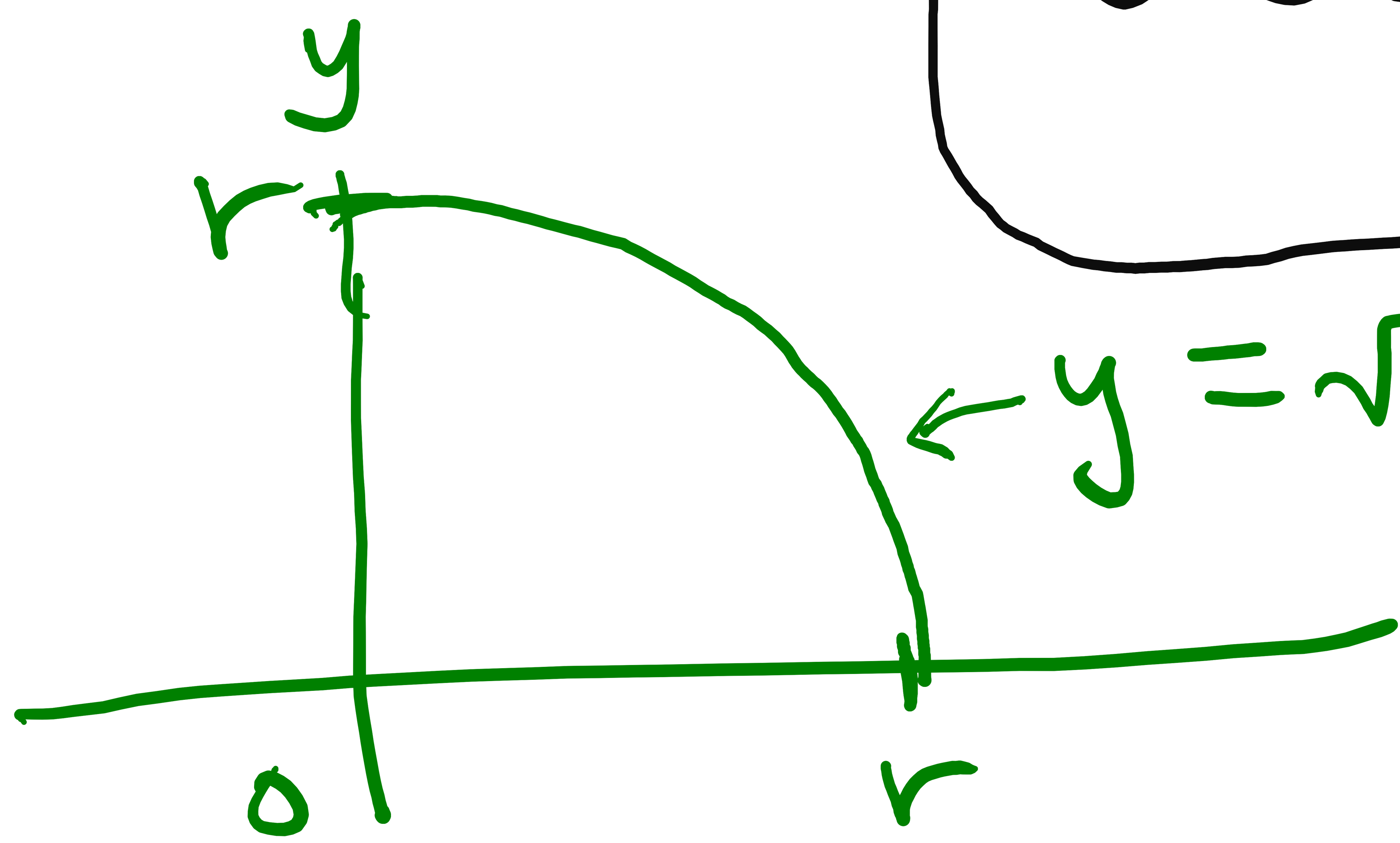
$$= \frac{1}{2}\left[\tan^{-1}(x+1) + \frac{x+1}{(x+1)^2+1}\right] + C$$

$$= \frac{1}{2}\left[\tan^{-1}(x+1) + \frac{x+1}{x^2+2x+2}\right] + C.$$

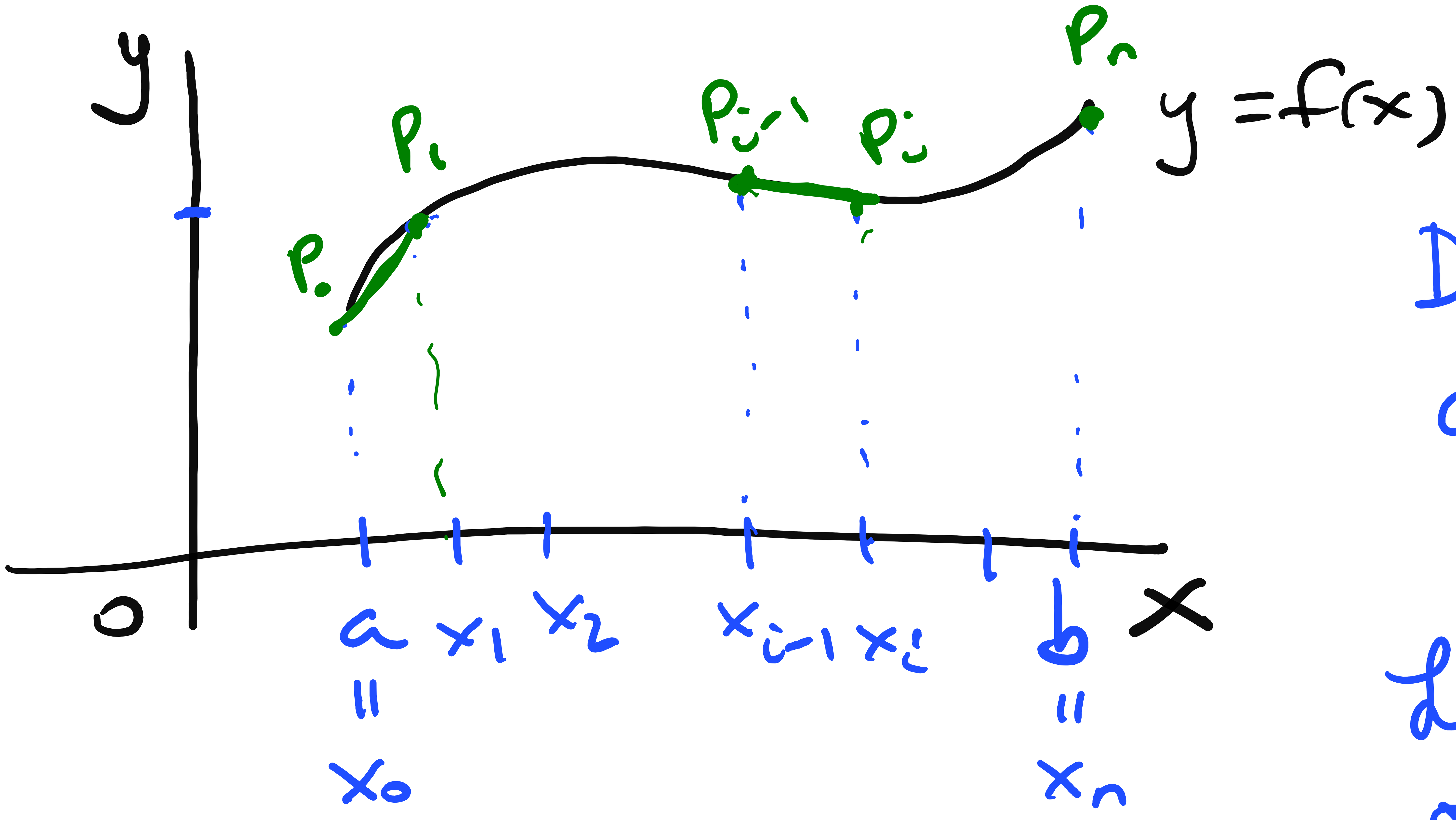
§8.1 Arc Length.

$$x^2+y^2=r^2$$

$$y=\sqrt{r^2-x^2}$$



Arc length of $y=f(x)$ for x from 0 to r is $\pi r/2$, i.e., $\frac{1}{4}$ the circumference of the circle.



Defn Let C be a curve with equation $y=f(x)$, $a \le x \le b$.

Let P be a partition of $[a,b]$.

$$a = x_0 < x_1 < x_2 < \dots < x_{i-1} < x_i < \dots < x_n = b.$$

Let $y_i = f(x_i)$

Then $P_i(x_i, y_i)$ lies on C.

The length of C is defined by

$$L = \lim_{\|P\| \to 0} \sum_{i=1}^{n} |P_{i-1} P_i| \quad \text{(if the limit exists)}$$

where $|P_{i-1}P_i|$ denotes the length of the "inscribed polygon from P_{i-1} to P_i",

i.e. the length of the line segment from P_{i-1} to P_i.

ARC LENGTH FORMULA

If f' is continuous on $[a, b]$, the length of the curve $y = f(x)$, $a \le x \le b$, is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

OR

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

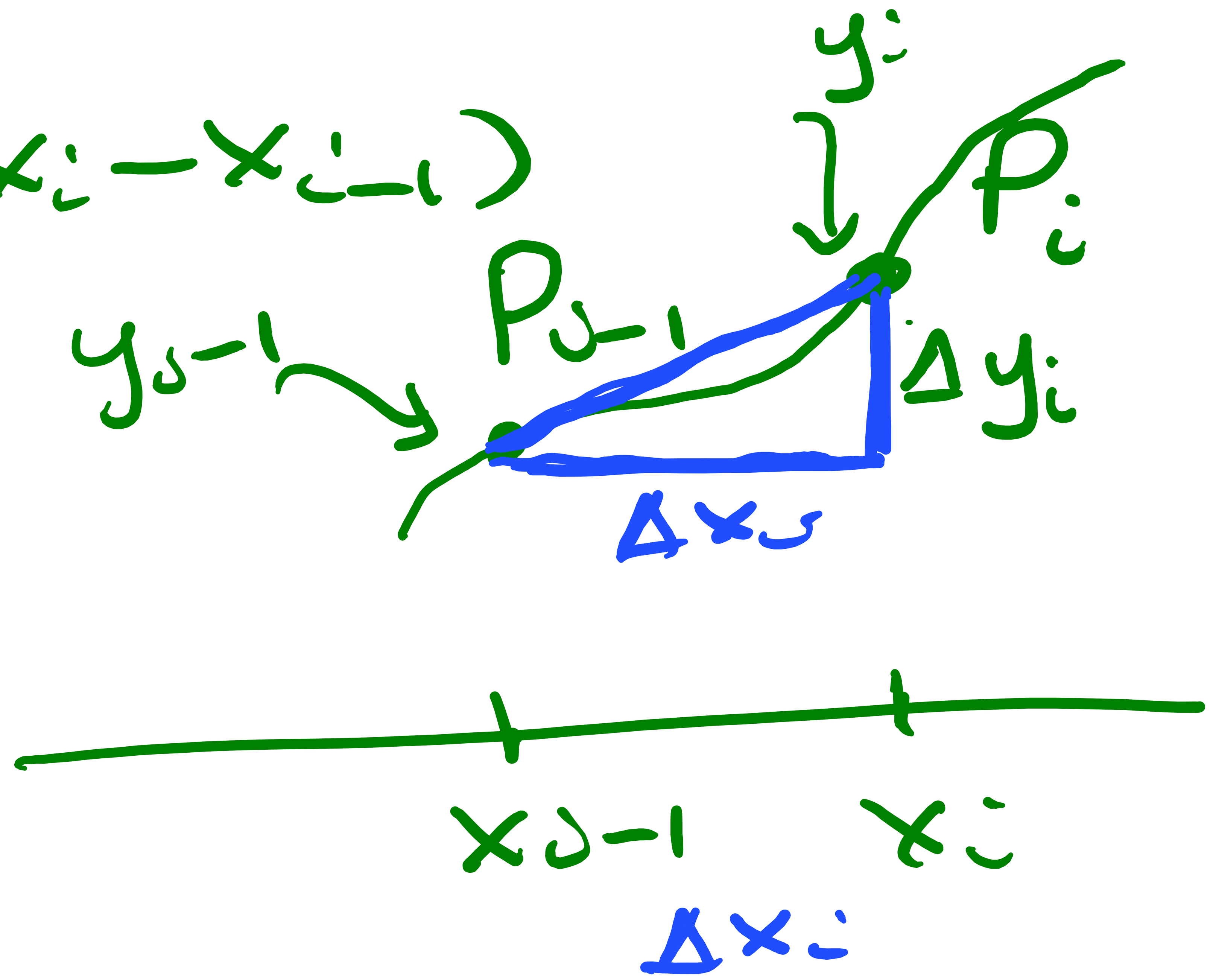
Proof: Apply the Mean Value Theorem for f on the interval $[x_{i-1}, x_i]$.

Then, there is a point $x_i^* \in (x_{i-1}, x_i)$ such that

$$\underbrace{f(x_i) - f(x_{i-1})}_{\parallel} = f'(x_i^*)(x_i - x_{i-1}).$$

$$\underbrace{y_i - y_{i-1}}_{\parallel} = f'(x_i^*)(x_i - x_{i-1})$$

$$\Delta y_i = f'(x_i^*)\Delta x_i$$



$|P_{i-1}P_i| = \sqrt{\Delta x_i^2 + \Delta y_i^2}$

Pythagorus

$$|P_{i-1}P_i|$$
$$= \sqrt{\Delta x_i^2 + \Delta y_i^2}$$
$$= \sqrt{\Delta x_i^2 + (f'(x_i^*)\Delta x_i)^2}$$
$$= \sqrt{1 + (f'(x_i^*))^2}\ \Delta x_i .$$

$$\therefore L = \lim_{\|P\| \to 0} \sum_{i=1}^{n} |P_{i-1} P_i|$$

$$= \lim_{\|P\| \to 0} \sum_{i=1}^{n} \sqrt{1 + (f'(x_i^*))^2}\, \Delta x_i$$

$$= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$, by the def'n of the definite integral.