Math 1A03 Calculus 1   Section C01   Dr. Wolkowicz

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Wednesdays 11:00-12:00 & Fridays 2:00-3:00   or by appointment

Course website: www.childsmath.ca/childsa/forms/main_login.php

My lecture notes will be posted after classes.

There are links to my notes from my website, as well as from the course website -> Course Information (on the left see Dr. Wolkowicz's Notes)

There is also lots of important information, including links to Announcements, Important Dates, Lecture Schedule, MSAF FAQ, Suggested Problems etc.

TUTORIALS Start week of Monday, Sept. 9.
Help Centre HH/104 starts Wednesday, Sept. 11.

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STUDENT AFFAIRS

Textbook: Calculus, Early Transcendentals, 8th Edition, James Stewart, Brooks/Cole

To prepare: Besides the **Pre-Calc Review** & **Calculus Warm-Up** on childsmath
see
page xxiii: To the Student
pages xxvi-xxx: Diagnostic tests

Motivation:
page 1 - 8 A Preview of Calculus
pages 9-54 Sections 1.1-1.4 Background that you are expected to know from High School
pages 77-104 Sections 2.1 - 2.3

We begin with a review of Trigonometry (see Appendix D pages A24-A33.)
& Section 1.5 Inverse Functions and Logarithms

Assignment 1 due Sept. 13

Trig identities.

$$\sin^2 x + \cos^2 x = 1.$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

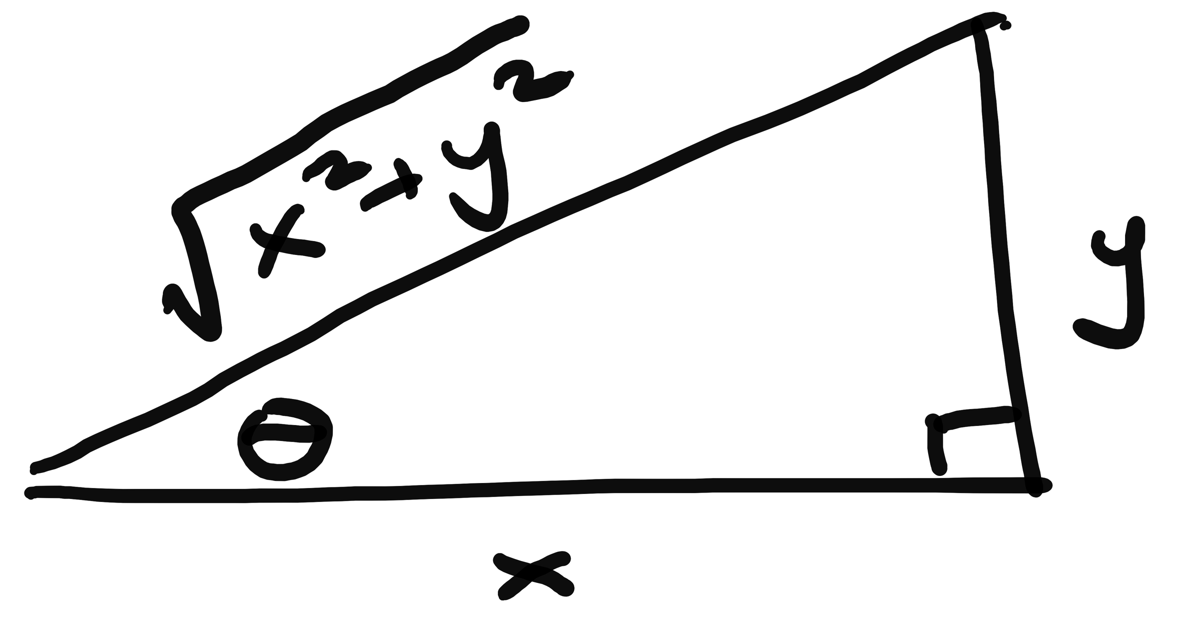
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b).$$

$\cos(2\theta) = \cos(\theta+\theta) = \cos^2\theta - \sin^2\theta$ in $\cos(a+b)$ above.
taking $a=b=\theta$

$= 1-\sin^2\theta - \sin^2\theta = 1-2\sin^2\theta.$

$\sin(2\theta) = 2\sin\theta\cos\theta$, taking $a=b=\theta$ in $\sin(a+b)$ above.

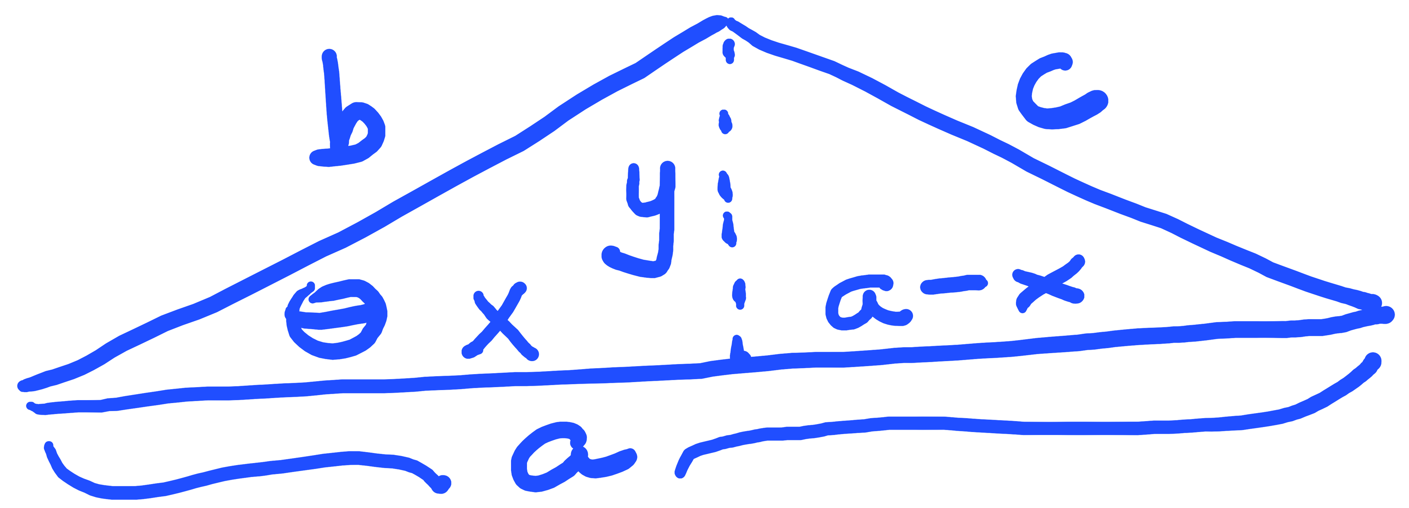
Proof that $\sin^2\theta + \cos^2\theta = 1$



$$\sin\theta = y/\sqrt{x^2+y^2} \qquad \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sin^2\theta + \cos^2\theta = \frac{y^2}{x^2+y^2} + \frac{x^2}{x^2+y^2} = \frac{x^2+y^2}{x^2+y^2} = 1.$$

Cosine Law



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Proof: $\cos\theta = x/b \Rightarrow x = b\cos\theta$

$$x^2 + y^2 = b^2 \Rightarrow y^2 = b^2 - b^2\cos^2\theta = b^2(1-\cos^2\theta)$$

$$(a-x)^2 + y^2 = c^2 \Rightarrow y^2 = c^2 - (a-x)^2$$
$$= c^2 - (a^2 - 2ax + x^2)$$
$$= c^2 - (a^2 - 2ab\cos\theta + b^2\cos^2\theta)$$

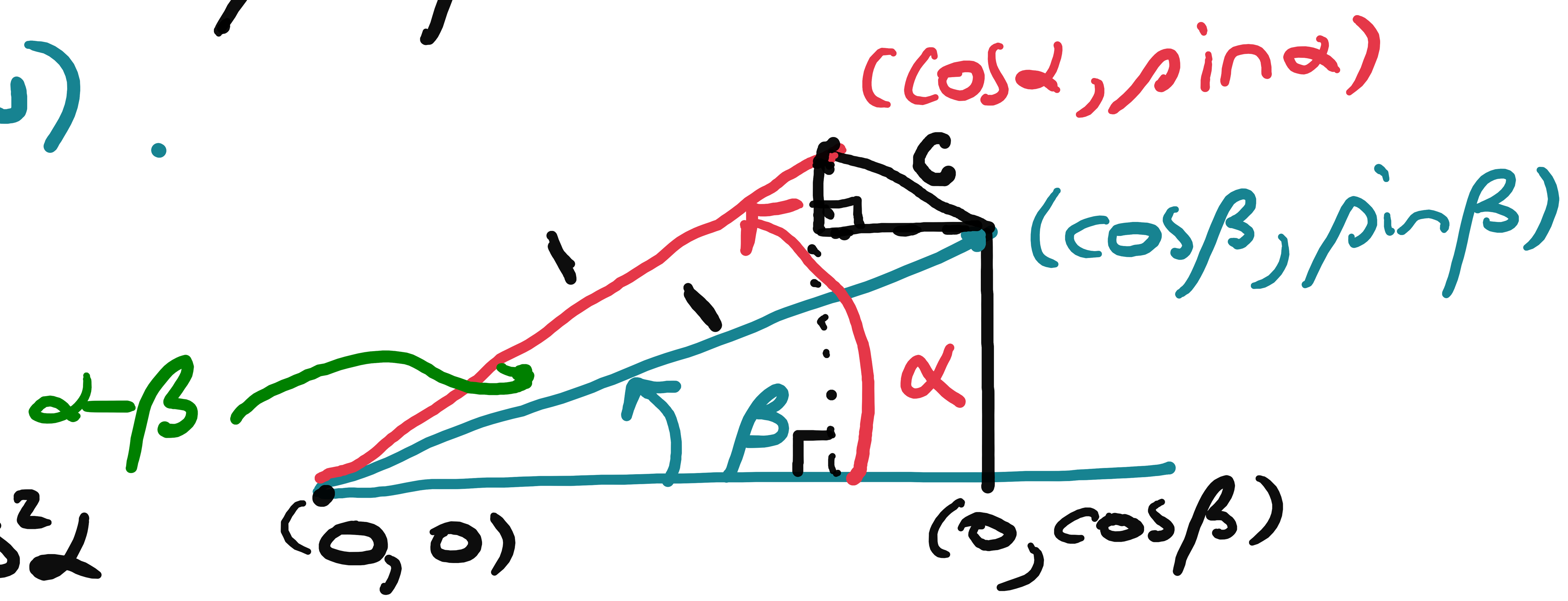
$$\therefore\ b^2(1-\cancel{\cos^2\theta}) = y^2 = c^2 - (a^2 - 2ab\cos\theta + \cancel{b^2\cos^2\theta})$$

$$\therefore\ c^2 = a^2 + b^2 - 2ab\cos\theta$$

$\not{}$

Proof of $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

(Use Pythagorus & Cosine Law).



Pythagorus: $c^2 = (\cos\beta - \cos\alpha)^2 + (\sin\alpha - \sin\beta)^2$

$= \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\alpha + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta$

$= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

Law of Cosines: $c^2 = 1^2 + 1^2 - 2(1)(1)\cos(\alpha-\beta)$

$= 2(1 - \cos(\alpha-\beta))$

$\therefore \cancel{2}(\cancel{1} - (\cos\alpha\cos\beta + \sin\alpha\sin\beta)) = c^2 = \cancel{2}(\cancel{1} - \cos(\alpha-\beta))$

$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

From this we can find $\cos(\alpha+\beta)$:

$\cos(\alpha+\beta) = \cos(\alpha - (-\beta))$

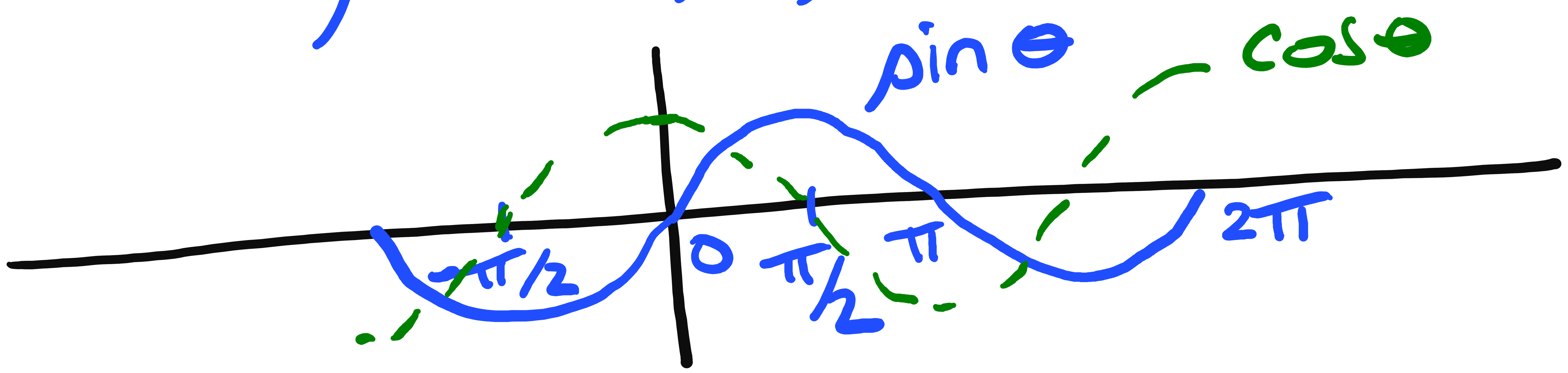
$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$, since cos is even, & sin is odd.

Now we have

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin(\alpha)\sin(\beta)$$

To find $\sin(\alpha \pm \beta)$, NOTE.



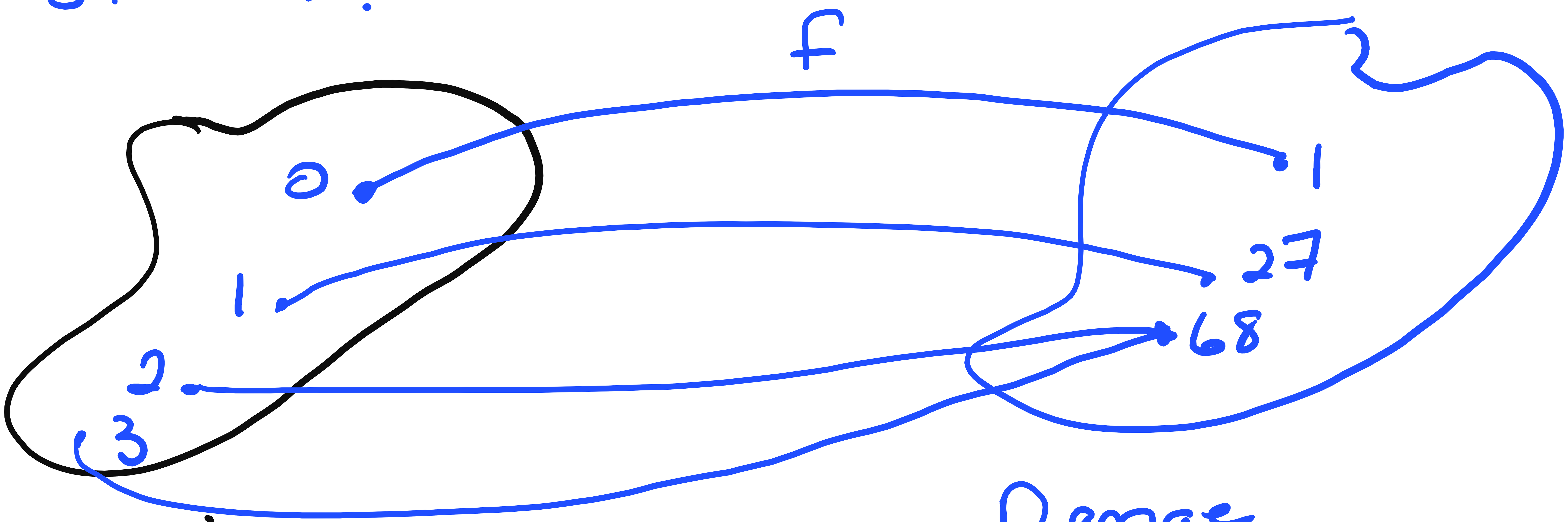
$$\cos(\theta - \tfrac{\pi}{2}) = \sin\theta$$
$$\sin(\theta - \tfrac{\pi}{2}) = -\cos\theta$$

$$\begin{aligned}
\sin(\alpha+\beta) &= \cos(\alpha + (\beta - \tfrac{\pi}{2})) \\
&= \cos\alpha\cos(\beta - \tfrac{\pi}{2}) - \sin(\alpha)\sin(\beta - \tfrac{\pi}{2}) \\
&= \cos\alpha\sin\beta - \sin(\alpha)(-\cos(\beta)) \\
&= \cos\alpha\sin\beta + \sin(\alpha)\cos\beta.
\end{aligned}$$

$$\begin{aligned}
\sin(\alpha-\beta) &= \cos(\alpha - \beta - \tfrac{\pi}{2}) \\
&= \cos((\alpha - \tfrac{\pi}{2}) - \beta) \\
&= \cos(\alpha - \tfrac{\pi}{2})\cos\beta + \sin(\alpha - \tfrac{\pi}{2})\sin\beta \\
&= \sin\alpha\cos\beta + (-\cos\alpha)\sin\beta \\
&= \sin\alpha\cos\beta - \cos\alpha\sin\beta.
\end{aligned}$$

§1.5 Inverse Functions & Logarithms

$y = f(x)$ is a function if each value of x in the domain of f (i.e. where f is defined) gives only one number (in the range of f).

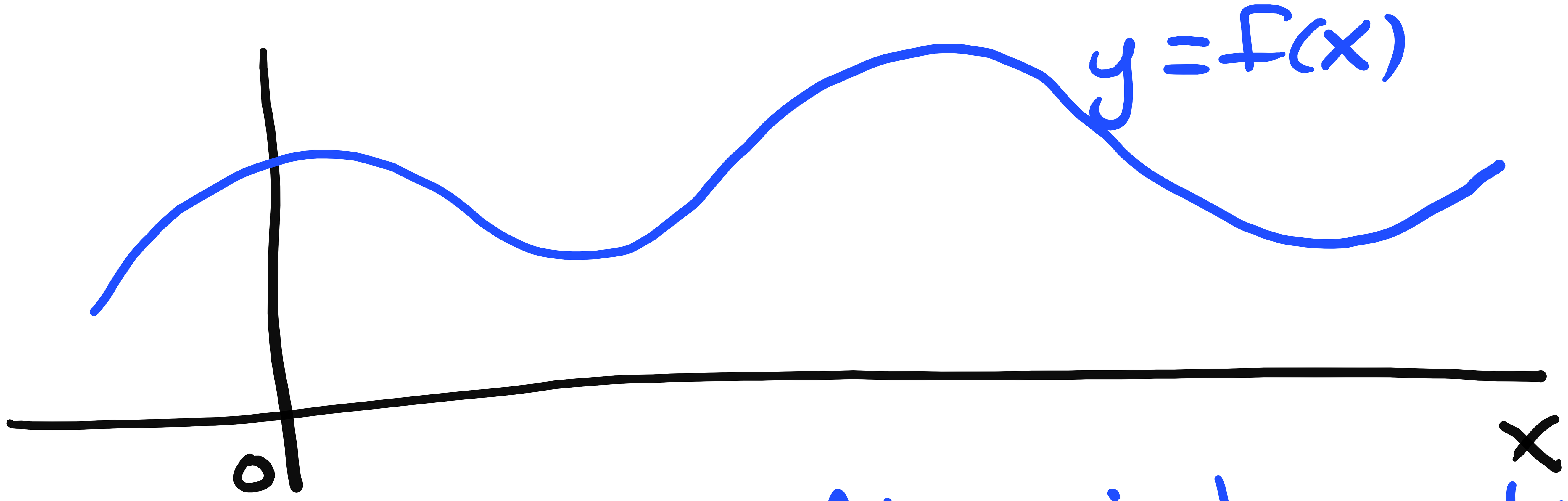


Domain
all values where f is defined
input; independent variable

Range
- all possible values of $f(x)$
output; dependent variable

Note $f(2) = 68 = f(3)$; and f is still a function.

Graph of a function $y = f(x)$



Not: the graph of a function

Any vertical line intersects $f(x)$ at most once.

NOT ALL functions have inverses..

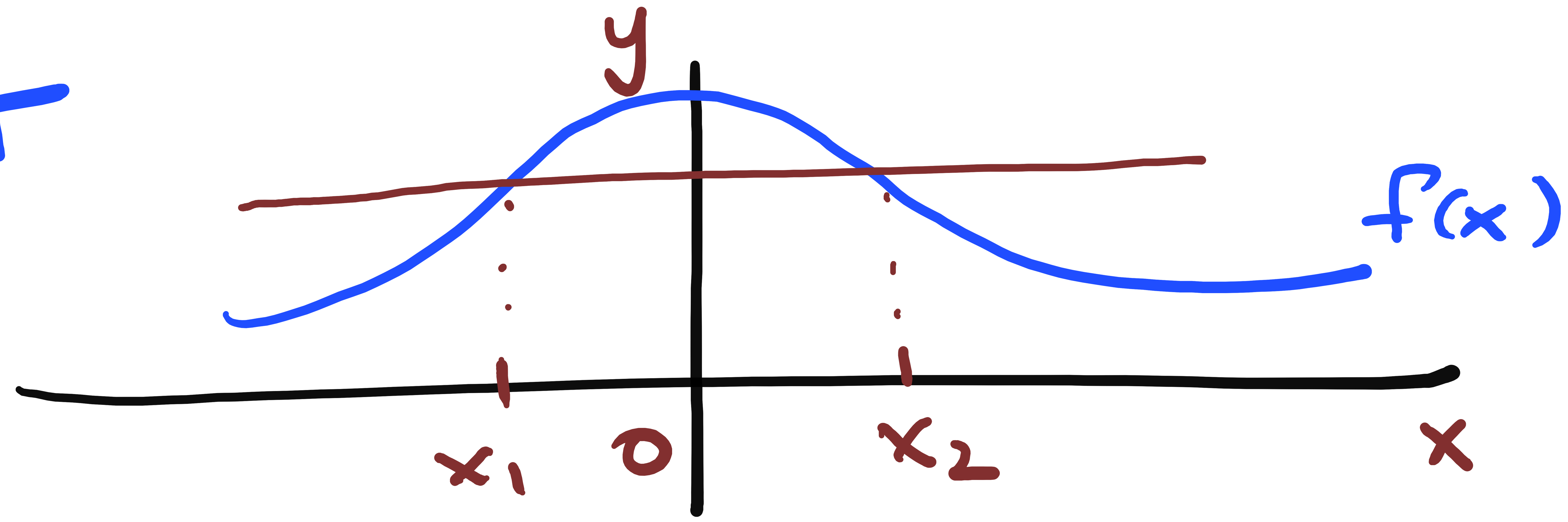
Def'n: A function is called <u>one-to-one</u> if it never takes the same value more than once.

i.e., $f(x_1) \neq f(x_2)$ if $x_1 \neq x_2$.

Horizontal TEST

$f(x_1) = f(x_2)$

and $x_1 \neq x_2$



This f is NOT one-to-one.

A function is one-to-one if no horizontal line intersects the graph at more than one point.

Only one-to-one functions have inverses.

If a function is increasing on its domain or is decreasing on its domain, it is one-to-one.