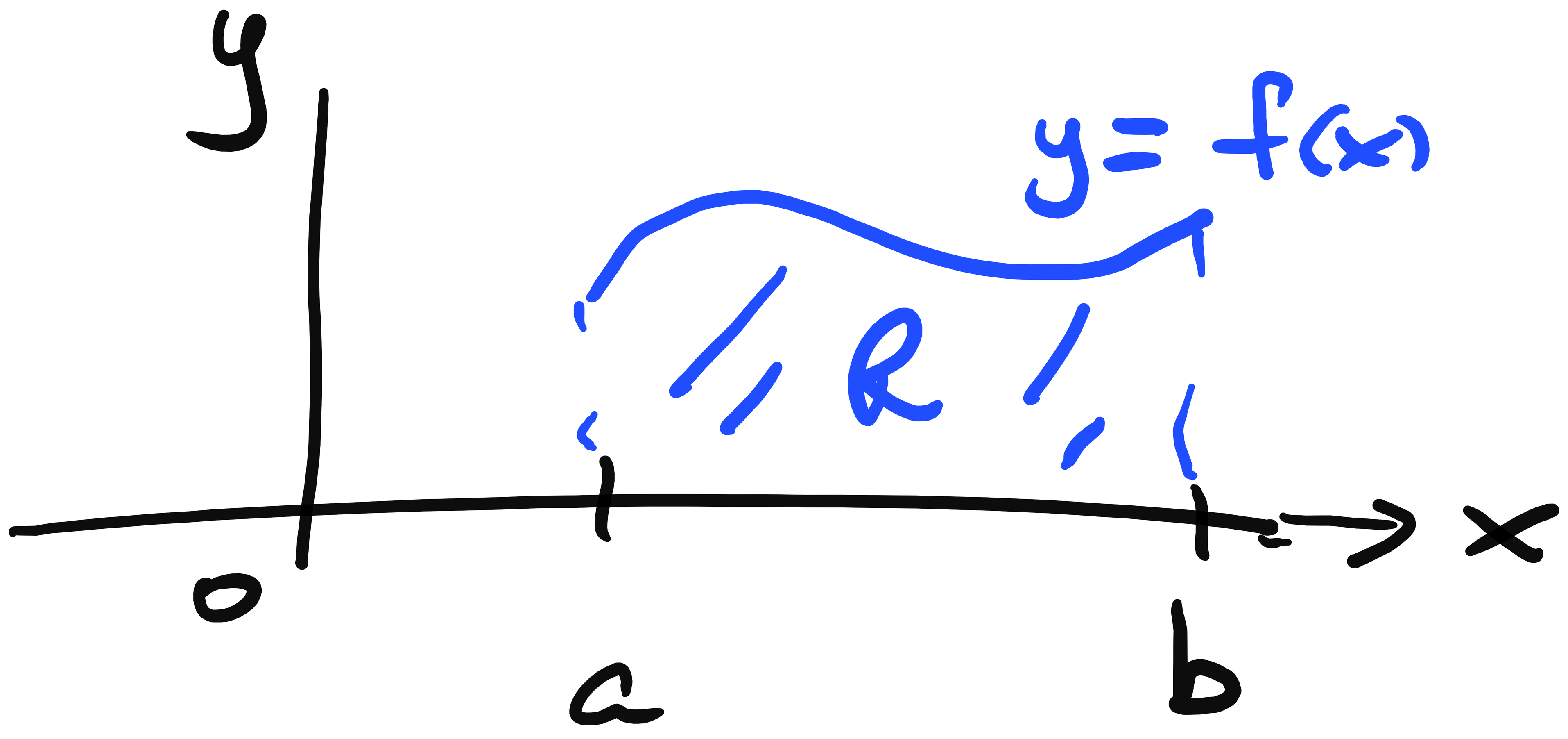
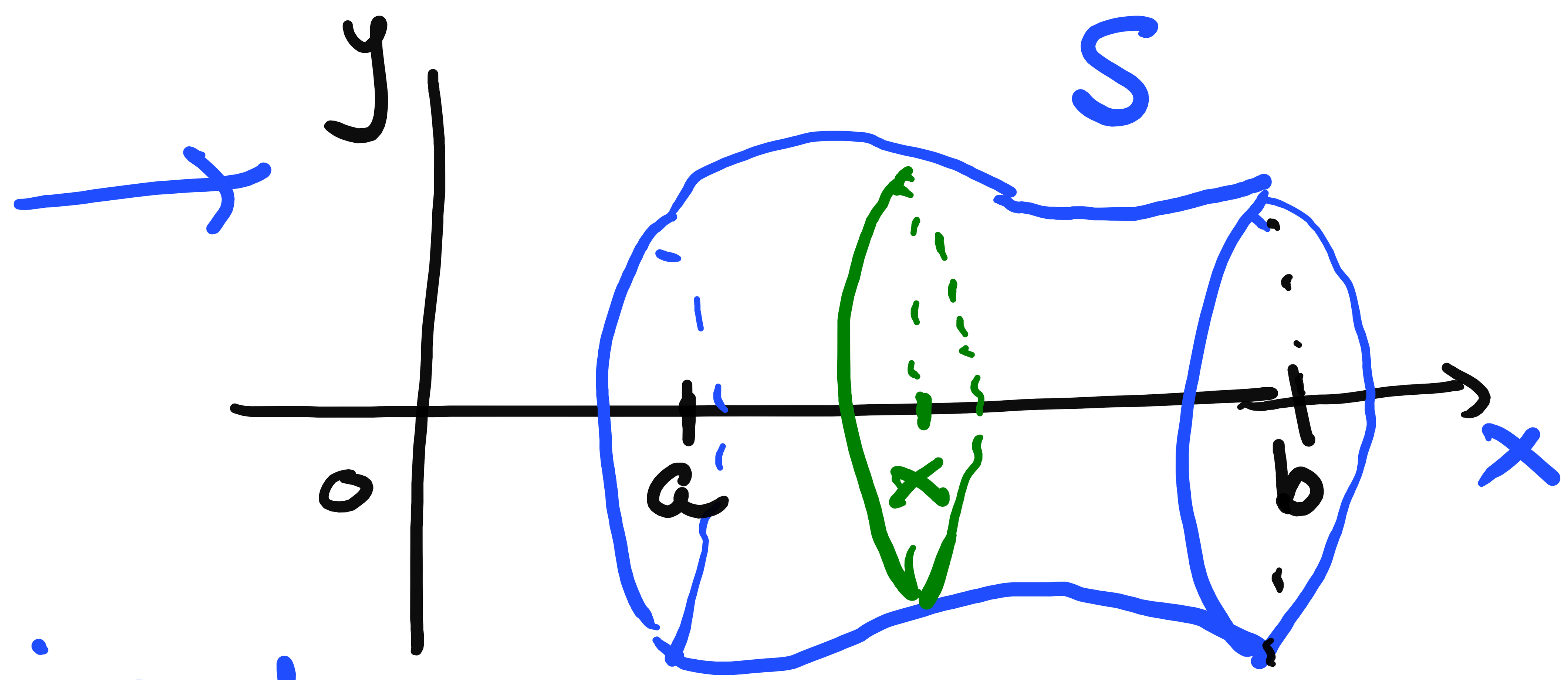
§6.2 cont'd Volume of a Solid of REVOLUTION.





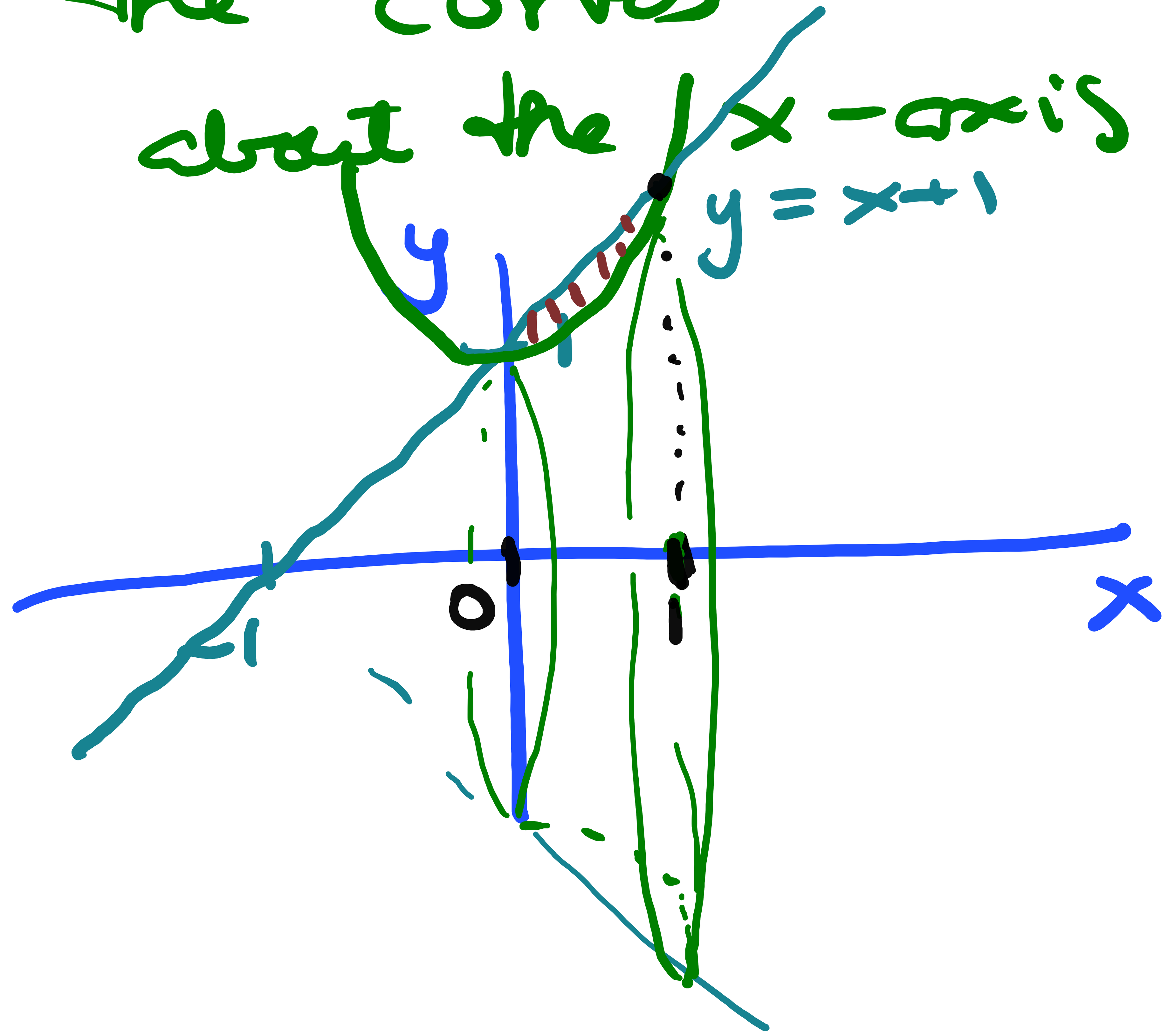
Let S be solid obtained by rotating the region R between the curve $y=f(x)$ and the x-axis from a to b about the x-axis.

Cross-section through each x perpendicular to the x-axis is a circular disc with radius $|y| = |f(x)|$ with cross-sectional area: $A = \pi (f(x))^2$.

$\therefore$ Volume of revolution of such a solid is:

$$V = \int_a^b \underbrace{\pi (f(x))^2}_{\text{area of moving cross-section}} dx .$$

<u>Example</u> Find the volume of the solid of revolution obtained by rotating the region R bounded by the curves $y = x^2+1$ & $y = x+1$ about the x-axis.



<u>Sol'n</u>. Draw graph.

Find where the curves intersect:

$$x+1 = x^2+1$$

$$x^2 = x$$

$$\underline{x=0} \text{ or } \underline{x=1}.$$

Method 1. V_1 is the volume of the solid obtained by rotating the region under $y=x+1$ (outter radius)

V_2 is the volume of the solid obtained by rotating the region under $y=x^2+1$ (inner radius).

Volume of the required ion

$$V = \underset{\text{outter}}{V_1} - \underset{\text{inner}}{V_2}$$

$$= \int_0^1 \pi \underbrace{(x+1)}_{\text{outter radius}}{}^2 \, dx - \int_0^1 \pi (\underbrace{x^2+1}_{\text{inner radius}})^2 \, dx$$

Method 2 A cross-section through x is an annulus (washer) like solid with inner radius x^2+1 and outter radius $x+1$.

$\therefore$ cross-sectional area is

$$A(x) = \pi \underbrace{(x+1)}_{\text{outter radius}}^2 - \pi \underbrace{(x^2+1)}_{\text{inner radius}}^2$$

$$\therefore V = \int_0^1 A(x)\,dx = \int_0^1 \pi(x+1)^2 - \pi(x^2+1)^2 \, dx.$$

__Either way:__ $V = \frac{7\pi}{15}.$

In general $V = \int_a^b \pi \left|(f(x))^2 - (g(x))^2\right| dx.$

and volume is ALWAYS __POSITIVE__ .

§6.4 __WORK__.

W work.
d distance
F force
a accelration
m mass.

gravitational constant

$g = 9.8\ m/sec^2.$

m metres.

s(t) displacement.
v(t) velocity
a(t) accelration.

Newton's 2nd Law of Motion $\frac{d^2 s(t)}{dt^2} = a(t) = \frac{dv(t)}{dt}$

$F = m \cdot a$

(mass × acceleration).

If acceleration is constant.

$W = F \cdot d.$ (force × distance).

Different unit systems.

SI metric.

m	mass	kg.	(kilograms)
d	distance	m	(meters)
t	time	s	(seconds)
F	force	N	(newtons)
			$N = \frac{kg \cdot m}{sec^2}$.
W	work	J	(Joules)
			J = newton-meter

American.

weight	lbs. (pounds)
distance	ft. (feet).

Note: Weight is a FORCE.

W = weight × distance

unit ft-lbs.

1 ft-lb ≈ 1.36 J

Example 1.

(a) How much work is done to lift a 2 kg. book off the floor onto a desk. 0.8 m high?

Sol'n.

$$F = m \cdot a = m \cdot g$$

gravitational constant. $a = g$

$$= (2\,kg)(9.8)\,m/sec^2 = 19.6\ N$$

$$W = F \cdot d$$

$$= (19.6\,N) \cdot (0.8\,m) = \underline{15.68\ J}.$$

(b). How much work is done lifting a 15 lb weight 4 ft off the ground.

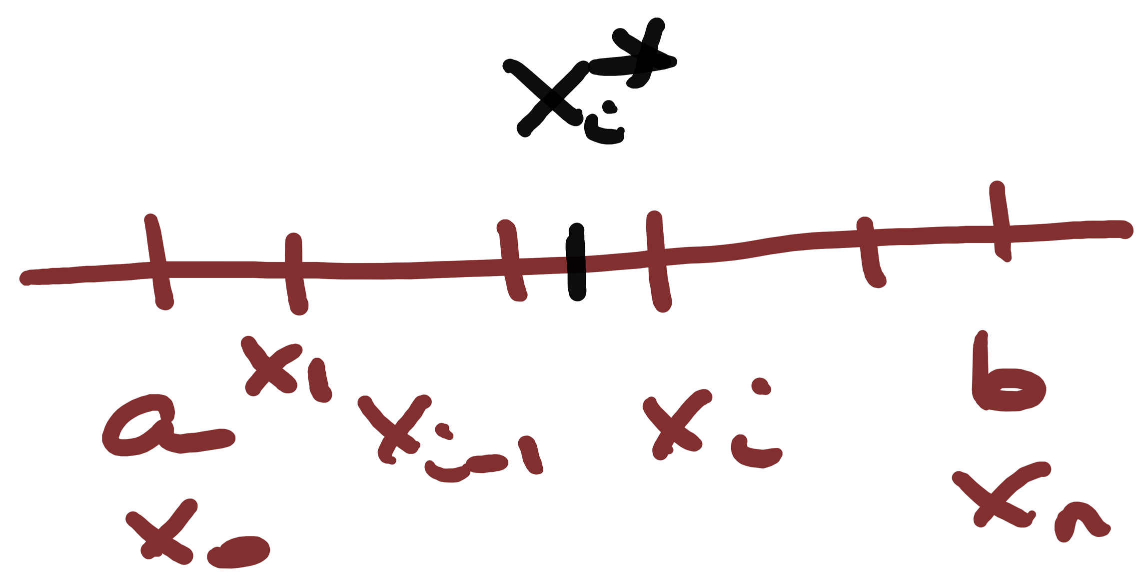
F = weight = 15 lb.

$$W = F \cdot d = (15\,lb)(4\,ft) = \underline{60\ ft\text{-}lb}.$$

<u>But</u> $W = F \cdot d$ assumes force is constant throughout the entire distance.

What if the force changes.

To move an object from x to a the force $f(x)$ acts on the object at each point x.



$f(x)$ force at point x.

(Assume $f(x)$ is <u>continuous</u> function)

Partition $[a,b]$ into n equal subintervals.

Assume $\Delta x = x_i - x_{i-1}$.
(equal partition)

$x_i^* \in [x_{i-1}, x_i]$.

$f(x_i^*)$ is the force acting on x_i^*.

Approximation of the work done to move the object in the section from $[x_{i-1}, x_i]$ is

$$W_i \approx f(x_i^*)\,\Delta x.$$ Assuming $f(x_i^*)$ is a good approx. of the force at all points in $[x_{i-1}, x_i]$

Approx work to move from a to b is

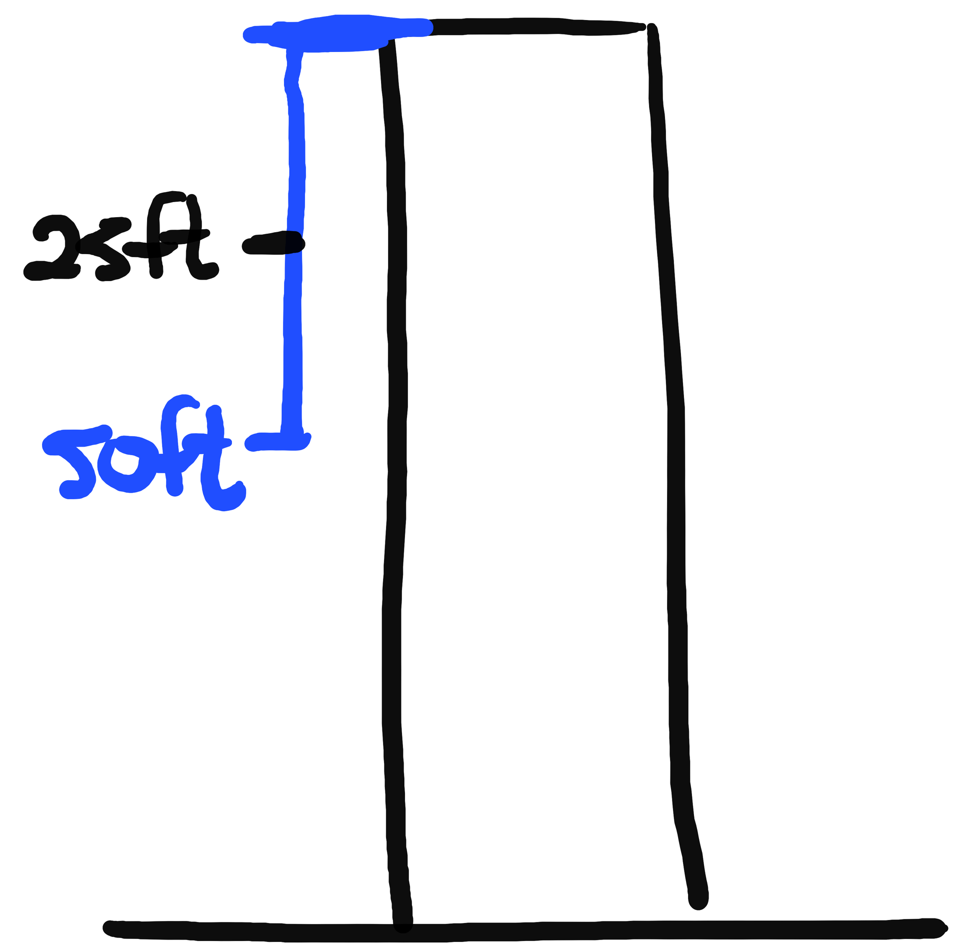
$$W_{approx} \approx \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

(force)(distance)

letting $n \to \infty$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \int_a^b f(x)\,dx.$$

Example A 150 lb cable is 50 ft long. It hangs vertically from the top of a tall building. How much work is required to lift half of the cable to the top of the building.



Sol^n: Cable weighs $\frac{150 \text{ lb}}{50 \text{ ft}} = 3 \text{ lb/ft}$.

Divide the cable into n equal parts of length Δx.

The weight of the i^{th} part (from x_{i-1} to x_i) is $3\Delta x$.

Approximation of work required to lift only the i^{th} section $[x_{i-1}, x_i]$ if Δx is "small" enough can be approximated by assuming the entire portion from x_{i-1} to x_i is a distance x_i^* from the top where $x_i^* \in [x_{i-1}, x_i]$.

$$W_i \approx F \cdot d$$
$$= \text{weight} \cdot d.$$
$$= (3\Delta x) \cdot x_i^*$$

Total work required to lift the top ½ section of the cable, i.e the top 25 ft is:

$$W = \lim_{n \to \infty} \sum_{i=1}^{n} 3x_i^* \Delta x = \int_0^{25} 3x \, dx = \frac{3x^2}{2}\Bigg|_{x=0}^{25} \text{ ft lb.}$$

But still need to consider the work needed.

to lift the rest of the cable 25 ft.
(i.e., the bottom half of the cable.

TO BE CONTINUED NEXT LECTURE.