§4.5 Curve Sketching cont'd.

$$f(x) = \frac{x+1}{\sqrt{x^2+1}} = y$$

(a) $\mathrm{dom}(f) = \mathbb{R}$

(b) y-intercept: $(x,y) = (0,1)$

x-intercept $(x,y) = (-1,0)$

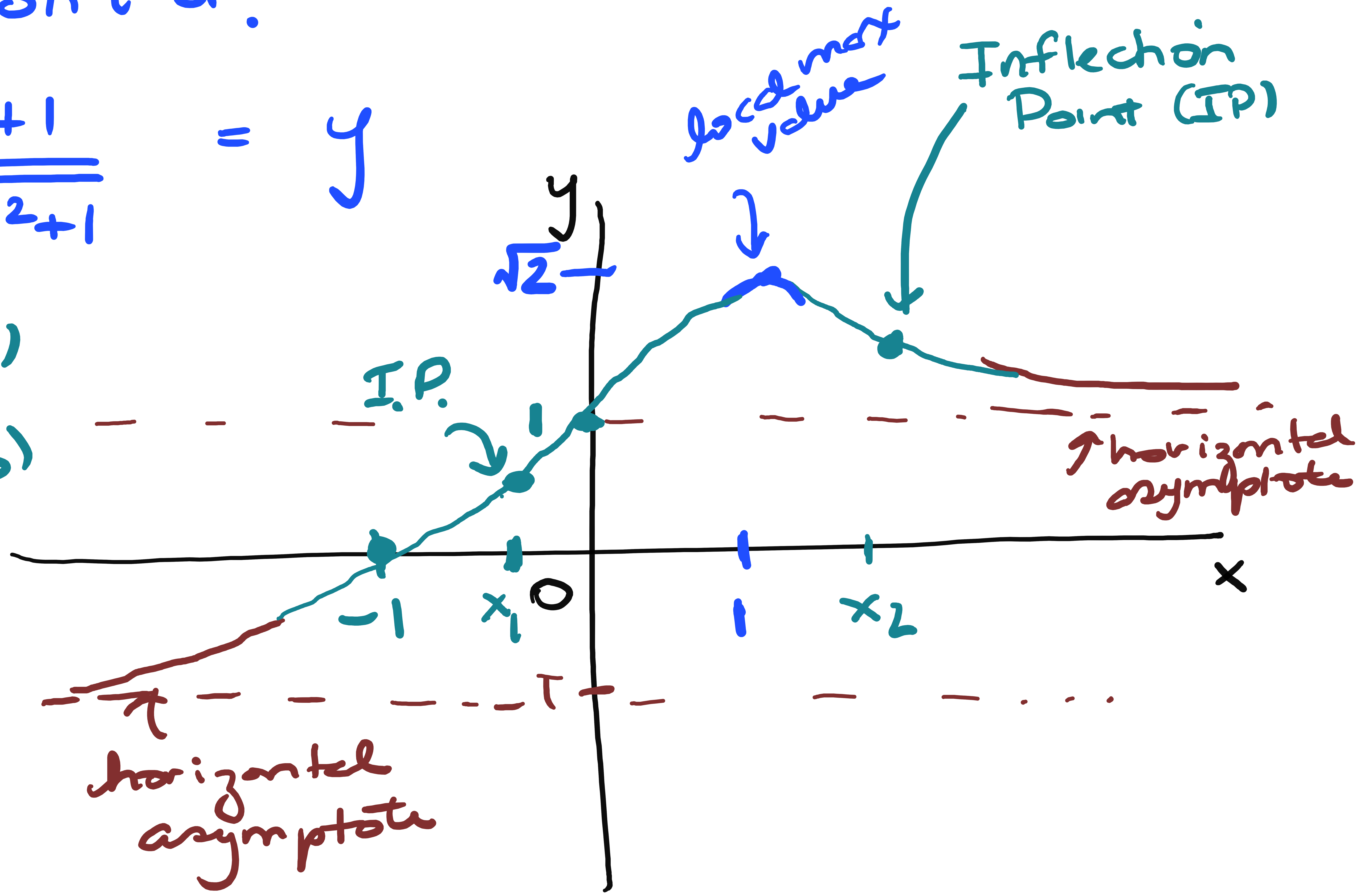
(c) no symmetry.

(d) asymptotes:

no vertical asym.

$$\lim_{x\to\infty} f(x) = 1$$

$$\lim_{x\to-\infty} f(x) = -1$$



(e) intervals where $f(x)\uparrow$ and $\downarrow$ $\left(f'(x) = \frac{1-x}{(x^2+1)^{3/2}}\right)$.

$f'(1) = 0$

$f'(x) > 0$ if $x < 1$, $f(x)\uparrow$

$f'(x) < 0$ if $x > 1$, $f(x)\downarrow$

(f) local min's + maxes.

$x=1$ was my candidate $\quad f(1) = \frac{2}{\sqrt{2}} = \sqrt{2}$.

By 1st deriv test $f(1)=\sqrt{2}$ is a local maximum value at $x=1$.

(g). Concavity & Inflection Pts : $f''(x) = \dfrac{2x^2 - 3x - 1}{(x^2+1)^{5/2}}$

- denominator always ⊕.
- numerator is quadratic.

f'': $\quad + \quad \big| \quad - \quad \big| \quad +$

$\frac{3-\sqrt{17}}{4} = x_1 \qquad \frac{3+\sqrt{17}}{4} = x_2$

x_1, x_2 inflection points (roots of the quadratic $\Rightarrow f''(x_i)=0$, $i=1,2$.

Concavity

$\left(-\infty, \frac{3-\sqrt{17}}{4}\right)$ $\cup$

$\left(\frac{3-\sqrt{17}}{4}, \frac{3+\sqrt{17}}{4}\right)$ $\cap$

$\left(\frac{3+\sqrt{17}}{4}, \infty\right)$ $\cup$

$-1 \qquad 0 \qquad 1$

$\frac{3-\sqrt{17}}{4} \overset{?}{\approx} -0.281 \qquad \frac{3+\sqrt{17}}{4} \overset{?}{\approx} 1.781$

Example: Sketch graph: $f(x) = \dfrac{x^2}{1-x^2}$.

(a) $\operatorname{dom}(f) = \{x : x \neq 1, x \neq -1\}$.

$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

discontinuities of f at $x=-1$ & $x=1$.
where f NOT defined.

(b) intercepts: $(0,0)$ is the only intercept.
(both x-intercept & y-intercept)

(c) symmetry. (not periodic).

$$f(-x) = \frac{(-x)^2}{1-(-x)^2} = \frac{x^2}{1-x^2} = f(x)$$

$\therefore$ $f(x)$ is <u>EVEN</u>.

$\therefore$ symmetry about the y-axis.

(d) asymptotes: (Since f EVEN only need to determine graph for $x \geq 0$. Symmetry tells what graph looks like for $x < 0$.

Horizontal asymptote:

$$\lim_{x \to \infty} \frac{x^2}{1-x^2} = -1$$

$\therefore$ the line $y=-1$ is a horizontal asymptote as $x \to \infty$.

By symmetry, since f is EVEN, $y=-1$ is also a horizontal asymptote as $x \to -\infty$.

$\left(\text{i.e. } \lim_{x\to-\infty} \frac{x^2}{1-x^2} = -1 \right)$

VERTICAL asymptotes: (at discontinuities $x = 1, -1$.)

$$\left.\begin{array}{l} \lim_{x\to 1^+} \frac{x^2}{1-x^2} = -\infty. \\ \lim_{x\to 1^-} \frac{x^2}{1-x^2} = +\infty \end{array}\right\} \text{The vertical line } x=1 \text{ is a vertical asymptote.}$$

By symmetry, since f is EVEN, we know $x=-1$ is also a vertical asymptote.

(e) Intervals where $f(x) \uparrow$ or $\downarrow$:

$$f'(x) = \frac{2x}{(1-x^2)^2}$$

(DNE at $x=1$, $x=-1$).

$f'(x) = 0$ iff $x = 0$.

(Critical #s, are $0, +1, -1$).

Interval of $\uparrow$ & $\downarrow$.

would know by symmetry

	x	$(1-x^2)^2$	f'	f $\uparrow$ OR $\downarrow$
$(-\infty, -1)$	$-$	$+$	$-$	$\downarrow$
$(-1, 0)$	$-$	$+$	$-$	$\downarrow$
$(0, 1)$	$+$	$+$	$+$	$\uparrow$
$(1, \infty)$.	$+$	$+$	$+$	$\uparrow$

(f) local mins + maxes.

$f'(0)=0$ iff $x=0$, $f(0)=0$

$f(1), f(-1)$ not defined.

$f'(1), f'(-1)$ " "

1st derivative test:

$\Rightarrow$ $f(0)=0$ is a local min value at $x=0$.

(g) Concavity + inflection points (I.P)

$$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$$

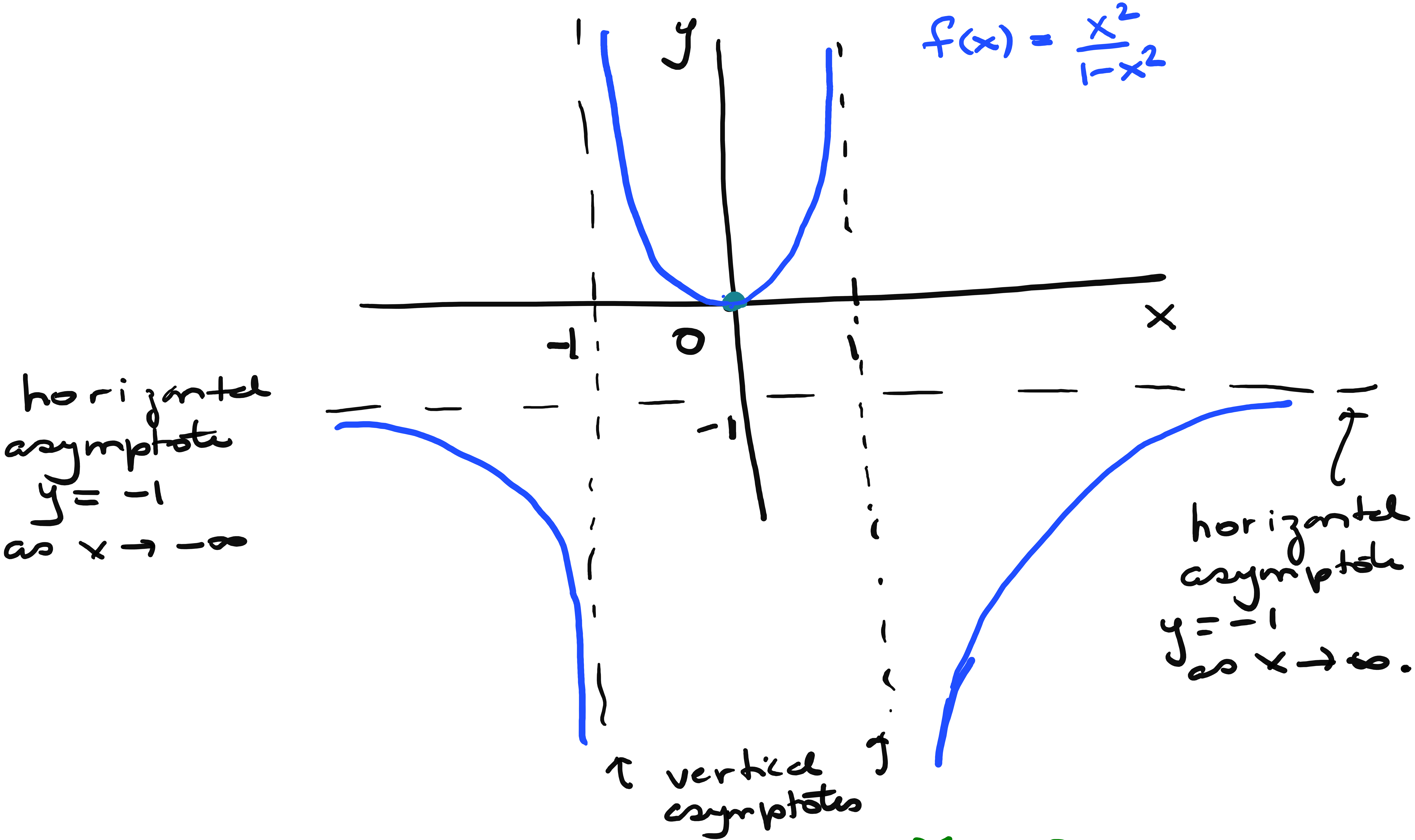
$f''(x)=0$

$\Longleftrightarrow 1+3x^2=0.$

NEVER.

$f''(x)$: $\cap$ $-$ | -1 | $\cup$ $+$ | 1 | $\cap$ $-$ x

Concavity change $\leftarrow$ Concavity can change where function or its derivative(s) not defined.



Example: Sketch graph: $y = xe^x = f(x)$.

(a) $\text{dom}(f) = \mathbb{R}$.

(b) intercepts: $(0,0)$ is the only intercept.

(c) no symmetry.

(d) $\lim_{x\to\infty} x e^{x} = \infty$. (No horizontal asymptote as $x \to \infty$. Instead $f(x) \to \infty$ as $x \to \infty$)

$\lim_{x\to-\infty} x e^{x}$, indeterminate form "$(-\infty)\cdot 0$"

$$= \lim_{x\to-\infty} \frac{x}{e^{-x}} \qquad \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\underset{\text{l'HR}}{=} \lim_{x\to-\infty} \frac{1}{-e^{-x}} = 0.$$

(gives $y=0$ is a horizontal asymptote as $x \to -\infty$.)

Function is continuous for all $x \in \mathbb{R}$
$\therefore$ no vertical asymptotes.

(e) Intervals where $f(x)\uparrow$ or $\downarrow$.

$$f'(x) = e^{x}(1+x).$$

$f'(x) = 0$ iff $1+x=0$, i.e., $x=-1$.

$f'(x) > 0$ if $x > -1$, $f\uparrow$ on $(-1,\infty)$

$ < 0$ if $x < -1$, $f\downarrow$ on $(-\infty,-1)$.

(f) local max or mins.

$f'(-1)=0$ (1st deriv test says $f(-1)$ is a local min).

$f(-1) = -e^{-1}$ is local min value.

(g) Concavity and I.P.'s.

$f''(x) = e^x(2+x)$ ($f''(x)=0$ iff $x=-2$, $f''(x)$ defined on $\mathbb{R}$).

$f''(x) < 0$ if $x < -2$

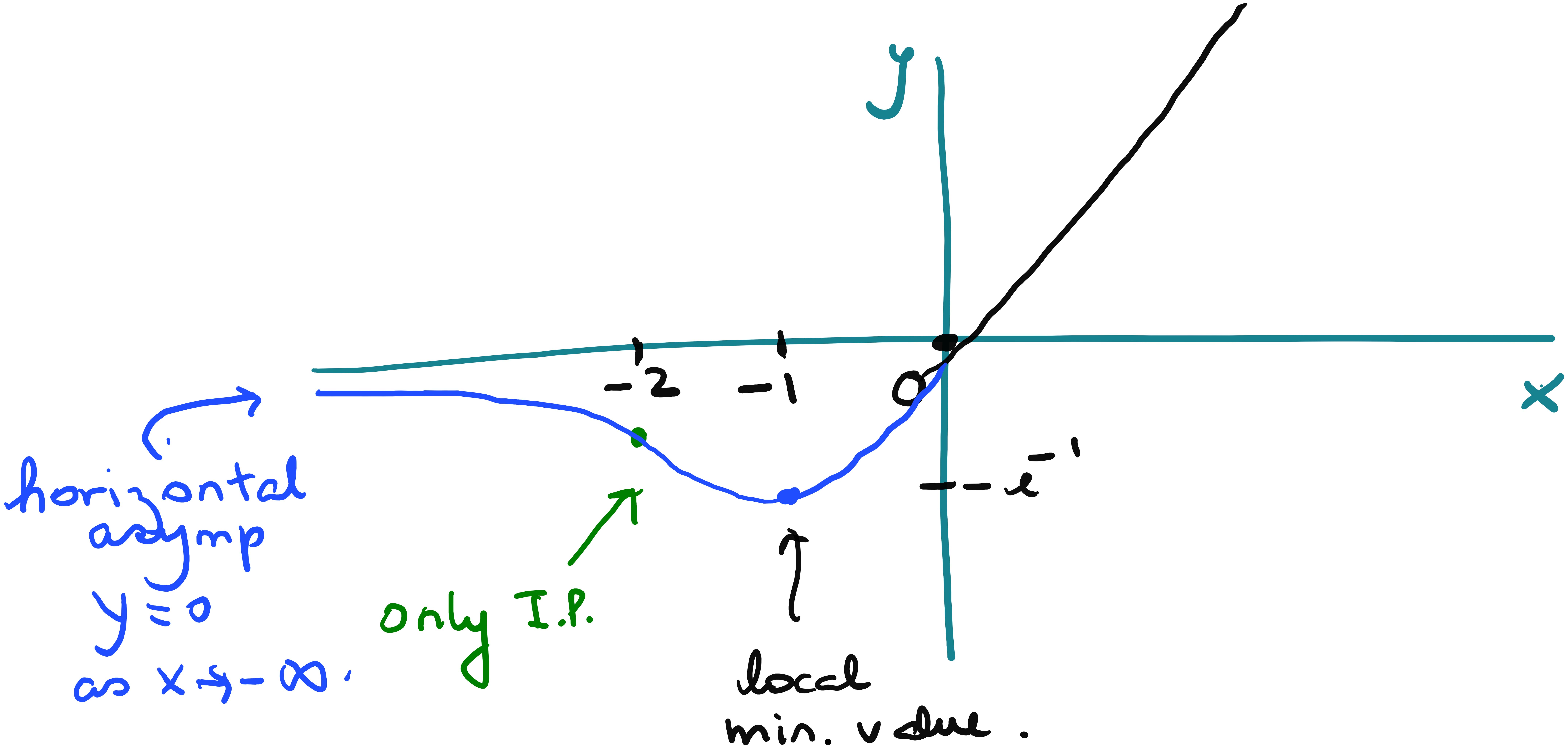
> 0 if $x > -2$.

$\therefore$ f is concave down, $\cap$, on $(-\infty, -2)$

up, $\cup$, on $(-2, \infty)$

At $x=-2$ we have an inflection point.

We put all this information together to obtain the graph:


y
x
-2
-1
0
$-e^{-1}$
horizontal asymp
$y=0$
as $x \to -\infty$.
only I.P.
local
min. value.