Test 1 REVIEW SESSIONS

Seating #1
Day: Fri. Oct. 4
Room: ITB 137
Time: 3:30-5:20

Seating #2
Day: Mon. Oct. 7
Room: PGCLL B138
Time: 4:30-6:20pm

Seating #3
Day: Mon. Oct. 7
Room: JHE 264
Time: 8:30-10:20pm

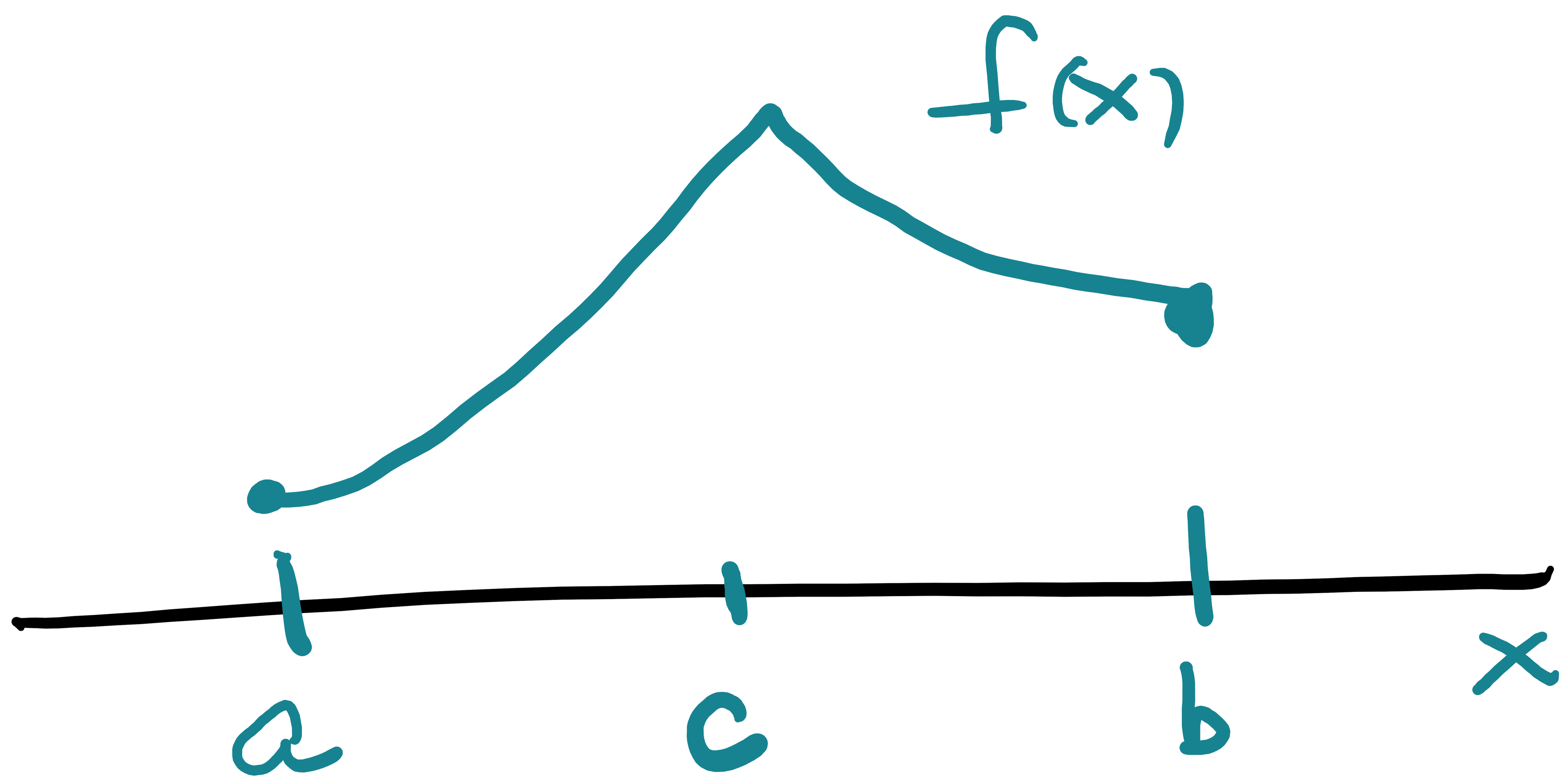
Test #1 Review Session Info ↑

§4.1 cont'd.

Thm (Extreme Value Theorem)

If f is a continuous function on a closed interval $[a,b]$, then f attains both its absolute maximum value $f(c)$, and its absolute minimum value $f(d)$, at some numbers $c, d \in [a,b]$.

Note: f does NOT have to be differentiable.



in theorem → d   → c

$f(c)$ is the absolute maximum value. attained at $x=c$

$f(a)$ is the absolute minimum value attained at $x=a$.

$f(c)$ is an absolute max value and a local max. value.
$f(a)$ is an absolute min value, but NOT a local min value.

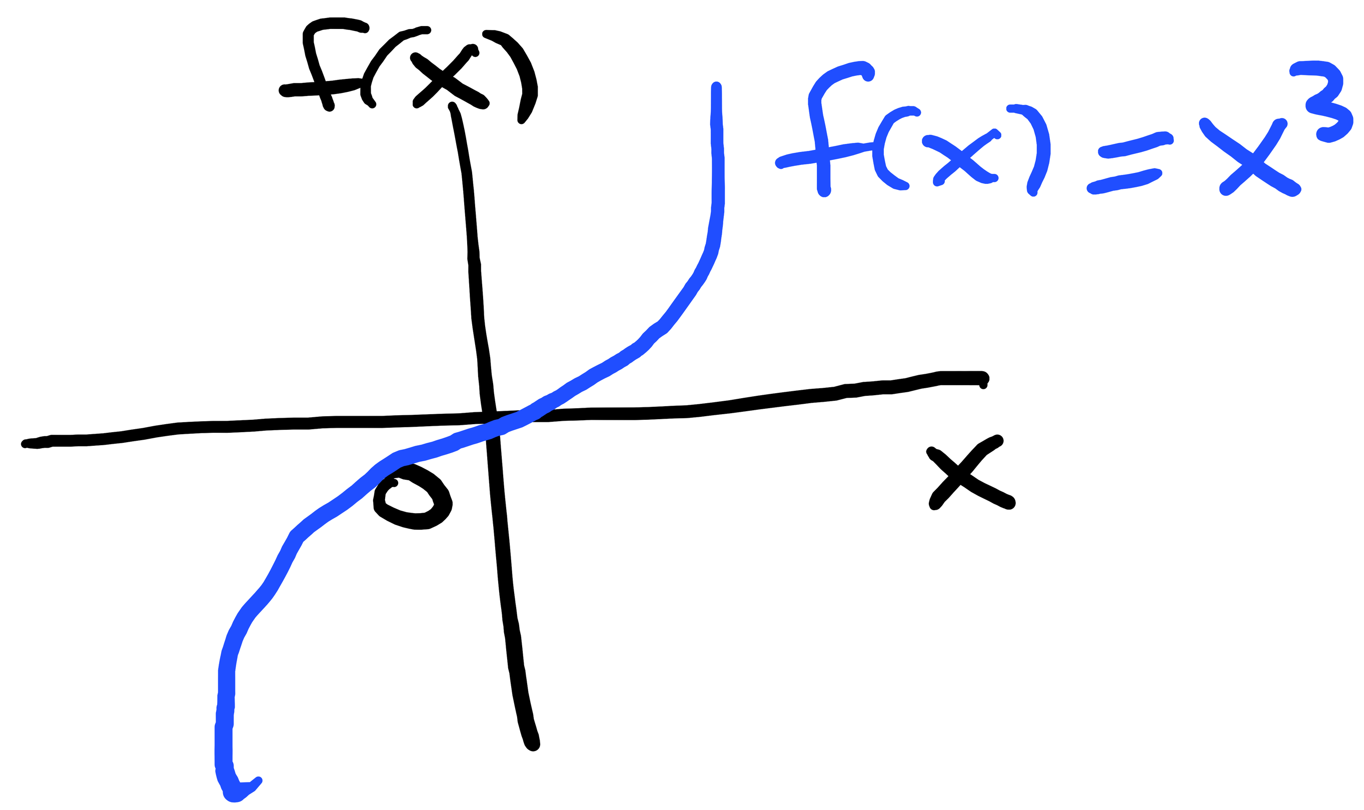
Thm (Fermat)

If f has a local maximum value or a local minimum value at c and $f'(c)$ exists, then $f'(c)=0$.

BEWARE:

1. If $f'(c)=0$, $f(c)$ does not have to be a local min value or a local maximum value.

E.g. $f(x)=x^3$. $f'(x)=3x^2$
$f'(0)=0$.

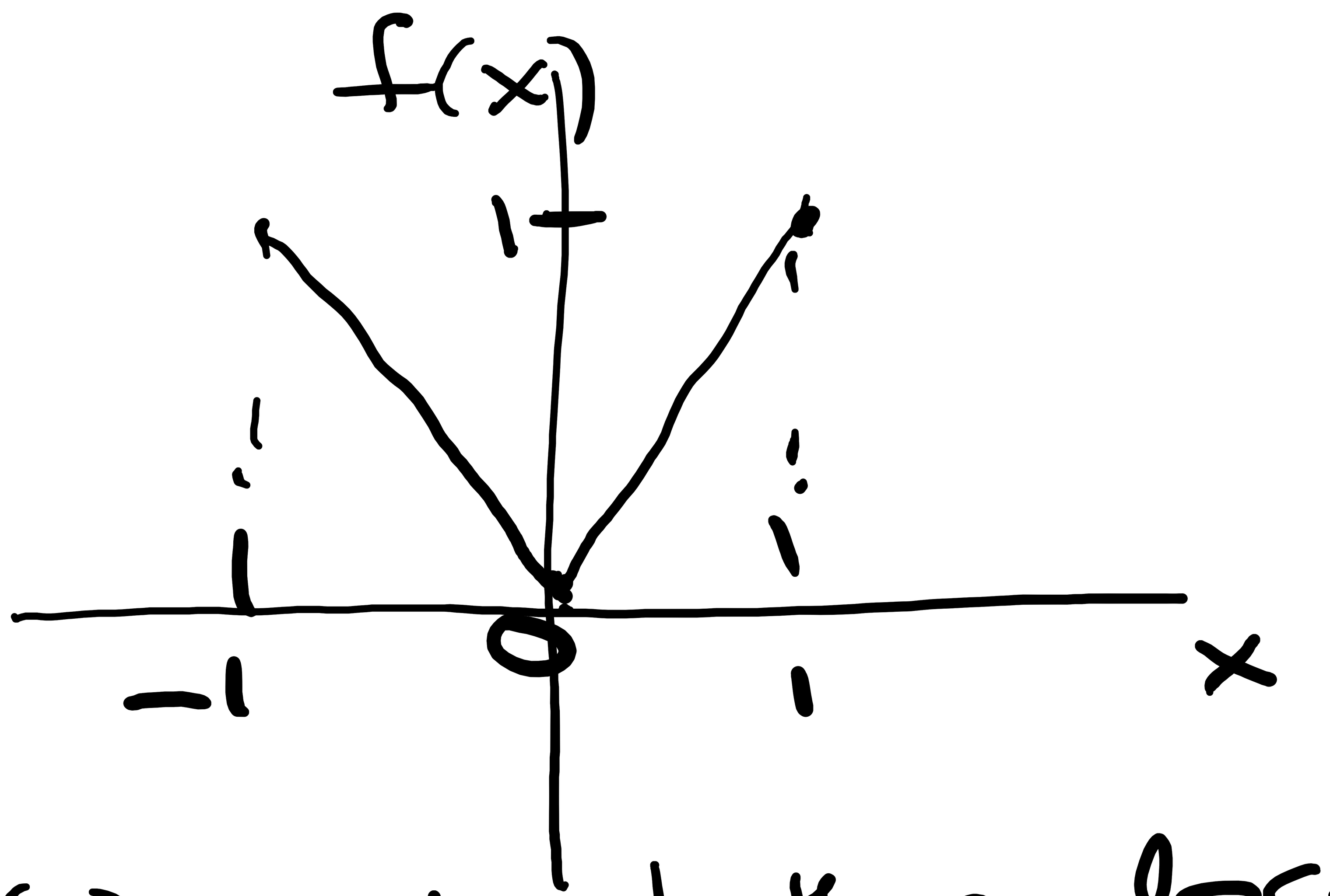


$f(0)=0$, is NOT a local maximum or a local minimum value.

② An extreme value (i.e. abs. max value OR abs. min value)

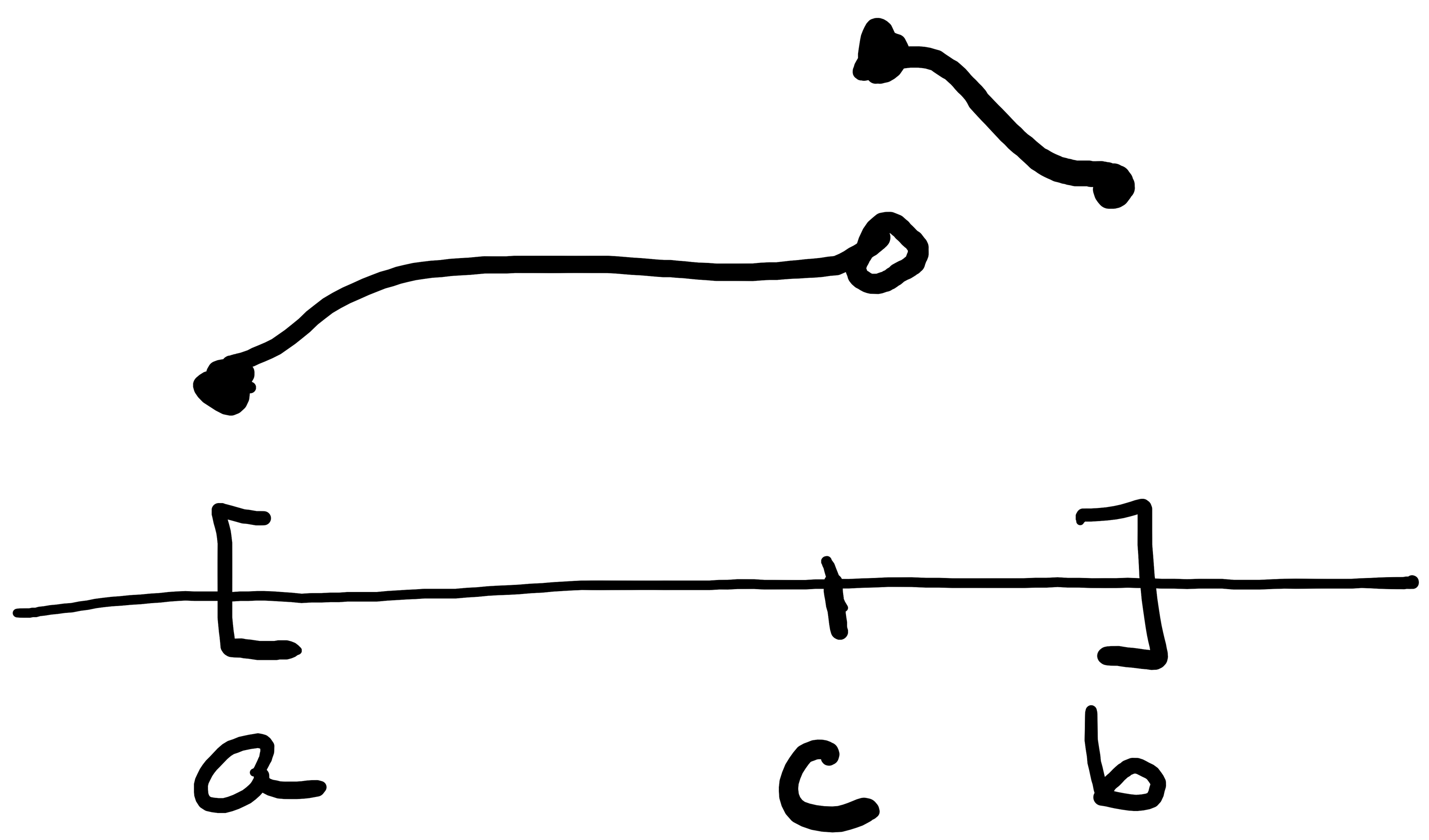
CAN exist at c, when $f'(c)$ DNE.

E.g. 1 $f(x) = |x|$, $x \in [-1, 1]$



$f(0) = 0$ is both a local & an absolute min value of f, but $f'(0)$ DNE.

OR



$f(c)$ is an absolute max value of f on $[a, b]$, but $f'(c)$ DNE.

Def'n A CRITICAL NUMBER of a function f is a number c in the domain of f such that either (i) $f'(c) = 0$

OR (ii) $f'(c)$ DNE.

FERMAT'S Thm (rephrased)

If f has a local maximum or local minimum value at c, then c is a critical number.

TO FIND ABSOLUTE MAX and ABSOLUTE MIN

values of a CONTINUOUS function on an interval $[a, b]$.

1) Find the values of f at the CRITICAL NUMBERS of f on $[a, b]$.

2) Find the values of f at the END POINTS of the interval.

3) The LARGEST of these values is the ABSOLUTE MAX VAL.
" SMALLEST " " " " " ABSOLUTE MIN VAL.

BEWARE: If f is NOT continuous, must also consider how f is defined at jump discontinuities or if

$$\lim_{x \to z} f(x) = \pm\infty, \text{ somewhere in } [a,b].$$

Example Find the absolute max & min values of $f(x) = x^2 - 2x - 1$. on $[-3, 0]$

Sol'n f is continuous on $[a,b] = [-3,0]$.

By the Extreme Value Th^m, there is an absol. max val & an absol. min. val. of f on $[-3,0]$.

① Find critical numbers.

$f'(x) = 2x+2$ exists $\forall x \in [-3,0]$

$f'(x) = 0 \Rightarrow 2x+2 = 0 \Rightarrow x = -1$.

$-1 \in [-3, 0]$, is a critical number

$$f(-1) = 1 - 2 - 1 = -2$$

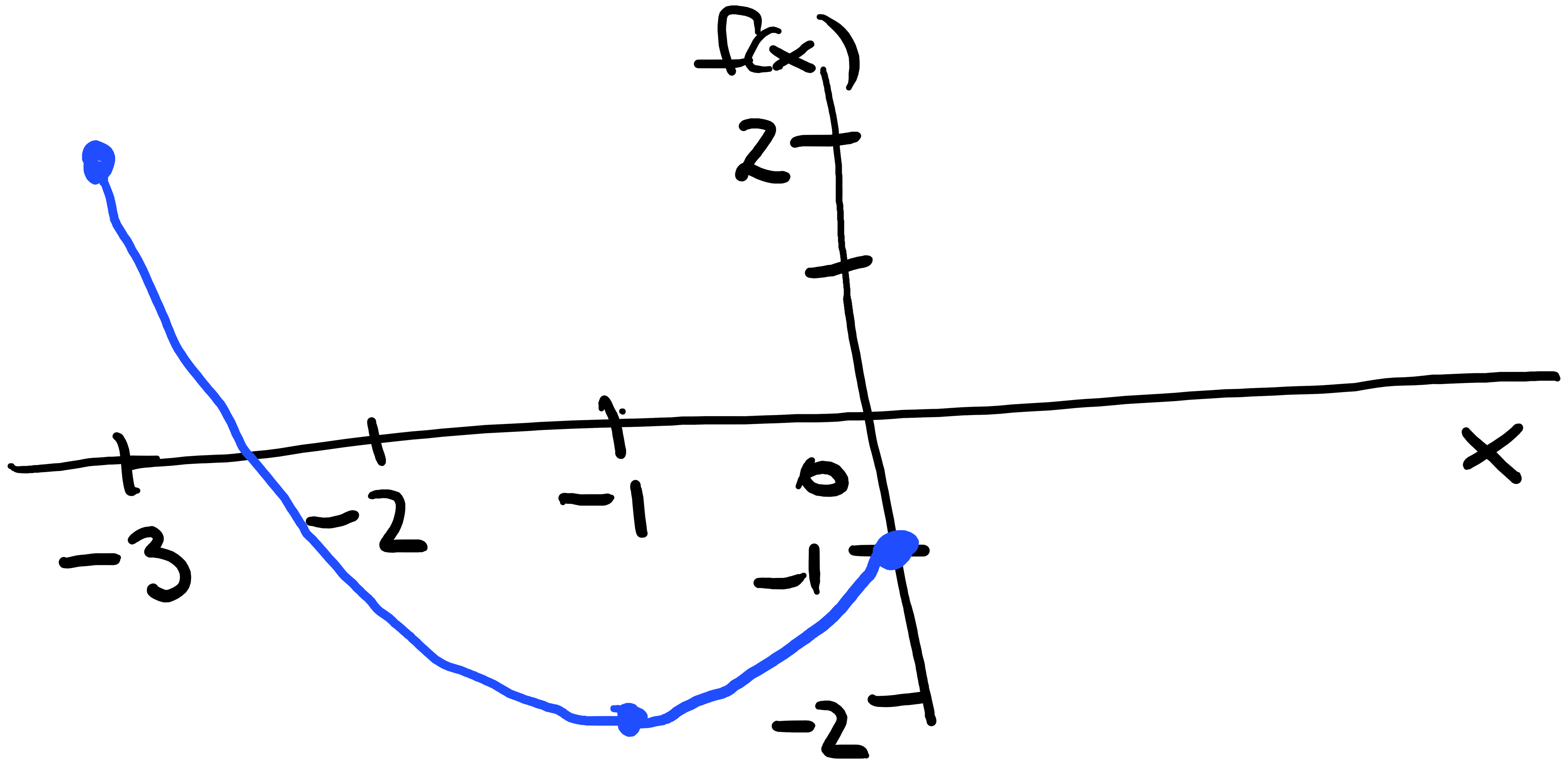
② End points: $x = -3, 0$.

$$\begin{cases} f(-3) = 9 - 6 - 1 = 2 \\ f(0) = 0 + 0 - 1 = -1. \end{cases}$$

$\therefore$ $f(-1) = -2$ is the abs. min. val. (at $x = -1$).

$f(-3) = 2$ is the abs. max. val. (at $x = -3$)

Graph.

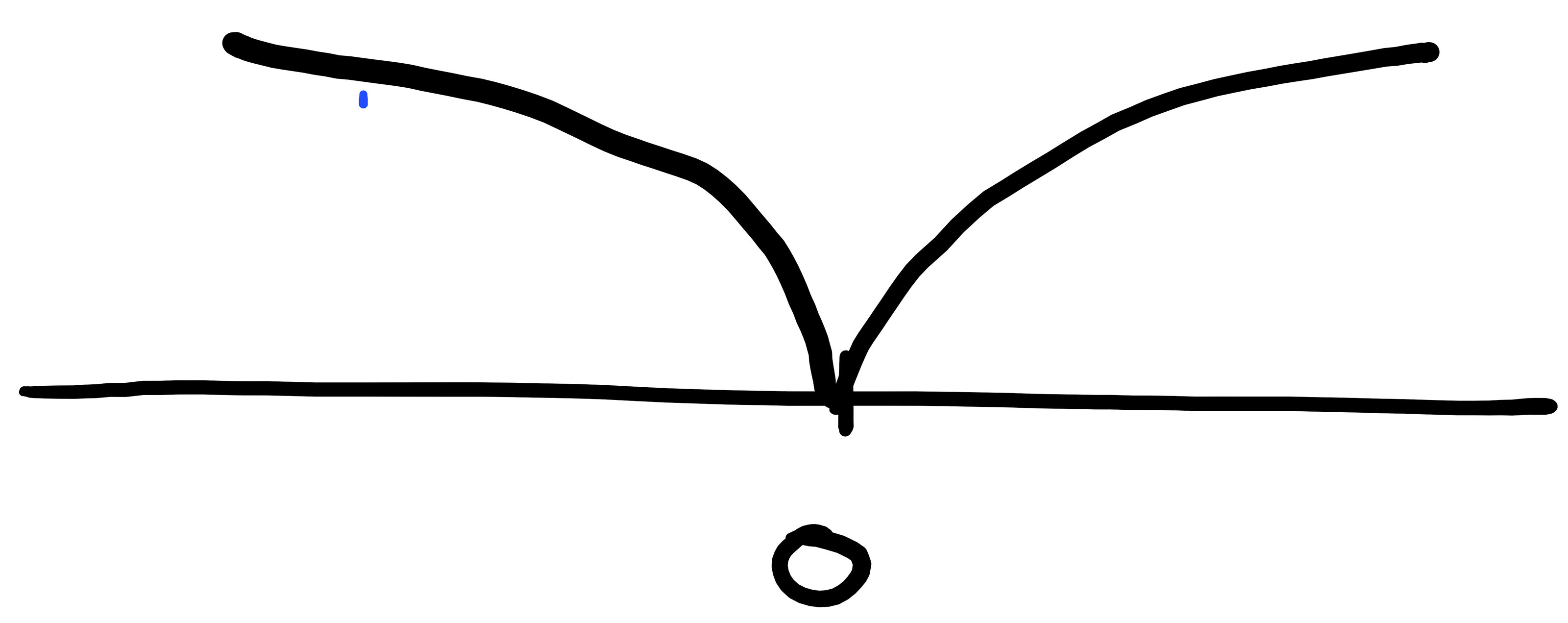


Example Find the abs max & min values of $h(x) = x^{2/3}$.

Sol'n: $h(x) = (x^2)^{1/3}$

domain $(-\infty, \infty)$
continuous on $(-\infty, \infty)$.

$h(x) = x^{2/3}$



To find the critical numbers:

$$h'(x) = \frac{2}{3} x^{-1/3}, \quad x \neq 0$$

$h'(x)$ DNE at $x = 0$.

$x = 0$ is the ONLY critical number

$h(0) = 0$. and "near" $h(0)$, $h(x) > h(0)$.

h has a local min value and an absolute min value of $h(0) = 0$ at $x = 0$.

$$\lim_{x\to\infty} x^{2/3} = \infty$$

$$\lim_{x\to-\infty} x^{2/3} = \infty.$$

No local or absolute max values on $(-\infty,\infty)$.

However, if considered
$h(x) = x^{2/3}$, $x \in [a,b]$ where a & b are finite
then by the Extreme Value Th^m,
since $h(x)$ is continuous on $[a,b]$,
(it is continuous for all $x \in (-\infty,\infty)$)
it would have both an absolute
max & absolute min value on $[a,b]$.

Next, § 4.2 Rolle's Th^m & the Mean Value Th^m.