§ 3.11 cont'd.

Show $\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$.

Pf (Done last day another way)

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\left(\frac{\sinh(x)}{\cosh(x)}\right)$$ quotient rule

$$= \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)}$$

$$= \frac{1}{\cosh^2(x)}$$ using hyperbolic identity $\cosh^2(x) - \sinh^2(x) = 1$.

$$= \operatorname{sech}^2(x)$$

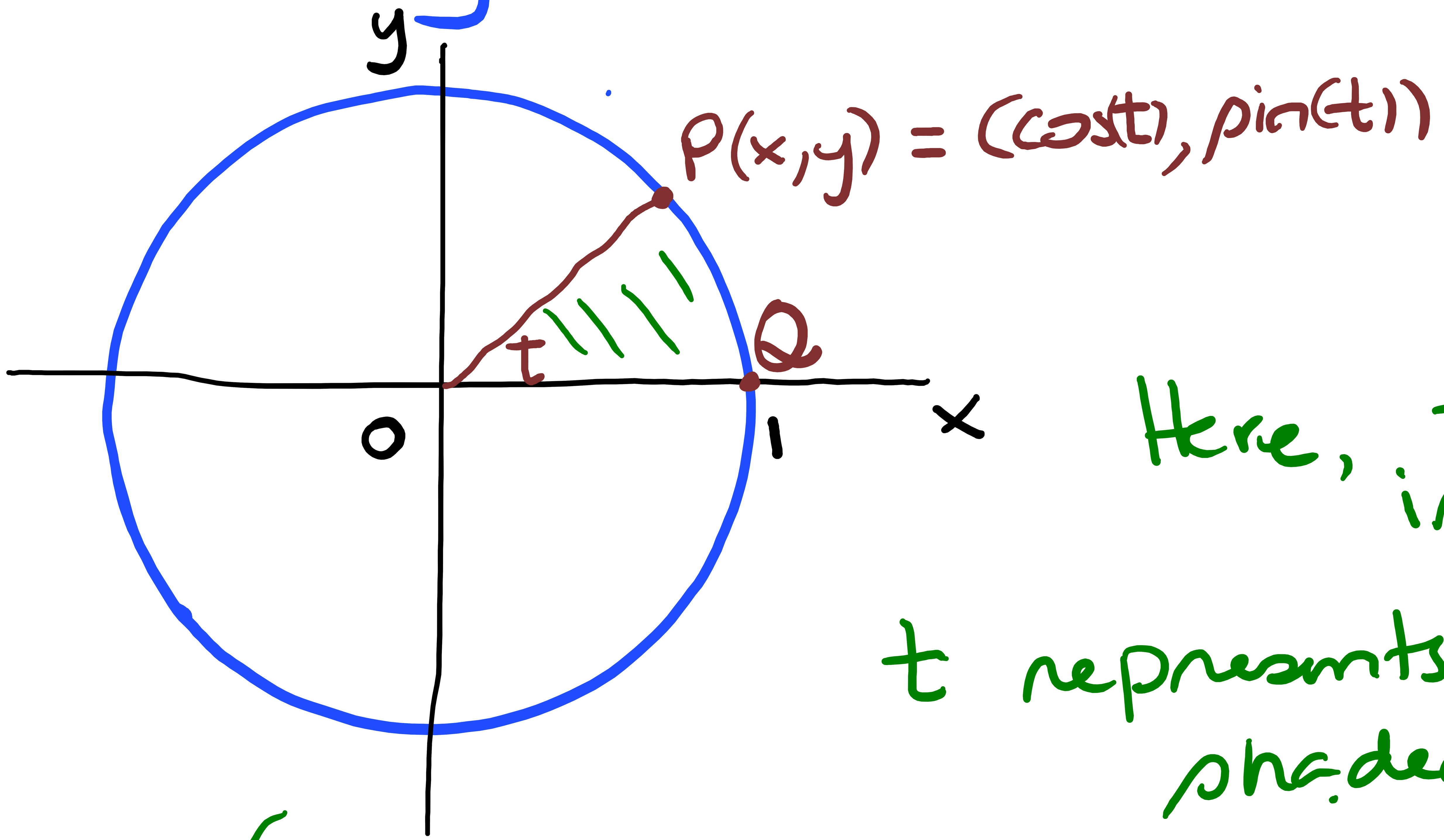
$\not\!\!\!\!\!\times$

Reason called HYPERBOLIC functions

RECALL: $\cosh^2(x) - \sinh^2(x) = 1.$

Let $t \in \mathbb{R}$, i.e. t is any real number.

Trig functions: $x^2 + y^2 = 1$, eq'n of the unit circle.



$\cos^2(t) + \sin^2(t) = 1$

$x = \cos(t)$, $y = \sin(t)$

Here, t is the angle ($\angle POQ$) in radians.

t represents <u>twice</u> the area of the shaded region.

(Recall: If $t = 2\pi$, we get the area of a circle: πr^2 (here $r=1$).

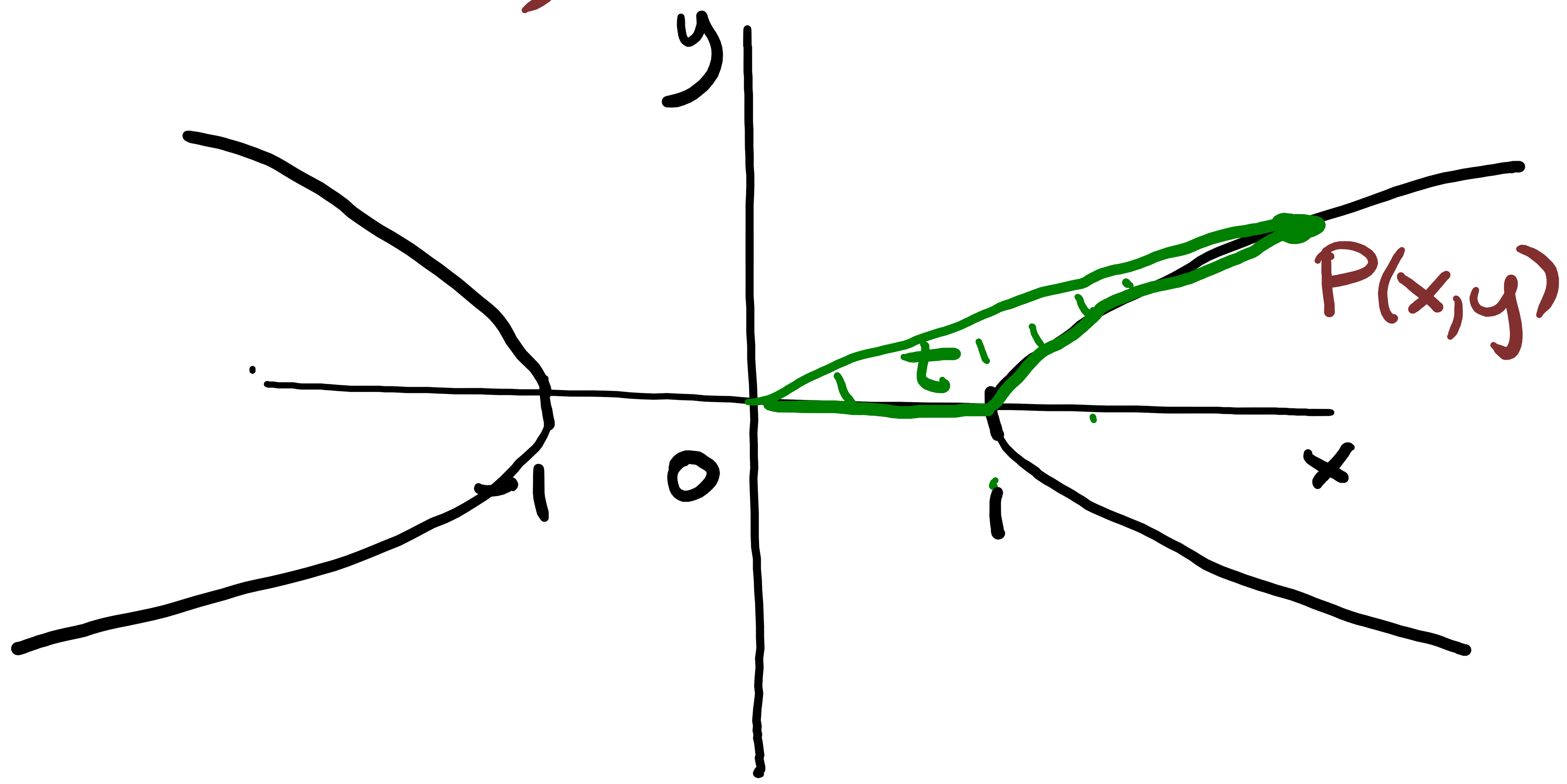
$\therefore$ The area of the unit circle is π

i.e. $t = 2\pi$ is twice π.

Similarly, the area of the shaded wedge is $t/2$,

i.e. t is twice the area of the shaded region.

Now, HYPERBOLIC FUNCTIONS.



$x^2 - y^2 = 1$ is a hyperbola.

$P(x,y) = (\cosh(t), \sinh(t))$

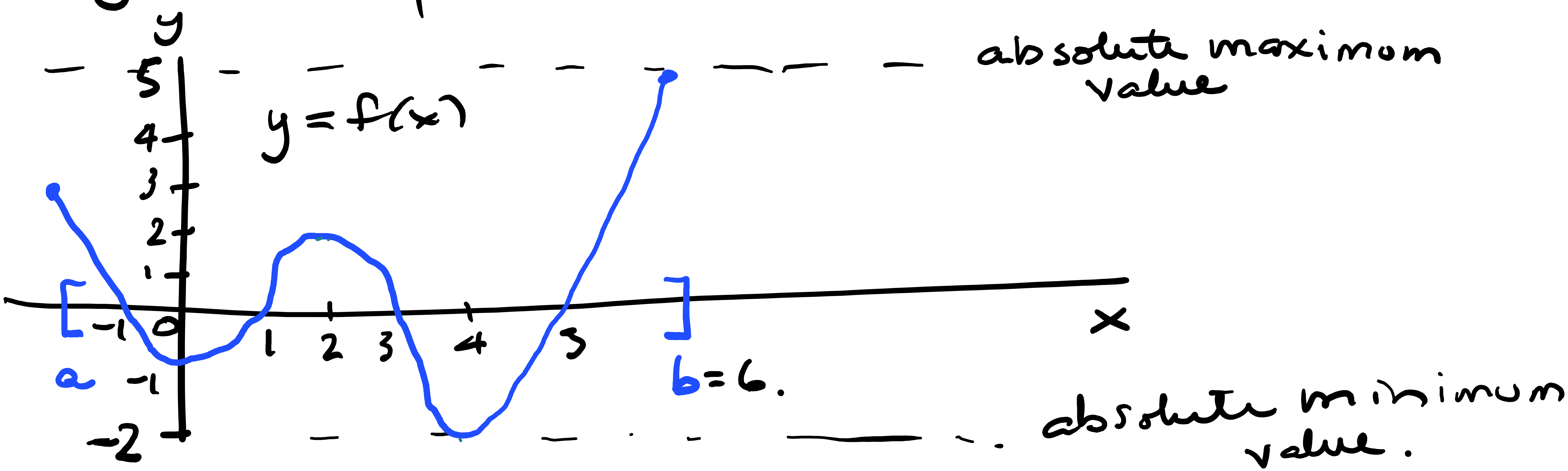
$\cosh^2(t) - \sinh^2(t) = 1.$

$\times$

Again,
t represents twice the area of the shaded region of the hyperbolic sector in the diagram.

Note: $\cosh(t) \geq 1,$
so we only consider the right half plane.

§ 4.1 Maximum & Minimum Values.



Absolute max value of $f(x)$ is 5 and it occurs at $x = 6$.

Absolute min value of $f(x)$ is -2 and it occurs at $x = 4$.

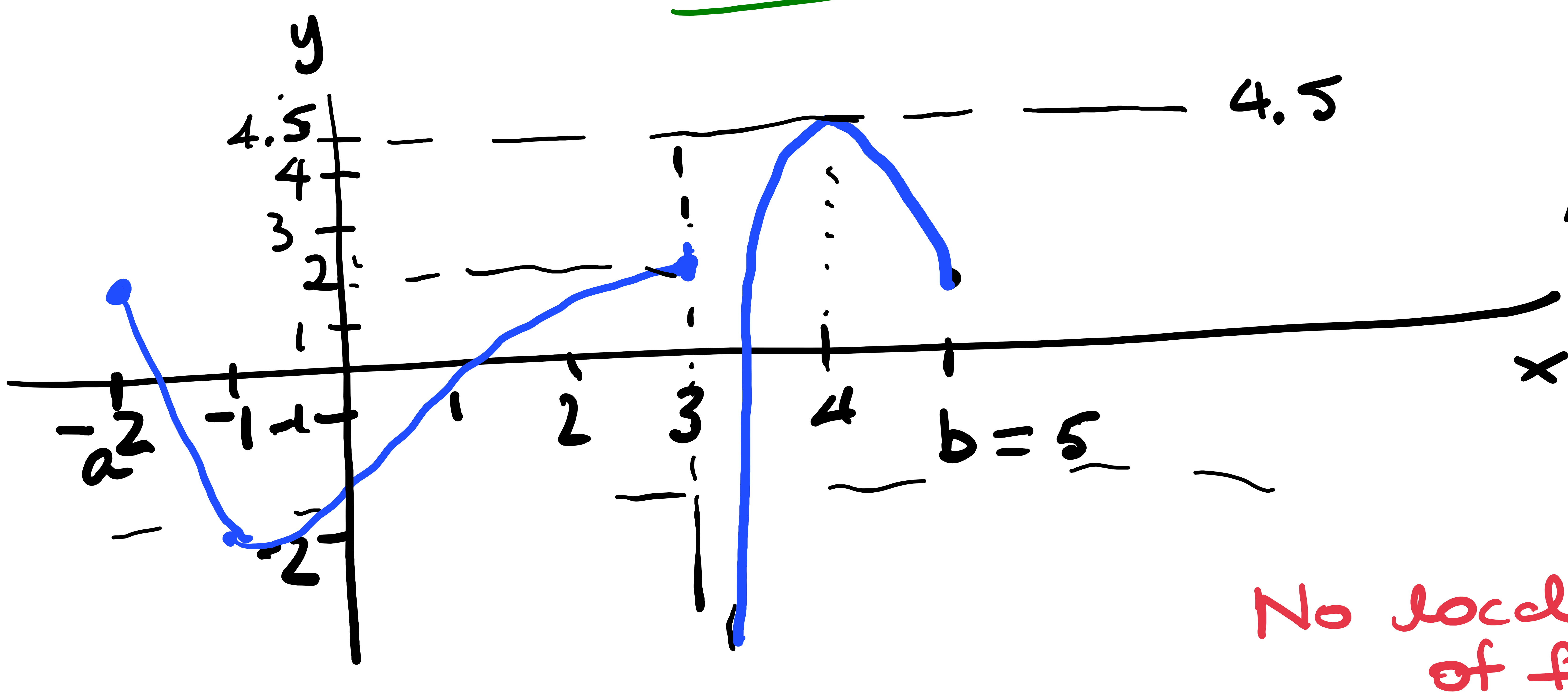
Local min values of $f(x)$ are -1 and -2 and they occur at $x = 0$ and $x = 4$, respectively, i.e. $f(0) = -1$, $f(4) = -2$.

Local max value of f(x) is 2 at $x=2$.

Defn Let c be a number in the domain D of f. Then $f(c)$ is

the absolute maximum value of f in D is $f(c)$ if $f(c) \geq f(x)$ for all $x \in D$.

the absolute minimum value of f in D is $f(c)$ if $f(c) \leq f(x)$ for all $x \in D$.



$[a,b] = [-2,5]$

Absolute max value of f on $[a,b]$ is 4.5 at $x=4$.

No absolute min. value of $f(x)$

No local minimum value of $f(x)$, since $\lim_{x \to 3^+} f(x) = -\infty$.

Local minimum value of f is -2 and it occurs at $x = 0$.

There are 2 local maximum values of $f(x)$

(i) local max 2 at $x = 3$

(ii) local max 4.5 at $x = 4$.

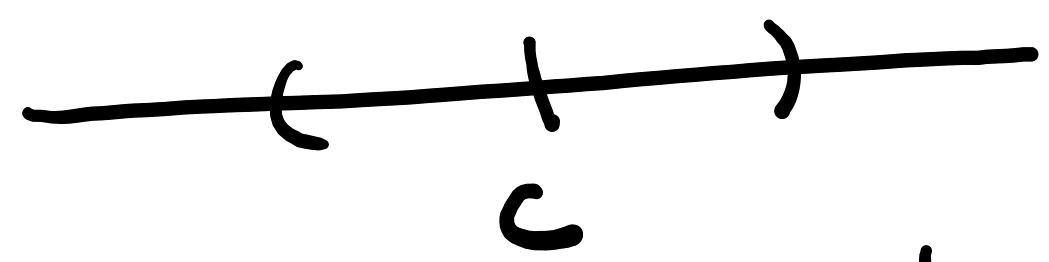
Def'n The number $f(c)$ is a **local minimum value** of $f(x)$ if

$f(c) \le f(x)$ for all x in some open interval containing c.

The number $f(c)$ is a local maximum value of $f(x)$ if

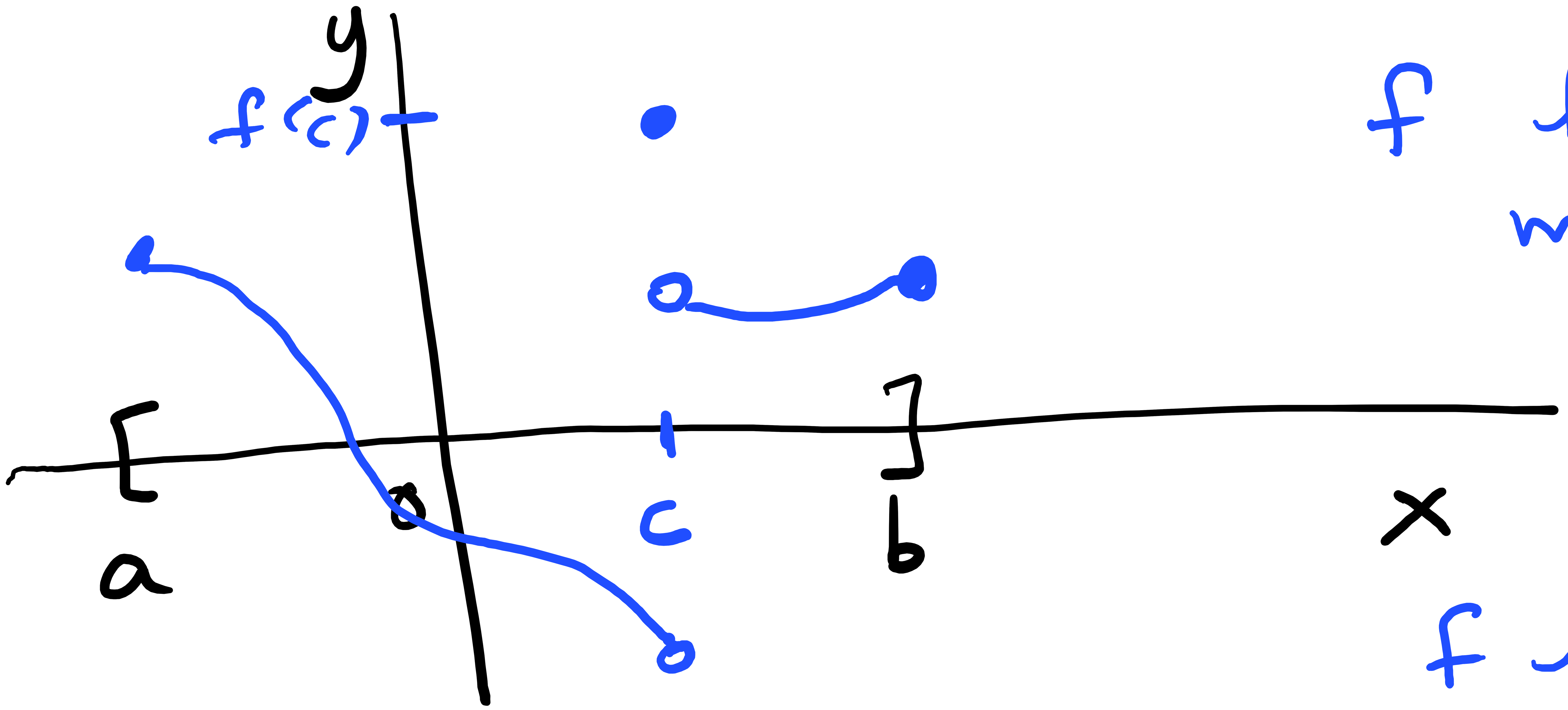
$f(c) \ge f(x)$ for all x in some open interval containing c.

NOTE: the entire open interval containing c must be in the domain of f including c.



Open interval

i.e points on both sides of c.



f has an absolute maximum value $f(c)$ at c.
(Note $f(c)$ is defined)

f has <u>no</u> absolute minimum value.

re: <u>END POINTS of interval.</u>

Absolute maximum and minimum values <u>can</u> occur at an end point of an interval.

Local max and min values <u>CANNOT</u> occur at an end point of an interval.

re: <u>Uniqueness</u>

The <u>absolute maximum</u> value is UNIQUE. However, it can occur at more than one point in the domain.
The absolute minimum value is UNIQUE.

Example:
$f(x) = \sin(x)$.

The absolute max val is 1, occuring at $x = \left(\frac{4n+1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

The absolute min val is -1, occuring at $x = \frac{(4n+3)}{2}\pi$, $n = 0, \pm 1, \pm 2, \dots$

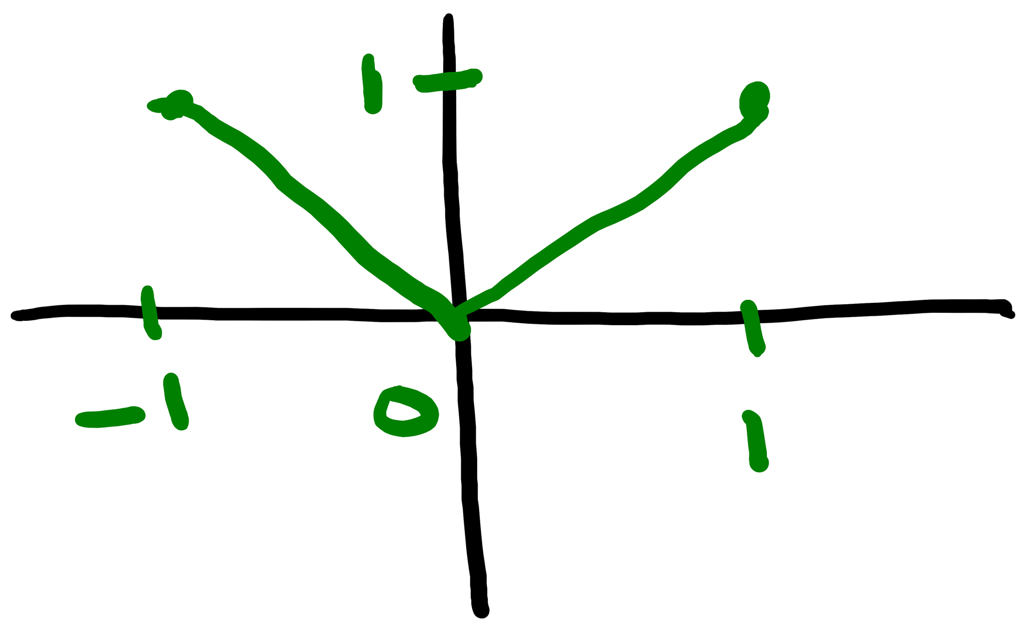
Thm (The Extreme Value Thm).

If f is CONTINUOUS on a closed interval $[a,b]$, then f attains an absolute maximum value $f(c)$, and an absolute minimum value $f(d)$, at some numbers $c, d \in [a,b]$.

In answer to a Question: BEWARE!

A function can have a local max or local min where the derivative is NOT DEFINED.

Ex. $f(x) = |x|$, $x \in [-1, 1]$.



$f(x)$ has a local min value of 0 at $x = 0$.

However, f is NOT differentiable at $x = 0$.