Network structure and systemic risk in banking systems*

Rama Cont[†] Amal Moussa[‡] Edson Bastos e Santos[§]

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Abstract

We present a quantitative methodology for analyzing contagion and systemic risk in a network of interlinked financial institutions, and apply this methodology to the study of the Brazilian financial system. Using a unique data set of all mutual exposures and capital levels of financial institutions in Brazil in 2007 and 2008, we show that the Brazilian financial system exhibits a complex network structure characterized by a strong degree of heterogeneity in connectivity and exposure sizes across institutions, which may be modeled as a directed scale-free weighted graph with heavy-tailed degree and weight distributions. Using a metric for the systemic importance of institutions –the Contagion Index– we study the potential for default contagion and systemic risk in the Brazilian system and analyze the contribution of balance sheet size and network structure to systemic risk. Our study reveals that, aside from balance sheet size, network-based measures of connectivity and concentration of exposures across counterparties –*counterparty susceptibility* and *local network frailty*– contribute significantly to the systemic importance of an institution. Requiring a minimum (aggregate) capital ratio is shown to reduce the systemic impact of defaults of large institutions; we show that the same effect may be achieved with less capital by imposing such capital requirements only on systemically important institutions and those exposed to them.

Keywords: default risk, domino effects, balance sheet contagion, scale-free network, small-world, default contagion, systemic risk, macro-prudential regulation, random graph.

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[†]IEOR Dept., Columbia University, New York.
[‡]Dept of Statistics, Columbia University, New York.
[§]Universidade de São Paulo and Banco Central do Brasil

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1 Introduction

The recent financial crisis has emphasized the importance of systemic risk, defined as macro-level risk which can impair the stability of the entire financial system. Bank failures have led in the recent years to a disruption of the financial system and a significant spillover of financial distress to the larger economy. Regulators have had great difficulties anticipating the impact of defaults partly due to a lack of visibility on the structure of the financial system as well as a lack of a methodology for monitoring systemic risk. The complexity of the contemporary financial systems makes it a challenge to define adequate indicators of systemic risk that could help in an objective assessment of the systemic importance of financial institutions and an objective framework for assessing the efficiency of macro-prudential policies.

One of the aspects of systemic risk which has been highlighted in the recent crisis has been the interconnectedness of financial institutions, which increases the probability of contagion of financial distress. Such externalities resulting from counterparty risk are a major concern for regulators (Haldane, 2009) and *network models* (Allen and Gale, 2000; Boss et al., 2004; Nier et al., 2007; Amini et al., 2010; Gai and Kapadia, 2010; Amini et al., 2011) provide an adequate framework for addressing them. Simulation studies based on network models have been extensively used by central banks for assessing contagion risk in banking systems; we refer to the pioneering work of Elsinger et al. (2006a) and the survey of Upper (2011).

In the present work we introduce and implement a quantitative methodology for analyzing contagion and systemic risk in a network of interlinked financial institutions, and apply this methodology to the study of the Brazilian financial system. Using a *unique* and *complete* data set of interbank exposures and capital levels provided by the Brazilian Central Bank, we analyze the *network structure* in the Brazilian financial system and the magnitude of contagion risk. Our analysis is based on a measure of the systemic importance of a financial institution, the Contagion Index, defined as the expected loss to the network triggered by the default of this institution when the system is subject to a market shock (Cont, 2009). The definition of this indicator takes into account both common market shocks to portfolios (correlation) and contagion through counterparty exposures (network effects). Contrarily to indicators of systemic risk purely based on market data (Acharya et al., 2010; Adrian and Brunnermeier, 2008), our metric of systemic importance make use of exposures, which represent potential losses, to simulate stress scenarios, resulting in a forward-looking measure of systemic risk. We build on methods proposed in Cont and Moussa (2010) for estimating and analyzing this indicator.

While most of the empirical studies on systemic risk and default contagion in interbank networks (Sheldon and Maurer, 1998; Furfine, 2003; Upper and Worms, 2004; Wells, 2004; Elsinger et al., 2006a,b; Mistrulli, 2007) have dismissed the importance of contagion, we find evidence that contagion is a significant factor of systemic risk in the Brazilian banking system. Our results do not necessarily contradict these findings but present them in a different light. Most of the aforementioned studies use indicators *averaged* across institutions: we argue that, given the heterogeneity of the systemic importance across institutions, the sample average gives a poor representation of the degree of contagion and *conditional* measures of risk should be used. Also, most of these studies are based on a generous recovery rate assumptions whereby all assets of a defaulting bank are recovered at pre-default value, which is far from reality especially in the short term where recovery rates are close to zero in practice. Finally, with the exception of Elsinger et al. (2006a,b), all these studies measure the impact of the idiosyncratic default of a single bank, whereas we use the more realistic setting where balance sheets are subjected to correlated market shocks in default scenarios. Similarly to previous studies (Elsinger et al., 2006a,b; Upper, 2011) we find that, while the probability of contagion is small, the loss resulting from contagion if it occurs can be very large in some cases.

1.1 Summary of main results

Our study reveals several interesting features on the structure of the Brazilian financial system and the nature of systemic risk and default contagion in this system:

- The Brazilian financial system exhibits a complex heterogeneous network structure: the distribution of in-degrees, out-degrees and mutual exposures are found to be heavy-tailed, exhibiting Pareto tails with exponents between 2 and 3. Furthermore, these statistical regularities are shown to be stable across time. These observations suggest to model it as a directed scale-free network.

- The network structure is qualitatively different from a small-world network. In particular, we observe many nodes with arbitrary small clustering coefficient.
- The systemic importance of institutions is quite heterogeneous: the cross-sectional distribution of the Contagion Index is found to be heavy-tailed. This implies that, while most financial institutions present only a negligible risk of contagion, a few of them may pose a significant risk of contagion.
- Ignoring the compounded effect of correlated market shocks and contagion via counterparty exposures can lead to a serious underestimation of contagion risk. Specifically, market shocks are found to increase the proportion of contagious exposures in the network, i.e. exposures that transmit default in all shock scenarios. We are thus led to question the conclusions of previous studies which dismissed the importance of contagion by looking at pure balance sheet contagion in absence of market shocks.
- Fundamental defaults due to market shocks are found to be the major source of aggregate losses for most periods. Nevertheless, contrarily to observations made in some previous studies, contagion is observed to be significant during periods of stress. This is explained by the fact that we measure the effect of contagion using conditional risk measures, whereas most previous studies examined cross-sectional averages, which underestimate the magnitude of contagion in a heterogeneous network.
- Balance sheet size matters when assessing systemic importance: the Contagion Index of a financial institution has a strong positive relationship with the total size of its interbank liabilities. However, size alone is not a good indicator for the systemic importance of financial institutions: network structure *does* matter when assessing systemic importance. Network-based measures of connectivity and concentration of exposures across counterparties –*counterparty susceptibility* and *local network frailty*– are shown to contribute significantly to the systemic importance of an institution.
- Using the Contagion Index as a metric for systemic impact allows a comparative analysis of various capital requirement policies in terms of their (reduction in) systemic impact. While a floor on the (aggregate) capital ratio is shown to reduce the systemic impact of defaults of large institutions, imposing more stringent capital requirements on the most systemic nodes and their counterparties is shown to be a more efficient procedure for immunizing the network against contagion.

1.2 Relation with previous literature

Our contribution builds on previous theoretical and empirical studies of default contagion in banking systems (see De Bandt and Hartmann (2000); Upper (2011) for a review of the literature), but also differs from them both in terms of the methodology used and in terms of the results obtained. In particular, we are led to revisit some of the conclusions in the previous literature on the magnitude of contagion risk in interbank networks.

Methodology *On the methodological side* previous studies on contagion in financial networks have mostly focused on the stability of the financial system as a whole, either in stylized equilibrium settings (Allen and Gale, 2000; Freixas et al., 2000; Battiston et al., 2009) or in simulation studies of default cascades (Upper and Worms, 2004; Mistrulli, 2007; Elsinger et al., 2006a,b; Nier et al., 2007). Nier et al. (2007) measure the average number of defaults when the institutions in the system are subject one at a time to an idiosyncratic shock which wipes out their external assets. Upper and Worms (2004) and Mistrulli (2007) consider various aggregate measures of contagion: the number of institutions that default by contagion and the loss as a fraction of the total assets in the banking system. Elsinger et al. (2006a) also measure contagion by counting the number of defaults due to counterparty exposure when the system is subject to correlated market shocks. These studies give insights on the global level of systemic risk in the entire network, but do not allow to measure the systemic importance of a given financial institution, which is our focus here. Our study adds to the existing literature by introducing the Contagion Index (Cont, 2009; Cont and Moussa, 2010) as a measure of the systemic importance of a single institution in the system, which allows to rank institutions in terms of the risk they pose to the system.

Recent studies such as Acharya et al. (2010); Zhou et al. (2010) have also proposed measures of systemic importance based on market data such as CDS spreads or equity volatility. By contrast to these methods which are based on historical market data, we use a forward-looking, simulation-based approach based on interbank *exposures* (Cont, 2009). Exposure data, which represent potential future losses, are available to regulators, should be used as an ingredient in evaluating systemic importance and interconnectedness. As argued in Cont (2009), since exposures are not publicly available, even if market variables correctly reflect public information they need not reflect the information contained in exposures, so exposures-based indicators are a useful complement to market-based indicators.

With the exception of Elsinger et al. (2006a,b), most simulation studies of contagion in banking networks examine the sole knock-on effects of the sudden failure of a single bank by considering an idiosyncratic shock that targets a single institution in the system. Upper and Worms (2004) estimate the scope of contagion by letting banks go bankrupt one at a time and measuring the number of banks that fail due their exposure to the failing bank. Sheldon and Maurer (1998) and Mistrulli (2007) also study the consequences of a single idiosyncratic shock affecting individual banks in the network. Furfine (2003) measures the risk that an exogenous failure of one or a small number of institutions will cause contagion. These studies fail to quantify the compounded effect of correlated defaults and contagion through network externalities. Our study, on the contrary, shows that common market shocks to balance sheets may exacerbate contagion during a crisis and ignoring them can lead to an underestimation of the extent of contagion in the network. We argue that, to measure adequately the systemic impact of the failure of a financial institution, one needs to account for the combined effect of correlation of market shocks to balance sheets and balance sheet contagion effects, the former increasing the impact of the latter. Our simulation-based framework takes into account common and independent market shocks to balance sheets, as well as counterparty risk through mutual exposures.

The loss contagion mechanism we consider differs from most network-based simulations, which consider the framework of Eisenberg and Noe (2001): this approach is based on a market clearing equilibrium defined by a clearing payment vector with proportional sharing of losses among counterparties in case of default (Eisenberg and Noe, 2001; Elsinger et al., 2006a,b; Müller, 2006). This leads to an endogenous recovery rate which corresponds to a hypothetical situation where all bank portfolios are simultaneously liquidated. Our approach is, by contrast, not an equilibrium approach but a stress-testing approach where, starting from the currently observed network structure, capital levels are stressed by random correlated shocks and a risk measure computed from the distribution of aggregate loss. We argue that, since bankruptcy procedures are usually slow and settlements may take up several months to be effective, creditors cannot recover the residual value of the defaulting institution according to such a hypothetical clearing mechanism, and write down their entire exposure in the short-run, leading to a short term recovery rate of zero. This seems a more reasonable approach in absence of a clearing mechanism.

Studies on simulated network structures have examined the variables that affect the global level of systemic risk in the network (Nier et al., 2007; Battiston et al., 2009) such as the connectivity, concentration, capital levels, but the main results (such as the level of contagion and the role of interconnectedness) strongly depend on the details of the model and the structure of the network, which have left open whether these conclusions hold in actual banking networks. On the other hand, most of the empirical studies have only partial information on the bilateral exposures in the network, and estimate missing exposures with a maximum entropy method (Sheldon and Maurer, 1998; Upper and Worms, 2004; Wells, 2004; Elsinger et al., 2006a,b; Degryse and Nguyen, 2007). However, the maximum entropy method is found to underestimate the possibility of default contagion (Mistrulli, 2007; van Lelyveld and Liedorp, 2006). Our study, by making use of empirical data on all bilateral exposures, avoids this caveat.

Results Our empirical findings on the network structure of the Brazilian financial system are –qualitatively and quantitatively– similar to statistical features observed in the Austrian financial system (Boss et al., 2004). This suggests that these features could be a general characteristic of interbank networks, and it would interesting to check whether similar properties are also observed in other interbank networks.

While most of the empirical studies on systemic risk and default contagion in interbank networks have dismissed the importance of contagion, our study reveals that the risk of default contagion is significant in the Brazilian financial system. We show examples in which the expected loss resulting from the default of an institution can exceed up to forty times the size of its interbank liabilities and some defaults combined with common shocks can initiate up to four additional defaults. In contrast with Elsinger et al. (2006a), we find that scenarios with contagion are more frequent than those without contagion when grouped by number of fundamental defaults. This difference in results is due to two reasons. First, our metric, the Contagion Index, measures the magnitude of loss *conditional* to the default of a given institution, instead of averaging across all defaults as in Elsinger et al. (2006a). We argue that these conditional measures provide a better assessment of risk in a heterogeneous system where the sample average may be a poor statistic. Second, we use a heavy-tailed model for generating the correlated shocks to balance sheets: we argue that this heavy-tailed model is more realistic than Gaussian factor models used in many simulation studies.

We find that market shocks can play an essential role in propagating default across the network. Specifically, we observe that the proportion of contagious exposures increases considerably when the system is subject to a market shock scenario, thus creating additional channels of contagion in the system. The Contagion Index, by compounding the effects of both market events and counterparty exposure, accounts for this phenomenon.

Our study also complements the existing literature by studying the contribution of network-based local measures of connectivity and concentration to systemic risk. Previous studies on simulated network structures have examined the contribution of aggregate measures of connectivity and concentration such as increasing the probability that two nodes are connected in an Erdös-Renyi graph, or increasing the number of nodes in the system (Battiston et al., 2009; Nier et al., 2007). However, they fail to detect the impact of connectivity and concentration *locally* around a single institution in the network. We thus introduce the *counterparty susceptibility* and *local network frailty* that measure respectively the susceptibility of the creditors of an institution to a potential default of the latter and the fragility of the entire network in the event of default of this institution. We find that the two measures can explain significantly default contagion.

The impact of capital requirements in limiting the extent of systemic risk and default contagion has not been explored systematically in a network context. Based on analogies with epidemiology and peer-to-peer networks (Cohen et al., 2003; Madar et al., 2004; Huang et al., 2007), we discuss *targeted* capital requirements and show that targeting the creditors of the most contagious institutions is a more effective procedure –in terms of the total capital it requires for the same level of systemic risk– than increasing capital ratios for all institutions in the network.

1.3 Outline

The paper is organized as follows. Section 2 describes the data set and provides an empirical analysis of the structure and statistical properties of the Brazilian financial network. Section 3 introduces a quantitative approach for measuring contagion and systemic risk, following Cont (2009). Section 4 applies this methodology to the Brazilian financial system. Section 5 investigates the role of different institutional and network characteristics which contribute to the systemic importance of Brazilian financial institutions. Section 6 analyzes the impact of capital requirements on these indicators of systemic risk and uses the insights obtained from the network model to examine the impact of *targeted* capital requirements which focus on the most systemic institutions and their counterparties.

2 The Brazilian financial system: a complex network

2.1 Data and consolidation procedure

The Brazilian financial system encompasses 2400 financial institutions chartered by the Brazilian Central Bank and grouped into three types of operation: Type I are banking institutions that have commercial portfolios, Type III are institutions that are subject to particular regulations, such as credit unions, and Type II represent all other banking institutions. Despite their reduced number (see table 1), financial institutions of Type I and II account for the majority (about 98%) of total assets in the Brazilian financial system (see table 2). We therefore consider in the Brazilian data set only Type I and Type II financial institutions which is a very good proxy for the Brazilian financial system. Most of the financial institutions belong to a conglomerate (75% of all financial institutions of Type I and II). Consequently, it is quite meaningful to analyze the financial system from a consolidated perspective where financial institutions are classified in groups that are held by the same shareholders. Only banking activities controlled by the holding company are considered in the consolidation procedure. The accounting standards for consolidation of financial statements were established by Resolutions 2,723 and 2,743, BCB (2000a,b), and they are very similar to IASB and FASB directives. If we regard financial institutions as conglomerates, the dimension of the exposures matrices reduces substantially, see table 1 for the number of financial conglomerates in the Brazilian financial system after the consolidation procedure.

These exposures, reported at six dates (June 2007, December 2007, March 2008, June 2008, September 2008 and November 2008) cover various sources of risk:

1. fixed-income instruments (certificate of deposits and debentures);
2. borrowing and lending (credit risk);
3. derivatives (including OTC instruments such as swaps);
4. foreign exchange and,
5. instruments linked to exchange-traded equity risk.

Derivatives positions were taken into account at their market prices when available, or at fair value when a model-based valuation was required.

The data set also gives the Tier I and Tier 2 capital of each institution, computed according to guidelines provided in Resolution 3,444 BCB (2007a) of the Brazilian Central Bank, in accordance with the Basel I and II Accords. Tier 1 capital is composed of shareholder equity plus net income (loss), from which the value of redeemed preferred stocks, capital and revaluation of fixed assets reserves, deferred taxes, and non-realized gains (losses), such as mark-to-market adjustments from securities registered as available-for-sale and hedge accounting are deducted. Tier 2 capital is equal to the sum of redeemed preferred stocks, capital, revaluation of fixed assets reserves, non-realized gains (losses), and complex or hybrid capital instruments and subordinated debt. We shall focus on Tier 1 capital as a measure of a bank's capacity to absorb losses in the short term.

Financial conglomerates in Brazil are subject to minimum capital requirements. The required capital is a function of the associated risks regarding each financial institution's operations, whether registered in their balance sheets (assets and liabilities) or not (off-balance sheet transactions), as defined in Resolution 3,490, BCB (2007b). The required capital is computed as $c_r = \delta \times$ Risk Base where the $\delta = 11\%$ is the so-called *Basel Index* and the risk base is the sum of credit exposures weighted by their respective risk weights, foreign currency and gold exposures, interest rate exposures, commodity exposures, equity market exposures, and operational risk exposures. It is important to highlight that the exposures considered in the computation of the *risk base* include not only interbank exposures but also exposures to all counterparties.

2.2 Network representation of the Brazilian financial system

Counterparty relations in financial system may be represented as a weighted directed graph, or a *network* , defined as a triplet $I = (V, E, c)$, consisting of

- a set V of financial institutions, whose number we denote by n,
- a matrix E of bilateral exposures: E_{ij} represents the exposure of node i to node j defined as the (mark-to-)market value of all liabilities of institution j to institution i at the date of computation. It is thus the maximal short term loss of i in case of an immediate default of j.
- $c = (c_i, i \in V)$ where c_i is the capital of the institution i, representing its capacity for absorbing losses.

Such a network may be represented as a graph in which nodes represent institutions and links represent exposures. Figure 1 illustrates the Brazilian interbank network in December 2007. It is observed to have

Type	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Dec-08**
Multiple Bank	135	135	135	136	139	139	140
Commercial Bank	20	20	21	20	20	18	18
Development Bank	4	4	4	4	4	4	4
Savings Bank	1	1	1	1	1	1	1
Investment Bank	17	17	17	18	18	18	17
Consumer Finance Company	51	52	51	56	55	55	55
Security Brokerage Company	113	107	114	107	107	107	107
Exchange Brokerage Company	48	46	48	46	46	45	45
Security Distribution Company	132	135	133	133	136	136	135
Leasing Company	40	38	41	37	36	36	36
Real Estate Credit Company and Savings and Loan Association	18	18	18	18	18	17	16
Mortgage Company	6	6	6	6	6	6	6
Development Agency	12	12	12	12	12	12	12
Total Banking Institutions of Type I and II	**597**	**591**	**601**	**594**	**598**	**594**	**592**
Credit Union	1.461	1.465	1.460	1.466	1.460	1.457	1.453
Micro-financing Institution	54	52	54	48	46	45	47
Total Banking Institutions Type III	**2.112**	**2.108**	**2.115**	**2.108**	**2.104**	**2.096**	**2.092**
Non-Banking Institutions	**332**	**329**	**333**	**324**	**317**	**318**	**317**
Total Banking and Non-Banking Institutions	**2444**	**2.437**	**2.448**	**2.432**	**2.421**	**2.414**	**2.409**

Table 1: Number of financial institutions by type of operation for the Brazilian financial system. Source: Sisbacen.

Assets in Billions of USD	**Jun-07**	%	**Dec-07**	%	**Mar-08**	%	**Jun-08**	%	**Sep-08**	%	**Dec-08**	%
Banking - Type I	1,064.8	87.1	1,267.7	87.8	1,366.9	87.9	1,576.0	87.7	1,433.2	88.0	1,233.6	87.5
Banking - Type II	129.6	10.6	142.7	9.9	152.7	9.8	179.4	10.0	160.1	9.8	148.3	10.5
Banking - Type I and II	**1,194.5**	**97.7**	**1,410.4**	**97.7**	**1,519.6**	**97.7**	**1,755.4**	**97.7**	**1,593.2**	**97.8**	**1,382.0**	**98.0**
Banking - Type III	17.7	1.5	21.5	1.5	23.7	1.5	28.3	1.6	24.1	1.5	19.1	1.4
Non-Banking	10.4	0.9	12.8	0.9	12.5	0.8	14.4	0.8	11.4	0.7	9.3	0.7
Total Financial System	**1,222.6**	**100.0**	**1,444.8**	**100.0**	**1,555.8**	**100.0**	**1,798.1**	**100.0**	**1,628.8**	**100.0**	**1,410.4**	**100.0**

Number of Conglomerates	**Jun-07**	%	**Dec-07**	%	**Mar-08**	%	**Jun-08**	%	**Sep-08**	%	**Dec-08**	%
Banking - Type I	102	5.4	101	5.4	101	5.4	101	5.4	103	5.5	101	5.4
Banking - Type II	32	1.7	32	1.7	32	1.7	33	1.8	34	1.8	35	1.9
Banking - Type I and II	**134**	**7.1**	**133**	**7.1**	**133**	**7.1**	**134**	**7.2**	**137**	**7.3**	**136**	**7.3**
Banking - Type III	1,440	76.8	1,440	77.0	1,436	77.0	1,441	77.0	1,442	76.9	1,438	77.0
Non-Banking	302	16.1	298	15.9	297	15.9	296	15.8	296	15.8	294	15.7
Total Financial System	**1,876**	**100.0**	**1,871**	**100.0**	**1,866**	**100.0**	**1,871**	**100.0**	**1,875**	**100.0**	**1,868**	**100.0**

Table 2: Representativeness of Brazilian financial institutions in terms of total Assets and number. The total assets were converted from BRL (Brazilian Reals) to USD (American Dollars) with the following foreign exchange rates (BRL/USD): 1.9262 (Jun-07), 1.7713 (Dec-07), 1.7491 (Mar-08), 1.5919 (Jun-08), 1.9143 (Sep-08), and 2.3370 (Dec-08). Source: Sisbacen.

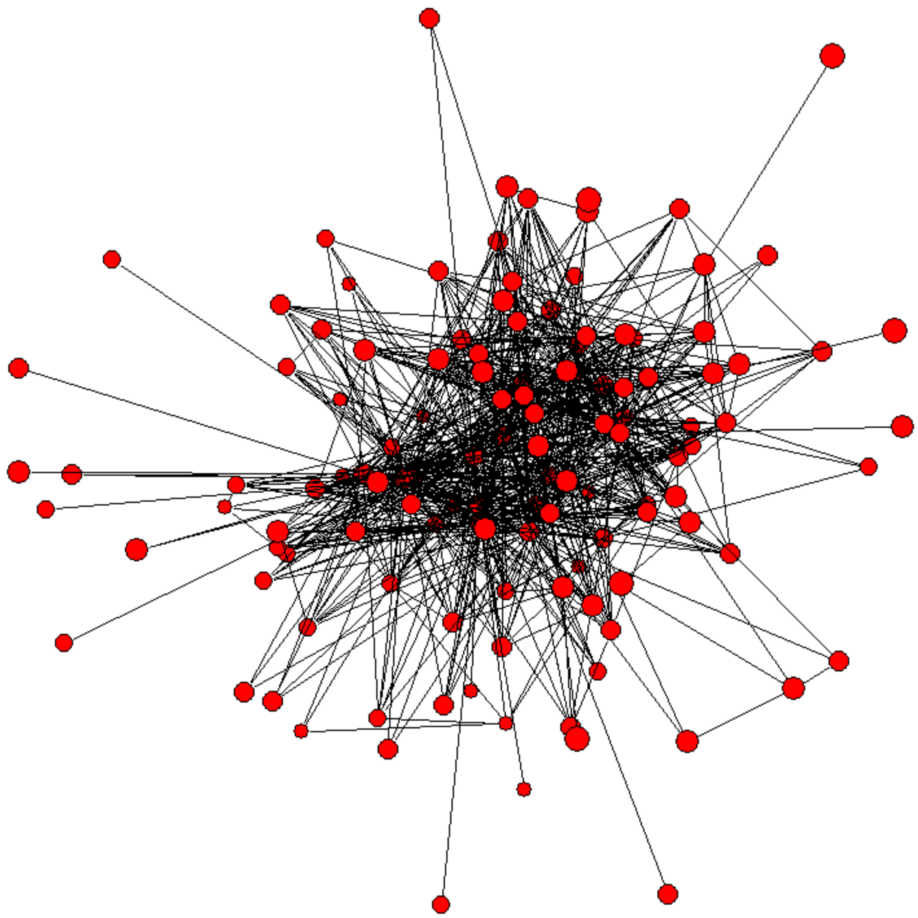


Figure 1: Brazilian interbank network, December 2007. The number of financial conglomerates is $n = 125$ and the number of links in this representation at any date does not exceed 1200.

a heterogeneous and complex structure, some highly connected institutions playing the role of "hubs" while others are at the periphery. We define the *in-degree* $k_{in}(i)$ of a node $i \in V$ as the number of its debtors and *out-degree* $k_{out}(i)$ the number of its creditors:

$$k_{in}(i) = \sum_{j \in V} \mathbb{1}_{\{E_{ij}>0\}}, \qquad k_{out}(i) = \sum_{j \in V} \mathbb{1}_{\{E_{ji}>0\}}, \tag{1}$$

The *degree* $k(i)$ of a node i is defined as $k(i) = k_{in}(i) + k_{out}(i)$ and measures its connectivity.

Although all institutions in the network are not banks, we will refer to the exposures as "interbank" exposures for simplicity. We denote $A(i)$ the interbank assets of financial institution i, and $L(i)$ its interbank liabilities:

$$A(i) = \sum_{j \in V} E_{ij}, \qquad L(i) = \sum_{j \in V} E_{ji}, \tag{2}$$

Table 3 presents some descriptive statistics of these variables.

In-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	8.56	8.58	8.75	8.98	8.99	7.88
Standard Deviation	10.84	10.86	10.61	11.15	11.32	11.02
5% quantile	0	0	0	0	0	0
95% quantile	30.50	29.30	30.45	31	32	30.60
Maximum	54	54	51	57	60	62
Out-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	8.56	8.58	8.75	8.98	8.99	7.88
Standard Deviation	8.71	8.82	9.02	9.43	9.36	8.76
5% quantile	0	0	0	0	0	0
95% quantile	26	26	27.90	29.25	30.20	27.40
Maximum	36	37	39	41	39	44
Exposures (in billions of BRL)	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	0.07	0.05	0.05	0.05	0.05	0.08
Standard Deviation	0.77	0.32	0.32	0.30	0.38	0.54
5% quantile	0.00	0.00	0.00	0.00	0.00	0.00
95% quantile	0.20	0.17	0.17	0.18	0.19	0.35
Maximum	23.22	9.89	9.90	9.36	12.50	15.90
Relative Exposures ($E_{ij}/c(i)$)	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	0.23	0.20	0.04	0.04	0.03	0.05
Standard Deviation	1.81	1.62	0.16	0.17	0.06	0.21
5% quantile	0.00	0.00	0.00	0.00	0.00	0.00
95% quantile	0.70	0.59	0.20	0.21	0.16	0.18
Maximum	49.16	46.25	4.57	5.17	0.69	6.02
Distance	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	2.42	2.42	2.38	2.38	2.33	2.35
Standard Deviation	0.84	0.85	0.84	0.82	0.77	0.78
5% quantile	1	1	1	1	1	1
95% quantile	4	4	4	4	3	4
Maximum (Diameter)	5	6	6	6	5	6

Table 3: Descriptive statistics of the number of debtors (in-degree), number of creditors (out-degree), exposures, relative exposures (ratio of the exposure of institution i to institution j to the capital of i), and distance between two institutions (nodes) in the network.

2.3 A heterogeneous network

2.3.1 Distribution of connectivity

Casual inspection of the graph in figure 1 reveals the existence of nodes with widely differing connectivity. This observation is confirmed by further analyzing the data on in-degrees and out-degrees of nodes.

Figures 2 and 3 show, respectively, the double logarithmic plot of the empirical complementary cumulative distribution for the in-degree $\hat{\mathbb{P}}(K_{in} \geqslant k)$ and out-degree $\hat{\mathbb{P}}(K_{out} \geqslant k)$ for $k \geqslant 1$. We notice that the tails of the distributions exhibit a linear decay in log-scale, suggesting a heavy Pareto tail.

This observation is confirmed through semiparametric tail estimates. Maximum likelihood estimates for the tail exponent α and tail threshold k_{min} (Clauset et al., 2009) are shown in Table 4 for the in-degree, out-degree and degree distributions. Maximum likelihood estimates for $\hat{\alpha}$ range from 2 to 3. The results are similar to the findings of Boss et al. (2004) for the Austrian network.

In-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	2.19	2.70	2.20	3.36	2.16	2.13	2.46
$\hat{\sigma}(\hat{\alpha})$	0.48	0.46	0.47	0.53	0.47	0.44	0.48
$\hat{k}_{in,min}$	6	13	7	21	6	5	9.7
Out-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	1.98	3.41	3.40	2.91	2.43	2.88	2.83
$\hat{\sigma}(\hat{\alpha})$	0.63	0.59	0.48	0.43	0.41	0.49	0.51
$\hat{k}_{out,min}$	5	15	16	12	9	11	11.3
Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	2.61	3.37	2.29	2.48	2.27	2.23	2.54
$\hat{\sigma}(\hat{\alpha})$	0.52	0.47	0.48	0.41	0.43	0.35	0.44
$\hat{k}_{min}$	17	34	12	15	12	10	16.7
Exposures*	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	1.97	2.22	2.23	2.37	2.27	2.52	2.27
$\hat{\sigma}(\hat{\alpha})$	0.02	0.60	0.21	0.69	0.38	0.98	0.48
$\hat{E}_{min}$	39.5	74.0	80.0	101.7	93.4	336.7	120.9

*values in millions of BRL (Brazilian Reals)

Table 4: Statistics and maximum likelihood estimates for the distribution of in/out degree: tail exponent α, tail threshold for in-degree $k_{in,min}$, out-degree $k_{out,min}$, degree k_{min}, and exposures E_{min}.

We test the goodness-of-fit of the power law tails for in-degree, out-degree and degree via the one-sample Kolmogorov-Smirnov test with respect to a reference power law distribution. The results in figures 2 and 3 provide evidence for the Pareto tail hypothesis at the 1% significance level.

2.3.2 Stationarity of degree distributions

The precise pattern of exposure across institutions may vary a priori in time: it is therefore of interest to examine whether the large scale structure of the graph, as characterized by the cross-sectional distributions of in- and out-degrees, is stationary, that is, may be considered as time-independent. Comparing quantiles of the degree distributions at different dates (figure 4) shows that the empirical distribution of the degree, in-degree and out-degree are in fact stable over time, even though the observations span the turbulent period of 2007-2008. This is confirmed by a two-sample Kolmogorov-Smirnov test for consecutive dates, which produces p-values all greater than 0.6, suggesting that the null hypothesis that the samples are drawn from the same distribution cannot be rejected.

2.3.3 Heterogeneity of exposure sizes

The distribution of interbank exposures is also found to be heavy-tailed, with Pareto tails. Figure 5 shows the existence of a linear decay in the tail of the double logarithmic plot for the empirical distribution of exposure sizes. Maximum likelihood estimates for the tail exponent α and the tail cutoff k_{min} for the distribution of exposures are shown in Table 4. Note that an interbank asset for an institution is an interbank liability for its counterparty, thus, the distribution of interbank liability sizes is the same. The only difference is how these exposures are allocated among the financial institutions in the network. Figure 5 shows evidence for Pareto tails in the exposure distributions at all dates.

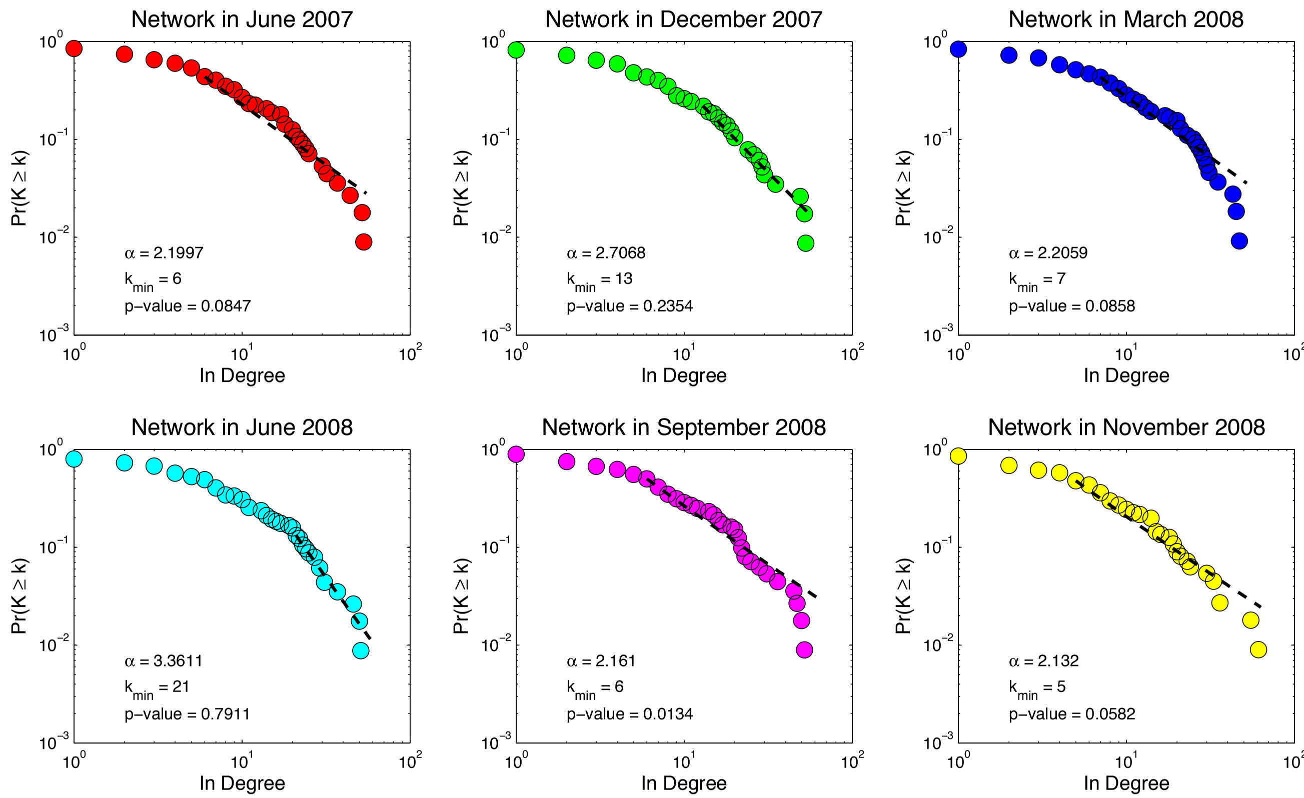


Figure 2: Brazilian interbank network: distribution of in-degree.

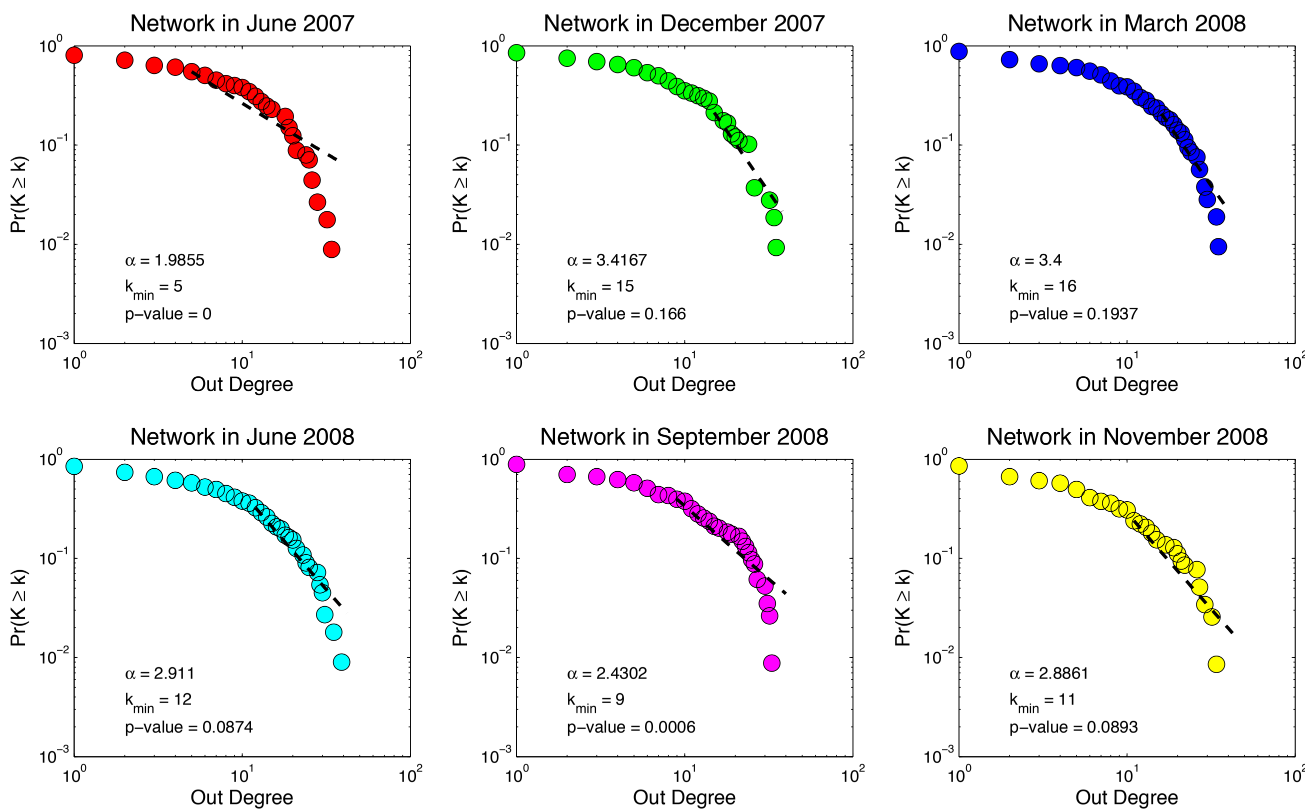


Figure 3: Brazilian interbank network: distribution of out-degree.

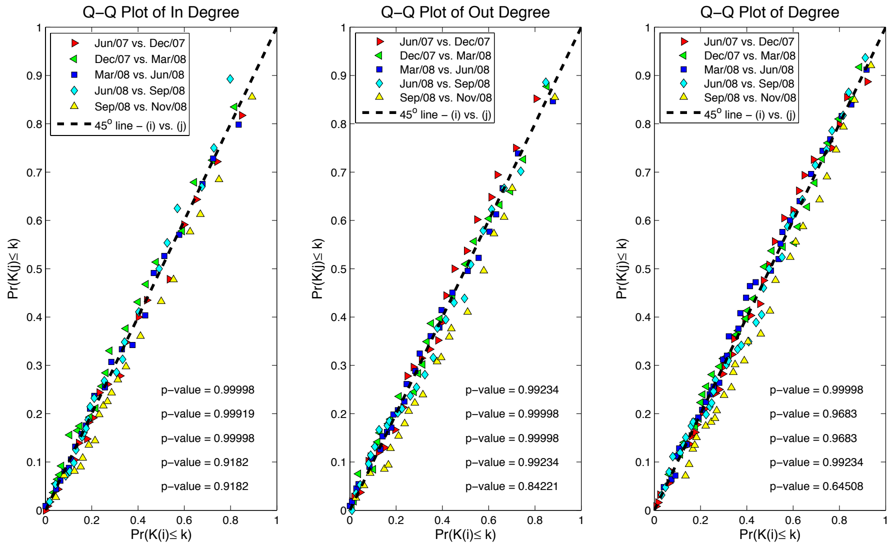


Figure 4: Scatterplot of the the empirical cumulative distributions at consecutive dates for the degree, in-degree and out-degree in the Brazilian interbank network.

It is interesting to measure the sizes of these exposures in terms of each institutions' (Tier 1) capital. The linear regression of the interbank assets size against the Tier 1 capital gives a positive slope smaller than 1, indicating that financial institutions in Brazil have on average sufficient Tier 1 capital to cover their interbank exposures. Figure 6 shows that in June 2007 the ratio of interbank exposures to Tier 1 capital exhibits a heterogenous distribution: most financial institutions hold much more Tier 1 capital than their interbank exposures, which means that they have a strong capacity to absorb losses. However, some institutions have interbank exposures more than a hundred times their Tier 1 capital. Thus, these ones can be very fragile to losses and may present a significant risk of default. We will see in section 5.2 that these fragile nodes are counterparties of the five most systemic institutions in the network and play a crucial role in the propagation of default across the network.

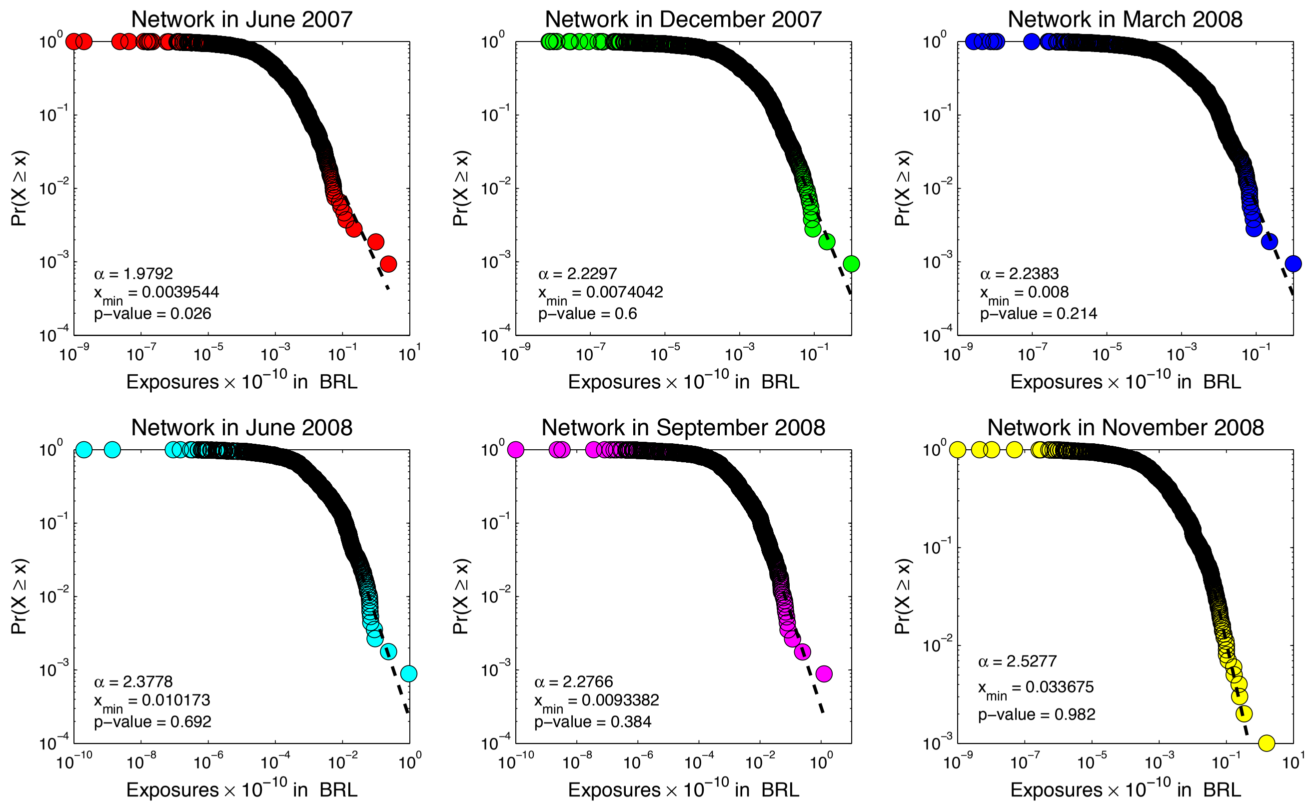


Figure 5: Brazilian interbank network: distribution of exposures in BRL.

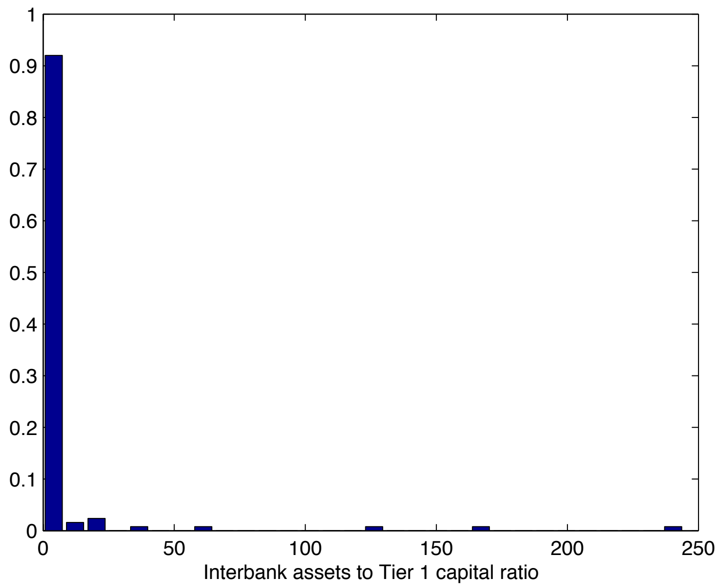


Figure 6: Distribution of the ratio of the interbank assets size to the Tier 1 capital in the network in June 2007.

Model: $A = \beta_0 + \beta_1 c + \epsilon$			
Coefficients	**Standard error**	**t-statistic**	R^2
$b_0 = -0.00$	0.07	-0.00	36%
$b_1 = 0.60$ **	0.07	8.37	
* significant at 5% confidence level			
** significant at 1% confidence level			

Table 5: Linear regression of interbank assets size on the Tier 1 capital in the network in June 2007.

2.3.4 Relation between exposure size and connectivity

Another interesting observation is that more (less) connected financial institutions have larger (smaller) exposures. We investigate the relationship between the in-degree $k_{in}(i)$ of a node i and its average exposure size $A(i)/k_{in}(i)$ and also examine the relation between the out-degree $k_{out}(i)$ and the average liability size $L(i)/k_{out}(i)$ and between $k(i)$ and $A(i)/k(i)$ by computing the Kendall tau for each of these pairs. Table 6 displays the Kendall tau $\tau_{Kendall}$ coefficients that measure the statistical dependence between the variables, and their respective p-values. The results show that the in-degree and the average interbank asset size, as well as the out-degree and the average interbank liability size, show positive dependence.

2.3.5 Clustering

The clustering coefficient of a node is defined as the ratio of the number of its links between its neighbors to the total number of possible links among its neighbors (Watts and Strogatz, 1998): this ratio, between 0 and 1, tells how connected among themselves the neighbors of a given node are. In complete graphs, all nodes have a clustering coefficient of 1 while in regular lattices the clustering coefficient shrinks to zero with the degree.

A property often discussed in various networks is the *small world* property (Watts and Strogatz, 1998) which refers to networks where, although the network size is large and each node has a small number of direct neighbors, the distance between any two nodes is very small compared to the network size. Boss et al. (2004) report that in the Austrian interbank network any two nodes are on average 2 links apart,

k_{in} **vs.** A/k_{in}	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
$\tau_{Kendall}$	0.28	0.25	0.22	0.26	0.24	0.21
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
k_{out} **vs.** L/k_{out}	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
$\tau_{Kendall}$	0.27	0.28	0.31	0.32	0.34	0.30
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
k **vs.** A/k	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
$\tau_{Kendall}$	0.24	0.24	0.21	0.23	0.23	0.23
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 6: Brazilian interbank network: Kendall $\tau_{Kendall}$ coefficients for in-degree k_{in} vs. interbank assets A, out-degree k_{out} vs. interbank liabilities L, and degree k vs. exposures w.

and suggest that the Austrian interbank network is a small-world. However, a small graph diameter does not characterize the small world property: indeed, complete networks are not small worlds and have diameter one. Another signature of the small world property is that, while the diameter is bounded or slowly increasing with the number of nodes, the *clustering coefficient* of nodes remain bounded away from zero (Cont and Tanimura, 2008). In the Brazilian financial system, we observe nodes with an arbitrary small clustering coefficient across all time periods (Figure 7). This absence of uniform clustering shows that the Brazilian financial system is not a small world network.

Figure 7 shows the relationship between the local clustering coefficient and number of degrees for the Brazilian interbank network. The negative slope of the plots shows that financial institutions with few connections (small degree) have counterparties that are very connected to each other (large clustering) while financial institutions with many connections (large degree) have counterparties with sparsely connected neighbors.

3 Systemic risk and default contagion

We now define indicators of default contagion and systemic impact for a financial institution, following Cont (2009). These indicators aim at quantifying the impact of the default of a given institution in terms of the (expected) loss it incurs for other institutions in the network, taking into account both balance sheet contagion and common shocks affecting balance sheets.

3.1 Default mechanism

Default occurs when an institution fails to fulfill a legal obligation such as a scheduled debt payment of interest or principal, or the inability to service a loan.

When discussing default modeling, it is important to differentiate between *insolvency* and *illiquidity.* Insolvency happens when the net worth of an institution is reduced to zero, i.e. losses exceed capital, while illiquidity occurs when reserves in liquid assets, such as cash and cash equivalents, are insufficient to cover short term liabilities. Illiquidity leads to default while, in principle insolvency may not necessarily entail default as long as the institution is able to obtain financing to meet payment obligations. Nevertheless, in the current structure of the financial sector where financial institutions are primarily funded through short-term debt, which must be constantly renewed, insolvent institutions would have great difficulties in raising liquidity as their assets lose in value. Indeed, renewal of short term funding is subject to the solvency and creditworthiness of the institution. Thus, in practice, insolvency leads to illiquidity which in turn leads to default.

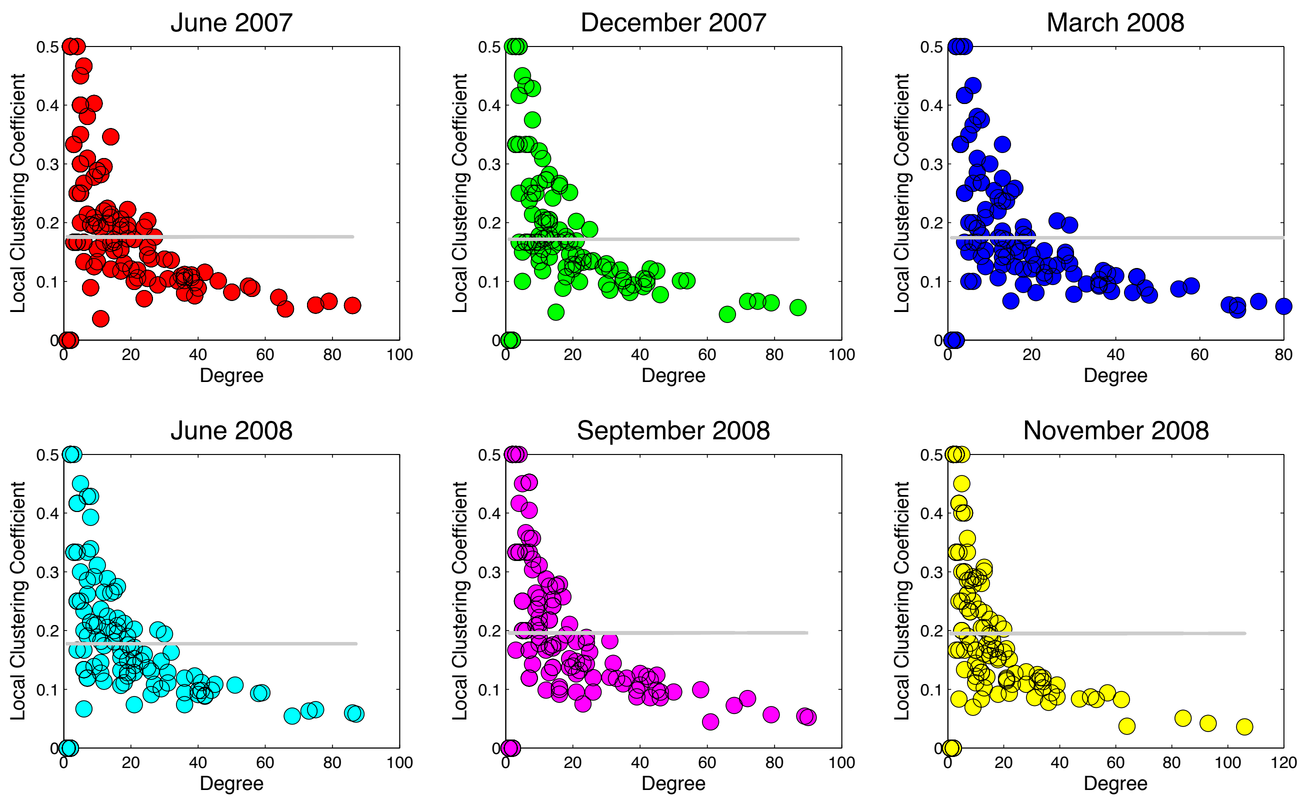


Figure 7: Degree vs. clustering coefficient for the Brazilian interbank network. The grey line is the average clustering coefficient.

Thus, in line with various previous studies, we define *default* as the event when the losses incurred by a financial institution render it insolvent. In practice, this may be defined as a scenario where losses in asset value exceed Tier 1 capital. If Tier 1 capital is wiped out, the institution becomes insolvent which is very likely to generate a loss of short term funding leading to default.

We recognize that institutions may default due to lack of liquidity even when just a portion of their Tier 1 capital is wiped out: the example of Bear Stearns is illustrative in this sense (Cox, 2008). However, given the current funding structure of financial institutions through short term debt, it is difficult to argue the opposite: absent a government bailout, insolvency due to market losses which exceed the level of capital will most probably lead to a loss of funding opportunities and credit lines and entail default. Thus, our estimates for the extent of default contagion will, if anything, lead to lower bounds for its actual extent in absence of government intervention.

3.2 Loss contagion

When a financial institution (say, i) defaults, it leads to an immediate writedown in value of all its liabilities to its creditors. These losses are imputed to the capital of the creditors, leading to a loss of E_{ji} for each creditor j. If this loss exceeds the creditor's capital i.e. $E_{ji} > c_j$ this leads to the insolvency of the institution j, which in turn may generate a new round of losses to the creditors of j. This *domino effect* may be modeled by defining a *loss cascade*, updating at each step the losses to balance sheets resulting from previously defaulted counterparties:

Definition 1 (Loss cascade)**.** *Consider an initial configuration of capital reserves* $(c(j), j \in V)$*. We define the sequence* $(c_k(j), j \in V)_{k\geq 0}$ *as*

$$c_0(j) = c(j) \qquad \text{and} \quad c_{k+1}(j) = \max(c_0(j) - \sum_{\{i, c_k(i)=0\}} (1 - R_i)E_{ji}, 0), \tag{3}$$

where R_i *is an exogenous recovery rate at the default of institution* i*.* $(c_{n-1}(j), j \in V)$*, where* $n = |V|$ *is the number of nodes in the network, then represents the remaining capital once all counterparty losses have been accounted for. The set of insolvent institutions is then given by*

$$\mathbb{D}(c, E) = \{j \in V : c_{n-1}(j) = 0\} \tag{4}$$

Remark 1 (Fundamental defaults vs defaults by contagion)**.** *The set* $\mathbb{D}(c, E)$ *of defaulted institutions may be partitioned into two subsets*

$$\mathbb{D}(c, E) = \underbrace{\{j \in V : c_0(j) = 0\}}_{\text{Fundamental defaults}} \bigcup \underbrace{\{j \in V : c_0(j) > 0, \quad c_{n-1}(j) = 0\}}_{\text{Defaults by contagion}}$$

where the first set represents the initial defaults which trigger the cascade –we will refer to them as fundamental defaults– and the second set represents the defaults due to contagion.

The default of an institution can therefore propagate to other participants in the network through the contagion mechanism described above. To measure the systemic importance of the institution (say, i) triggering the loss cascade, we introduce the *Default Impact* $DI(i)$ of i that measures the loss incurred by the network in the default cascade triggered by the default of institution i:

Definition 2 (Default Impact)**.** *The* Default Impact $DI(i, c, E)$ *of a financial institution* $i \in V$ *is defined as the total loss in capital in the cascade triggered by the default of* i*:*

$$DI(i, c, E) = \sum_{j \in V} c_0(j) - c_{n-1}(j), \tag{5}$$

where $(c_k(j), j \in V)_{k\geq 0}$ *is defined by the recurrence relation* (3)*, with initial condition is given by*

$$c_0(j) = c(j) \quad \text{for} \quad j \neq i \quad \text{and} \quad c_0(i) = 0.$$

It is important to note that the Default Impact does not include the loss of the institution triggering the cascade, but focuses on the loss this initial default inflicts to the rest of the networks: it thus measures the loss due to contagion.

The contagion mechanism described above is similar to the one presented in Furfine (2003); Upper and Worms (2004); Mistrulli (2007). Since liquidation procedures are usually slow and settlements may take up several months to be effective, creditors cannot recover the residual value of the defaulting institution according to such a hypothetical clearing mechanism, and write down their entire exposure in the short-run, leading to a short term recovery rate of zero. In absence of a clearing mechanism, this approach seems more reasonable than the one proposed by Eisenberg and Noe (2001) which corresponds to a hypothetical situations where all portfolios are simultaneously liquidated. Finally, we note that this model does not capture medium- or long-term contagion: maintaining exposures constant over longer term horizons, as in (Elsinger et al., 2006a) is unrealistic since exposures and capital levels fluctuate significantly over such horizons.

3.3 Contagion Index of a financial institution

Economic downturns, market-place events, or decreased liquidity conditions can be a major source of systemic losses. It would be then interesting to introduce a metric of systemic importance that considers not only credit risk- such as the Default Impact- but also systemic events, such as market shocks that could affect the capital of all institutions at the same time.

We adopt the same approach as Cont and Moussa (2010). We introduce correlated negative market shocks $\epsilon_i, i = 1..n$ that reduce the Tier 1 capital of all institutions in the network with a severity that depends on the credit worthiness of each institution: institutions with higher default probabilities are affected by larger market shocks.

Each scenario of market shocks leads an initial set of institutions to default, and the default can propagate across the network through the loss cascade mechanism described in the previous section; that is the loss cascade starting with initial capital levels $\max(c(i) + \epsilon_i, 0)$. We introduce the *Contagion Index* $CI(i, c, E)$ of an institution i as a measure of the expected loss incurred by the network in the default cascade triggered by the default of institution i when the entire network is subject to correlated market shocks, conditional on the event that institution i has defaulted due to the market shock. Thus, while the Default Impact is a deterministic measure of the loss generated by an exogenous default of i, the Contagion Index is a measure of the expected loss generated by the failure of i in a stressed market, compounding the effects of both credit and market risks.

Given a (statistical) model for the market shocks ϵ generating stress scenarios, we now define, following Cont (2009), the

Definition 3 (Contagion Index)**.** *The Contagion Index $CI(i, c, E)$ of institution $i \in V$ is defined as its expected Default Impact in a market stress scenario:*

$$CI(i, c, E) = \mathbb{E}\left[DI(i, (c + \epsilon)_+, E) | \, c(i) + \epsilon_i < 0\right] \tag{6}$$

$CI(i, c, E)$ is the expected loss –measured in terms of capital– inflicted to the network in the default cascade triggered by the initial default of i. The averaging is done over scenarios where the market shocks trigger the default of i. As argued in Cont (2009), the Contagion Index measures the systemic impact of the failure of an institution.

The computation of this index involves a model of correlated market shocks affecting balance sheets. This correlation has been found to be significant in banking systems across different countries Lehar (2005). Different specifications –static or dynamic, factor-based or copula-based– are possible (see Cont and Moussa (2010)).

In this paper we consider a factor model of market shocks:

$$\epsilon_i = f_i(S, Z_i) \tag{7}$$

where S is a common factor and the Z_i's are IID random variables representing idiosyncratic shocks.

In the examples below, we model the joint distribution of shocks to balance sheets using a multivariate model with heavy-tailed marginals and a dependence structure described by a Cauchy copula (Cont and Moussa, 2010): The shock to the balance sheet of $i-th$ institution is given by:

$$\epsilon_i = \sigma_i F^{-1} \circ G\left(\rho S + (1-\rho) Z_i\right) \tag{8}$$

$$\tag{9}$$

where G is the cumulative distribution function of a standard Cauchy. F is the cumulative distribution function of a negative heavy-tailed random variable; as an example we use the law of a Cauchy variable conditioned to be negative. Here ρ is a dependence parameter, S represents a common factor and $Z_i, i = 1..n$ independent idiosyncratic factor with a standard Cauchy distribution. σ_i is a scale factor which is calibrated to the probability of default p_i of institution i:

$$\sigma_i = -\frac{c(i)}{F^{-1}(p_i)} \tag{10}$$

The probabilities of default are calibrated to historical default rates given by Standard & Poors ratings for the firms. Unlike the standard Gaussian copula model, where large shocks to a given balance sheet can only result from large systemic shocks, this model allows for defaults either due to large idiosyncratic shocks *or* large system-wide shocks.

The Contagion Index is computed by Monte Carlo simulation. It should be noted that, since we condition of "rare events", a naive Monte Carlo simulation is quite inefficient. We use an improved simulation procedure of the Contagion Index, based on a stratification method, presented in Cont and Moussa (2010).

4 Is default contagion a significant source of systemic risk?

Most empirical studies of interbank networks have pointed to the limited extent of default contagion (Sheldon and Maurer, 1998; Furfine, 2003; Upper and Worms, 2004; Wells, 2004; Elsinger et al., 2006a,b; Mistrulli, 2007). However, almost all these studies (with the exception Elsinger et al. (2006a,b)) examine the sole knock-on effects of the sudden failure of a single bank by an idiosyncratic shock, thus ignoring the compounded effect of both correlated market events and default contagion. A correlated market shock affecting the capital of all institutions in the network can considerably reduce the capital of the network, which makes it more vulnerable to potential losses and increases the likelihood of large default cascades. We explore in this section the extent of default contagion in the Brazilian financial system (section 4.1), and study the role market shocks have in generating channels of contagion across the network (section 4.2). We also analyze the contribution of *fundamental* defaults and *defaults by contagion* to the systemic risk of the network as whole (section 4.3). We compare the latter to the results obtained in Elsinger et al. (2006a) for the Austrian banking system.

4.1 Evidence for contagion

When studying the contagion risk an institution i may pose to the financial system, two interesting questions arise:

- How much would the financial system suffer if institution i fails?
- How many institutions in the system would become insolvent if institution i fails?

The Contagion Index provides an answer to the first question by measuring the contagion loss induced by the failure of institution i in a stressed market. The second question relates to the number of institutions that default by contagion in the cascade triggered by a default of institution i.

Definition 4. *We define the size $\kappa(i, c, E)$ of the default cascade initiated by the default of institution i as the expected number of defaults by contagion generated when the system is subject to correlated market* *shocks given that the shock triggers the default of i.*

$$\kappa(i, c, E) = \mathbb{E}\left[\sum_{j=1}^{n} 1_{c(j)+\epsilon_j>0, c_{n-1}(j)=0} | c(i) + \epsilon_i < 0\right] \tag{11}$$

We find that the size of default cascades varies across the institutions that trigger the cascade: most institutions do not seem to generate other defaults due to contagion in the system (see figure 8), however some institutions can trigger up to 4 defaults which represents about 3% of the financial system. This means that domino effects should not be measure by averaging across the entire network: one should condition on the event of default of each individual institution in the system.

This presence of contagion is confirmed by comparing the Contagion Index of each institution to its interbank liabilities: a Contagion Index which exceeds the institution's interbank liabilities is a signature of contagion. As shown in figure 9 the Contagion Index can significantly exceed (up to forty times) the interbank liabilities for the most systemic nodes. This indicates that default contagion is a significant component of systemic risk for these systemically important institutions.

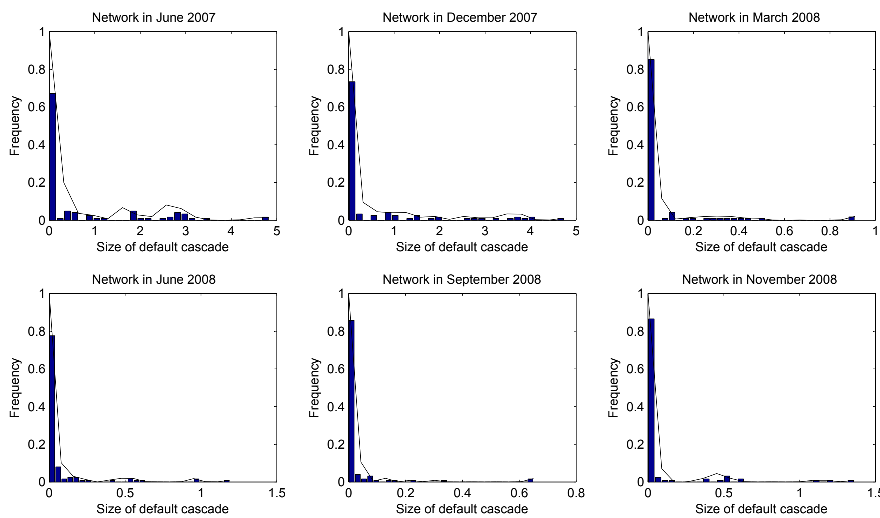


Figure 8: Distribution of the size of default cascade.

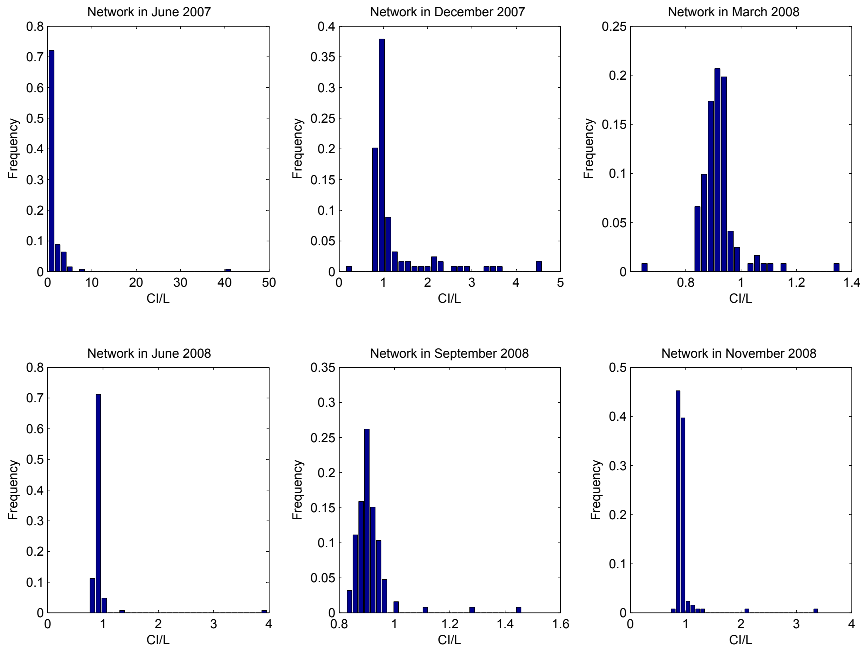


Figure 9: Ratio of the Contagion Index to the interbank liabilities: the Contagion Index can be up to forty times the size of interbank liabilities.

4.2 The role of correlated market shocks

Both the Default Impact and the Contagion Index exhibit heavy tailed distributions (see figures 11, 12) indicating the existence of few institutions that present a high contagion risk to the financial system (up to 10% of the total capital of the network) while most institutions exhibit a small risk. We also note that the probability of observing a large Default Impact and a large Contagion Index is the highest during June 2007 and December 2007 (see figure 10). These periods correspond to the appearance of the subprime mortgage crisis in the United States.

Figure 13 displays the cross-sectional distribution of the ratio of the Contagion Index to the Default Impact, in June 2007. We observe that the Contagion Index may, for some nodes, significantly exceed the Default Impact. Thus correlated shocks to balance sheets seem to *amplify* contagion. This comes from the fact that market shocks reduce the capital available to financial institutions and render them more susceptible to default.

Exposures that are not covered by an adequate amount of capital to sustain their loss in the event of default constitute channels of contagion across the system. We will call such exposures *contagious exposures*:

Definition 5 (Contagious Exposure)**.** *An exposure of institution i to j is called* contagious *if its size exceeds the capital of i: $E_{ij} > c(i)$.*

If the link $i \to j$ represents a contagious exposure, the default of j leads to the default of i in all stress scenarios. Thus, the subgraph constituted of contagious exposures will be a primary support for the propagation of default cascades: the larger this subgraph, the larger the extent of contagion. In a stress scenario in which balance sheets are subjected to negative market shocks, new contagious exposures may appear, leading to a higher degree of contagion. Figure 14 shows the graph of contagious exposures (black) in the Brazilian network in June 2007, with, in red, the exposures that become contagious once a (particular) set of correlated market shocks is applied to balance sheets. The role of contagious exposures is further explored in Amini et al. (2010) from a theoretical point of view.

Figure 15 presents the proportion of contagious exposures in the Brazilian system, their expected proportion under stress test scenarios, and their expected proportion in scenarios where the level of common downward shocks to balance sheets exceeds its 5% quantile. We find that correlated market shocks may increase the proportion of contagious exposures considerably, so ignoring market risk when assessing contagion effects can lead to a serious underestimation of the extent of default contagion.

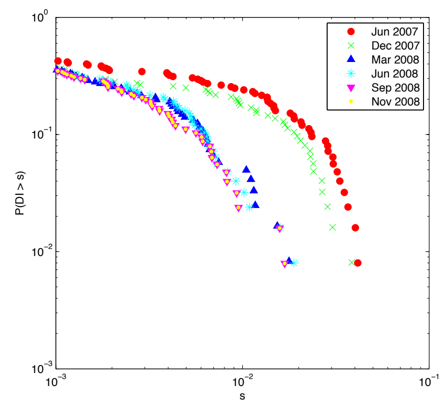


Figure 10: Brazilian interbank network: distribution of the default impact and the Contagion Index on the logarithmic scale. The highest probabilities of having a large Default Impact and a large Contagion Index are observed in June 2007 and December 2007.

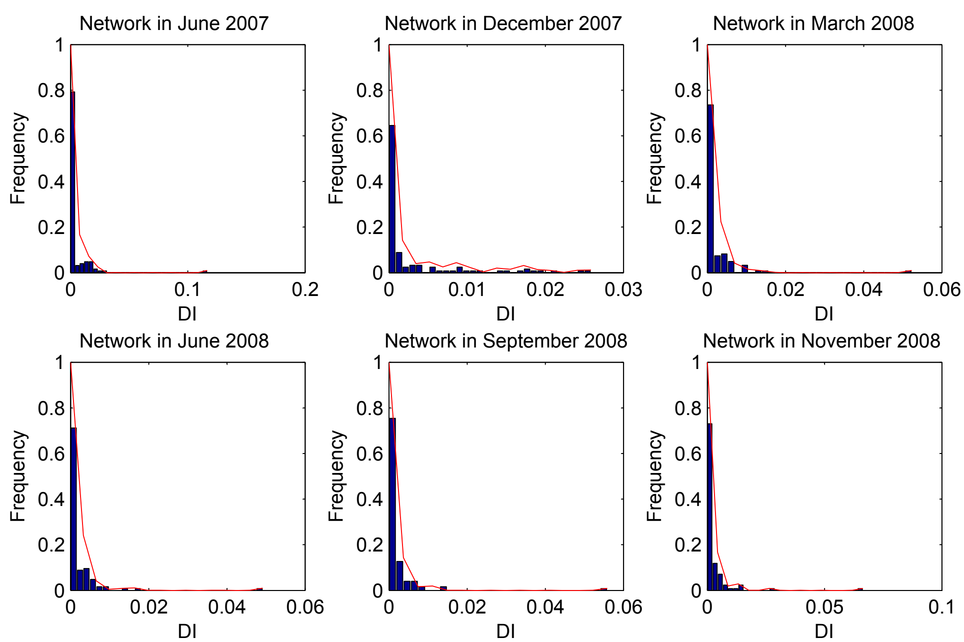


Figure 11: Distribution of the Default Impact. Most institutions have a small Default Impact, however, some can have an impact up to 10% of the total network capital (June 2007).

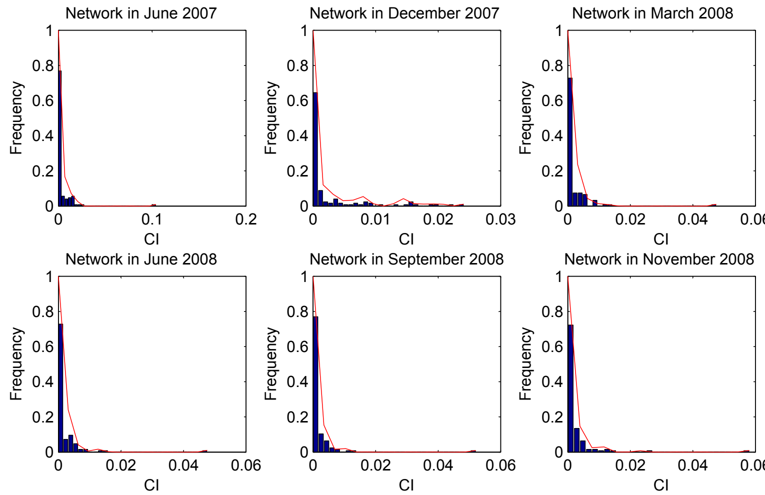


Figure 12: Distribution of the Contagion Index. Most institutions have a small Contagion Index, however, some can have an impact up to 10% of the total network capital (June 2007).

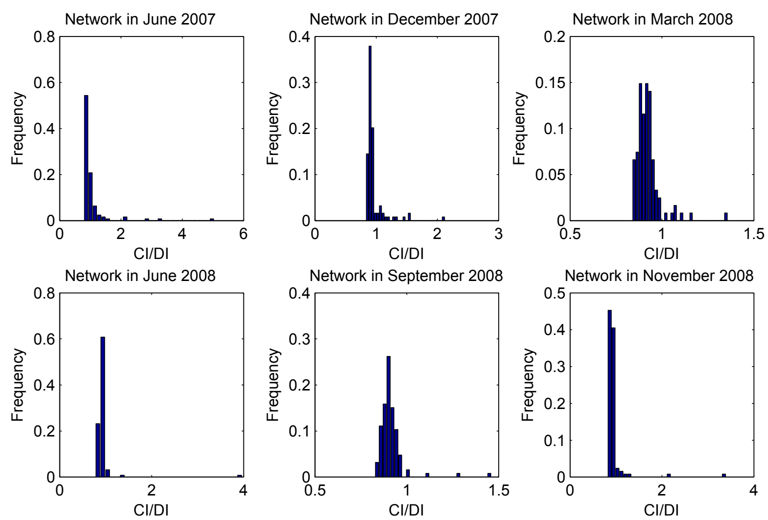


Figure 13: Default impact vs Contagion Index: the Contagion Index can be four times the Default Impact for some nodes.

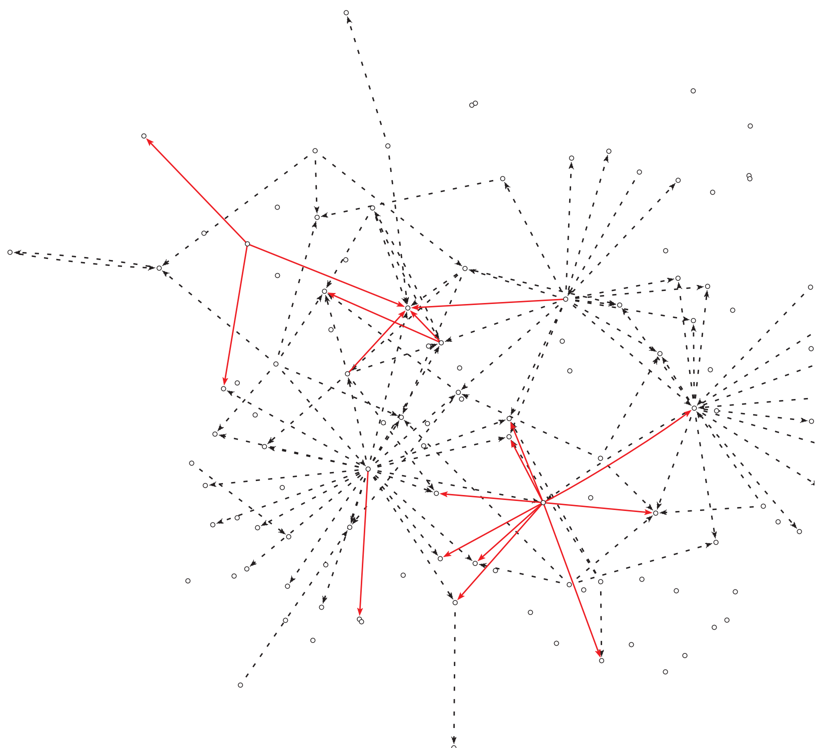


Figure 14: Network of contagious exposures before (dashed lines) and after (dashed and red lines) market shocks.

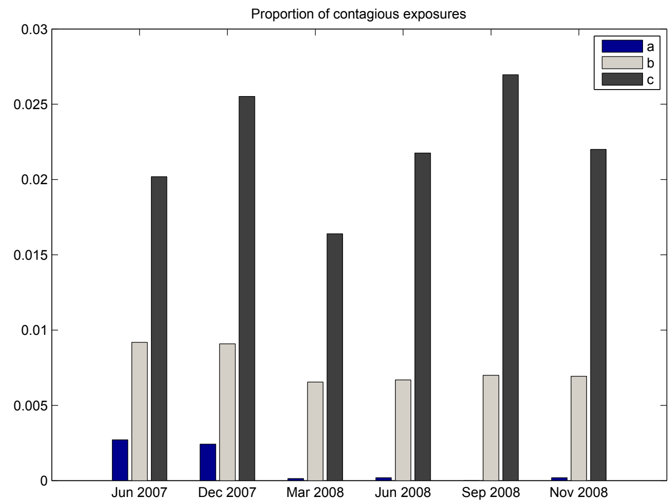


Figure 15: Proportion of contagious exposures (a) in the initial network, (b) averaged across market shock scenarios, (c) averaged across scenarios where common factor falls below 5% quantile level.

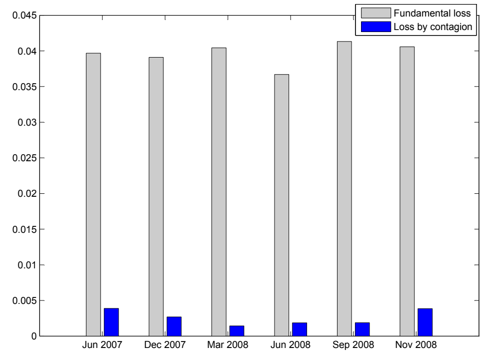
4.3 Fundamental losses vs losses by contagion

Elsinger et al. (2006a) distinguish *fundamental defaults* -due to exogenous market shocks- from defaults by contagion and perform a simulation study of the respective contributions to systemic risk of fundamental defaults and contagion effects. In their study on the Austrian banking network, fundamental defaults are found to be more frequent than contagion effects, which leads them to conclude that the main source of systemic risk is the correlation among risk factors influencing balance sheets.

We conduct a similar analysis to study the contribution of default contagion to systemic risk, albeit with a different metric, the Contagion Index. We classify all simulated default events into those resulting from large market shocks and those resulting from contagion. We define the *expected loss* (EL) incurred by the institutions at the end of the default cascade when the system is subject to market shocks given that the common factor of the market shocks falls below its 5%-quantile level. We decompose the (expected) losses into losses resulting from fundamental market shocks L_F and those resulting from contagion $L_C = EL - L_F$:

$$EL = \sum_{v=1}^{n} \mathbb{E}\left[c(v) - c_{n-1}(v)|S < S_{0.05}\right] \qquad L_F = \sum_{v=1}^{n} \mathbb{E}\left[c(v) - c_0(v)|S < S_{0.05}\right] \tag{12}$$

where $c_0(v) = (c(v) + \epsilon_v)_+$ and $S_{0.05}$ is the 5%-quantile of S. Figure 16 shows that the losses due to fundamental defaults are significantly larger (by a factor of 10) than the loss due to contagion. However, the number of defaults by contagion (Figure 16, below) is comparable to the number of fundamental defaults especially in June and December 2007. Thus, although fundamental defaults seem to be a major source of systemic risk, one cannot neglect the impact of contagion.



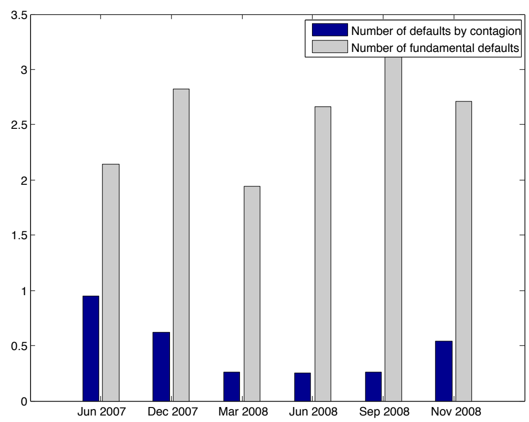


Figure 16: Fundamental loss vs loss by contagion when the system is subject to correlated market shocks, given that the common factor in the market shocks falls below its 5%-quantile. Above: fundamental loss vs loss by contagion in BRL. Below: expected number of fundamental vs contagious defaults.

As in Elsinger et al. (2006a), we also compute the probabilities of occurrence of contagion and the expected number of defaults due to contagion grouped by the number of fundamental defaults. We find much more scenarios with contagion than in Elsinger et al. (2006a): for more than two fundamental defaults, the scenarios with contagion are more frequent than those without contagion. Thus, default contagion cannot be ignored.

Fundamental defaults	Scenarios with no contagion (%)	Scenarios with contagion (%)	Number of defaults by contagion
0	47.67	0.00	0.00
1	25.52	10.11	0.78
2	6.81	6.23	1.49
3	1.18	1.90	2.00
4	0.13	0.28	2.79
5	0.02	0.10	5.44
6 and more	0.00	0.05	10.52
Total	81.34	18.66	

Table 7: Probabilities of occurrence of contagion and expected number of defaults due to contagion, grouped by the number of fundamental defaults.

5 What makes an institution systemically important?

Previous studies on contagion in financial networks (Allen and Gale, 2000; Battiston et al., 2009; Elsinger et al., 2006a; Nier et al., 2007) have examined how the network structure may affect the global level of systemic risk but do not provide metrics or indicators for localizing the source of systemic risk within the network. The ability to compute a Contagion Index for measuring the systemic impact of each institution in the network, enables us to locate the institutions which have the largest systemic impact and investigate their characteristics.

We first investigate (section 5.1) the effect of the size, measured in terms of interbank liabilities or assets on the Contagion Index. Then we examine (section 5.2) the effect of network structure on the Contagion Index and define, following Cont and Moussa (2010), network-based indicators of connectivity *counterparty susceptibility* and *local network frailty*, which are then shown to be significant factors for contagion.

5.1 The role of balance sheet size

Size is generally considered a factor of systemic importance. In our modeling approach, where losses flow in through the asset side and flow out through the liability side of the balance sheet, it is intuitive that, at least at the first iteration of the loss cascade, firms with large liabilities to other nodes will be a large source of losses for their creditors in case of default. Accordingly, interbank liabilities are highly correlated with any measure of systemic importance. A simple plot on the logarithmic scale of the Contagion Index against the interbank liability size reveals a strong positive relationship between the interbank liabilities of an institution in the Brazilian financial system and its Contagion Index (see figure 17). A linear regression of the logarithm of the Contagion Index on the logarithm of the interbank liability size supports this observation: interbank liabilities explains 96% of the cross-sectional variability of the Contagion Index.

Therefore, balance sheet size does matter, not surprisingly. However, the size of interbank liabilities does not entirely explain the variations in the Contagion Index across institutions: the interbank liability size does exhibit a strong positive relationship with the Contagion Index, but the ranking of institutions according to liability size does not correspond to their ranking in terms of systemic impact (see figure 17).

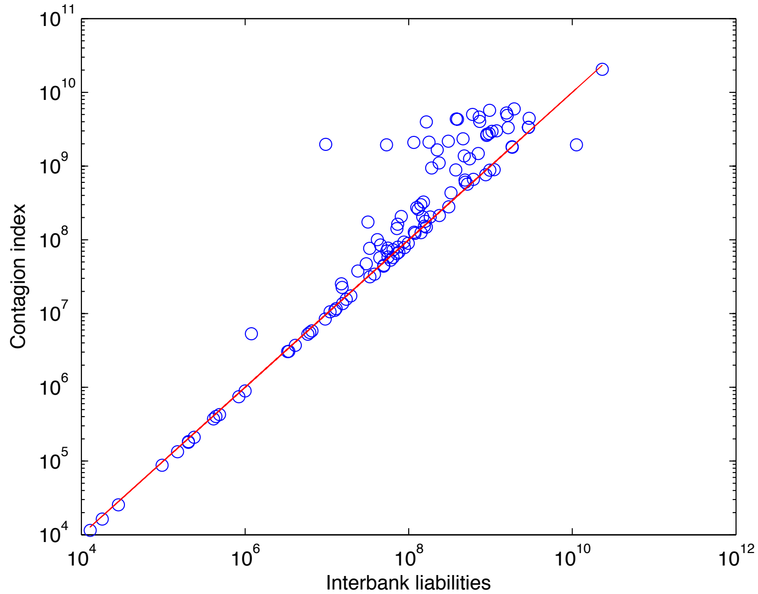


Figure 17: Scatterplot on the logarithmic scale of the Contagion Index versus the interbank liability size in June 2007.

Model: $log(CI) = \beta_0 + \beta_1 log(L) + \epsilon$			
Coefficients	**Standard error**	**t-statistic**	R^2
b_0 = -0.58	0.36	-1.41	96%
b_1 = 1.04**	0.02	51.75	

* significant at 5% confidence level
** significant at 1% confidence level

Table 8: Log-log cross-sectional regression of the Contagion Index (expressed in percentage of the total network capital) on the interbank liability in June 2007.

Table 9, where nodes are labeled according to their decreasing ranking in terms of the Contagion Index, shows that Node 5 has interbank liabilities less than the 90% quantile of the cross sectional interbank liability sizes. This suggests that factors other than size contribute to their systemic importance.

Ranking	**Contagion index** (in billion BRL)	**Number of creditors**	**Interbank liability** (in billion BRL)
1	20.77	8	23.27
2	4.95	32	1.57
3	4.58	13	2.96
4	3.85	14	1.95
5	3.40	21	0.97
Network median	0.10	5	0.07
90%-quantile	2.45	21	1.11

Table 9: Analysis of the five most contagious nodes in June 2007.

Plotting the Contagion Index against the interbank asset size (figure 18) shows that the contribution of the size of interbank assets to the Contagion Index is less significant. Note that interbank liabilities are not balanced with respect to interbank assets, due to deposits and other types of liabilities which are excluded from interbank liabilities.

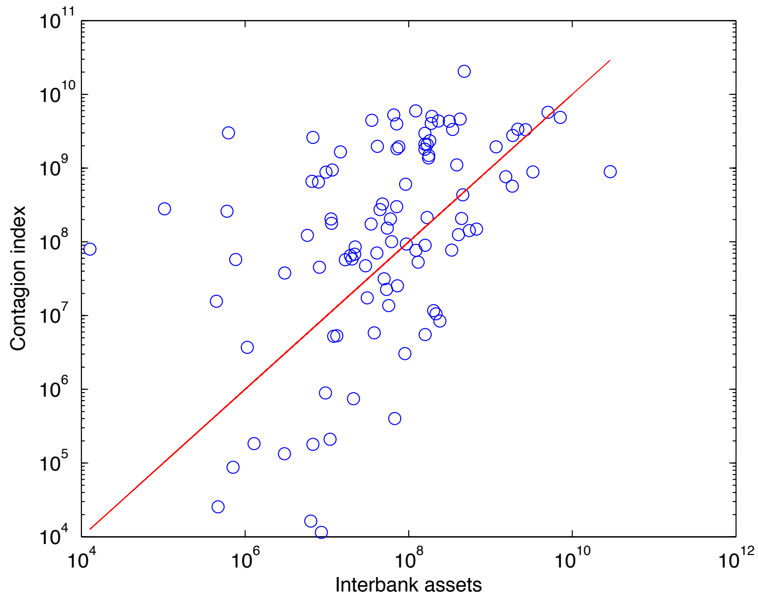


Figure 18: Scatterplot on the logarithmic scale of the Contagion Index versus the interbank assets size in June 2007.

Model: $log(CI) = \beta_0 + \beta_1 \log(A) + \epsilon$			
Coefficients	**Standard error**	**t-statistic**	**Adjusted** R^2
$b_0 = 8.00$ **	1.99	4.02	22%
$b_1 = 0.58$ **	0.11	5.20	

* significant at 5% confidence level
** significant at 1% confidence level

Table 10: Log-log cross-sectional regression of the Contagion Index on the size of interbank assets in June 2007: $R^2 = 22\%$.

5.2 The role of network structure

Table 9 shows that, while the sheer size of liabilities of the node with the highest Contagion Index can explain its ranking, the four other most systemic nodes have liability sizes roughly in line with the network average, so size effects alone do not explain the magnitude of their systemic impact. This points to the possible contribution of interconnectedness, or network structure, in explaining the magnitude of their Contagion Index. As shown in figure 19 the five most systemic nodes are not very connected and just have few contagious exposures (in red) but, as shown in figure 20, their *creditors* are heavily connected and many of their cross-exposures are contagious exposures (in the sense of Definition 5). This motivates to define indicators which go beyond simple measures of connectivity such as the degree (or weighted degree): following Cont and Moussa (2010), we define the following indicators which attempt to quantify the sensitivity of the counterparties of these nodes to their default:

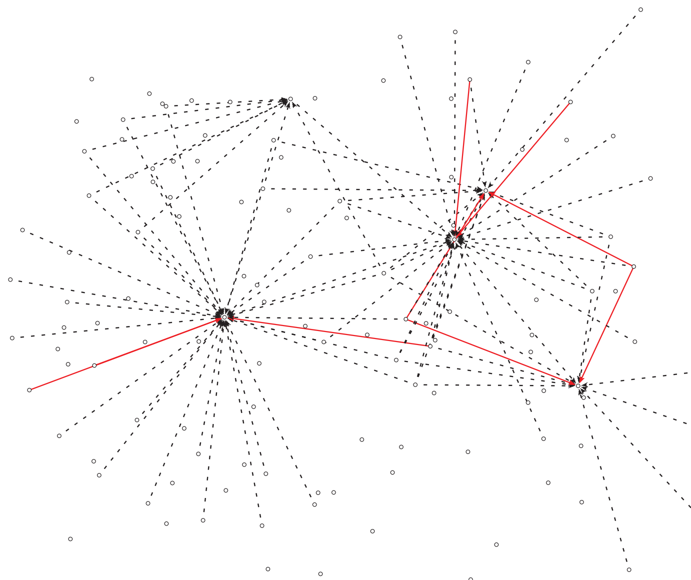


Figure 19: Subgraph of the five institutions with highest Contagion Index and their creditors in the network in June 2007. Non contagious exposures are dashed lines. Contagious exposures are full red lines.

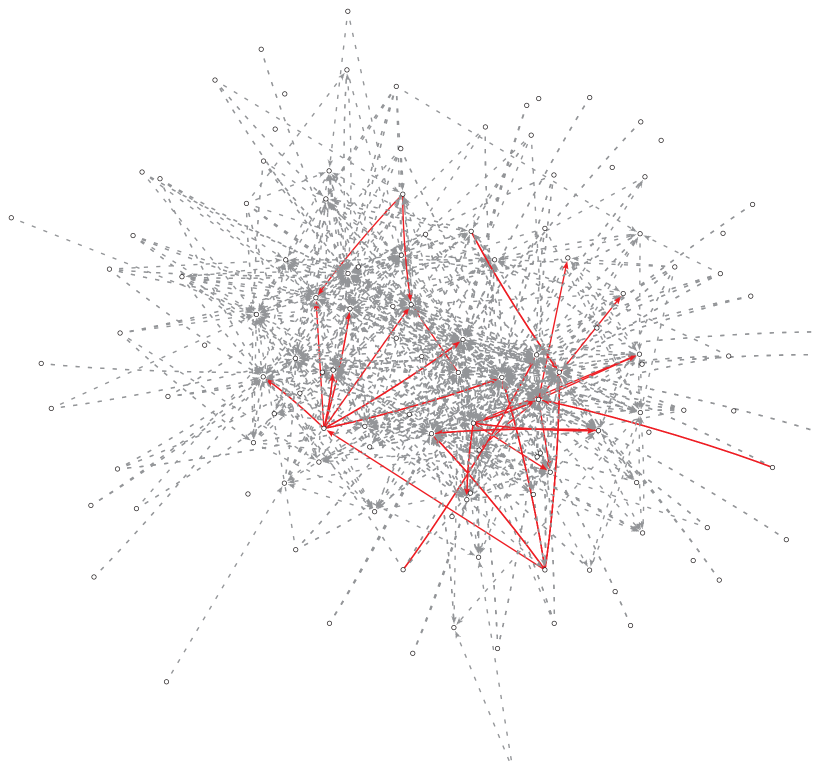


Figure 20: Subgraph of the five institutions with highest Contagion Index and their first and second-order neighbors in the network in June 2007. Non contagious exposures are dashed lines. Contagious exposures are full red lines.

Definition 6. *Susceptibility coefficient*
The susceptibility coefficient of a node is the maximal fraction of capital wiped out by the default of a single counterparty.

$$\chi(i) = \max_{j \neq i} \frac{E_{ij}}{c(i)}$$

A node with $\chi(i) > 100\%$ may become insolvent due to the default of a single counterparty. Counterparty risk management in financial institutions typically imposes an upper limit on this quantity.

Definition 7. *Counterparty susceptibility*
The counterparty susceptibility $CS(i)$ of a node i is the maximal (relative) exposure to node i of its counterparties:

$$CS(i) = \frac{\max_{j, E_{ji} > 0} E_{ji}}{c(j)}$$

$CS(i)$ is thus a measure of the maximal vulnerability of creditors of i to the default of i.

Definition 8. *Local network frailty*
The local network frailty $f(i)$ at node i is defined as the maximum, taken over counterparties exposed to i, of their exposure to i (in % of capital), weighted by the size of their interbank liability:

$$f(i) = \max_{j, E_{ji}>0} \frac{E_{ji}}{c(j)} \quad L(j)$$

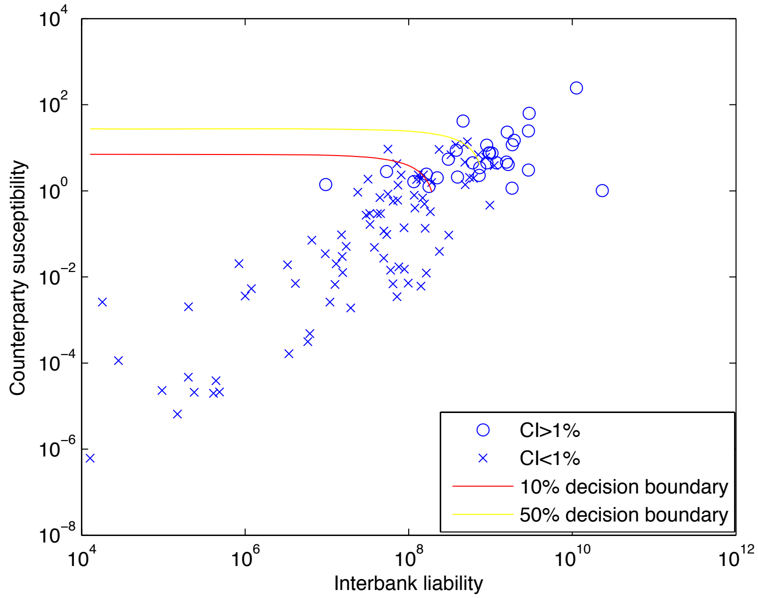
Thus, local network frailty combines two risk components: the risk that the counterparty incurs due to its exposure to node i, and the risk that the (rest of the) network incurs if this counterparty fails. A large value $f(i)$ indicates that i is a node whose counterparties have large liabilities *and* are highly exposed to i.

The analysis of the creditors of the five most systemic institutions in the network (see table 11) indicates that the number of creditors and the size of interbank liabilities of the counterparties, as well as the counterparty susceptibility and local network frailty, can explain a high Contagion Index of a financial institution when the size of its interbank liabilities fails to explain. We observe that the five most systemic nodes have each at least one very connected counterparty with a large interbank liability size. They exhibit in general a high counterparty susceptibility and local network frailty.

Ranking	$\max_{j,E_{ji}>0} k_{out}(j)$	$\max_{j,E_{ji}>0} L(j)$	$CS(i)$	$f(i)$
1	36	1.10	0.85	0.95
2	36	2.91	3.83	3.25
3	34	11.23	23.42	263.15
4	34	11.23	5.60	62.97
5	34	23.27	1.65	3.15
Network median	34	2.01	1.25	2.05
90%-quantile	36	11.23	3.04	6.89

Table 11: Analysis of the counterparties of the five most contagious nodes in June 2007. The counterparty interbank liability and local network frailty are expressed in billion BRL.

We are thus led to investigate whether we could rank systemically important institutions based on the measures of connectivity and centrality defined above. We classify institutions in the Brazilian system into those with a high Contagion Index (higher than 1% of the total network capital) and those with a small Contagion Index (smaller than 1% of the total network capital), according to their interbank liability, counterparty susceptibility and local network frailty. This can be achieved by conducting a logistic regression of the indicator of the Contagion Index being higher than 1% of the total network capital once on the interbank liability and counterparty susceptibility, and once on the interbank liability and local network frailty. Figure 21 displays the decision boundaries at the probabilities 10% and 50% when observing once the interbank liability size and the counterparty susceptibility and once the interbank liability size and the local network frailty: a node outside the 10% decision boundary has an estimated probability of 10% to have a Contagion Index higher than 1% of the network capital; a node outside the 50% decision boundary has an estimated probability of 50% to have a Contagion Index higher than 1% of the network capital. We note that institutions with a high Contagion Index tend to have a large interbank liability, local network frailty and counterparty susceptibility.



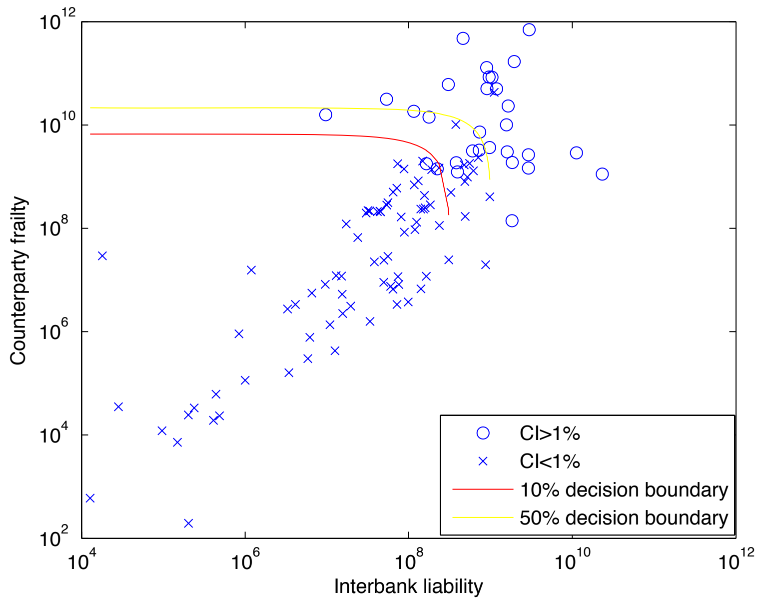


Figure 21: Counterparty susceptibility (upper figure) and local network frailty (lower figure) of the most systemic nodes (with a Contagion Index higher than 1% of the network capital) and the less systemic nodes (with a Contagion Index smaller than 1% of the network capital). Nodes above the 10% decision boundary have with 10% probability a Contagion Index higher than 1% of the network capital. The ones above the 50% decision boundary have with 50% probability a Contagion Index higher than 1% of the capital in the Brazilian system.

The outputs of the logistic regression are summarized in table 12. We observe that the counterparty susceptibility and the local network frailty contribute significantly to the variability of the probability of observing a large Contagion Index [1]: positive coefficients at the 1% significance level and a very high pseudo-R^2.

[1] The Adjusted Pseudo-R^2 in a logistic regression is defined as $1 - logL(M)/logL(0)((n-1)/(n-k-1))$ where $logL(M)$ and $logL(0)$ are the maximized log likelihood for the fitted model and the null model, n is the sample size and k is the number of regressors.

Model: $logit(p(CI > 1\%)) = \beta_0 + \beta_1 \log(L) + \beta_2 \log(CS) + \epsilon$		
Coefficients	**Standard error**	**Adjusted Pseudo-R^2**
$\widehat{\beta_0}$ = -20.85**	7.96	93.46%
$\widehat{\beta_1}$ = 0.96*	0.39	
$\widehat{\beta_2}$ = 0.98*	0.40	
Model: $logit(p(CI > 1\%)) = \beta_0 + \beta_1 \log(L) + \epsilon$		
Coefficients	**Standard error**	**Adjusted Pseudo-R^2**
$\widehat{\beta_0}$ = -29.24**	7.11	94.54%
$\widehat{\beta_1}$ = 1.39**	0.34	
Model: $logit(p(CI > 1\%)) = \beta_0 + \beta_1 \log(CS) + \epsilon$		
Coefficients	**Standard error**	**Adjusted Pseudo-R^2**
$\widehat{\beta_0}$ = -1.46**	0.37	43.36%
$\widehat{\beta_1}$ = 1.31**	0.33	
Model: $logit(p(CI > 1\%)) = \beta_0 + \beta_1 \log(L) + \beta_2 \log(f) + \epsilon$		
Coefficients	**Standard error**	**Adjusted Pseudo-R^2**
$\widehat{\beta_0}$ = -43.20**	11.06	97.76%
$\widehat{\beta_1}$ = 1.05**	0.39	
$\widehat{\beta_2}$ = 0.97**	0.29	
Model: $logit(p(CI > 1\%)) = \beta_0 + \beta_1 \log(f) + \epsilon$		
Coefficients	**Standard error**	**Adjusted Pseudo-R^2**
$\widehat{\beta_0}$ = -21.32**	4.75	93.79%
$\widehat{\beta_1}$ = 0.95**	0.22	

* significant at 5% confidence level
** significant at 1% confidence level

Table 12: Marginal contribution of the interbank liabilities, counterparty susceptibility and local network frailty to the Contagion Index.

We also test for the differences in median between the counterparty susceptibility of the institutions with a Contagion Index higher than 1% of the total network capital and the counterparty susceptibility of those with a Contagion Index smaller than 1% of the total network capital. The Wilcoxon signed-rank test rejects the hypothesis of equal medians at the 1% level of significance. The median of the counterparty susceptibility of the institutions with a high Contagion Index (2.29) is significantly higher than the median of the counterparty susceptibility of the institutions with a small Contagion Index (0.06). Similarly, the median of the local network frailty of the institutions with a high Contagion Index (18.79 billion BRL) is significantly higher than the median of the local network frailty of the institutions with a small Contagion Index (0.02 billion BRL).

6 Does one size fit all? The case for targeted capital requirements

Capital requirements are a key ingredient of bank regulation: in the Basel Accords, a lower limit is imposed on the ratio of capital to (risk-weighted) assets. It is clear that globally increasing the capital cushion of banks will decrease the risk of contagion in the network, but given the heterogeneity of systemic importance, as measured by the Contagion index, it is not clear whether a *uniform* capital ratio for all institutions is the most efficient way of reducing systemic risk. Indeed, recent debate has considered the option of more stringent capital requirements on systemically important institutions. One idea, which we explore here, is to impose higher capital requirements on institutions whose position in the network plays a key role in the network's resilience to contagion.

Studies in epidemiology or the spread of viruses in peer-to-peer networks (Cohen et al., 2003; Madar et al., 2004; Huang et al., 2007) have explored similar problems in the context of immunization of heterogeneous networks to contagion. Madar et al. (2004) study various immunization strategies in the context of epidemic modeling. They show that in *random immunization* schemes, where nodes are randomly chosen and vaccinated, the whole population must get vaccinated to effectively control epidemic propagation. They propose instead a *targeted immunization* strategy that consists in vaccinating first the nodes with largest degrees. A third approach, called *acquaintance immunization* (Cohen et al., 2003; Madar et al., 2004), which consists in immunizing randomly selecting individuals as well as their acquaintances, is shown to perform better than random immunization, especially in scale-free networks.

Based on these analogies, we consider *targeted capital requirement* policy which consists in imposing capital requirements on the the 5% most systemic institutions in the network and their *creditors*: this aims at reducing the number of contagious links (see Definition 5) emanating from the most systemic institutions, since these links play a major role in contagion of default in the network Amini et al. (2010).

We consider two different policies for setting capital requirements:

- Minimum capital-to-exposure ratio: in this case, we require institutions to hold a capital larger than $\bar{c}$ that could cover at least a portion θ of their interbank exposures:

$$\bar{c}(i) = \max(c(i), \theta A(i)) \tag{13}$$

- Cap on susceptibility: Counterparty susceptibility (Definition 7) and local network frailty (Definition 8) are a significant source of systemic risk. Thus, preventing large values of counterparty susceptibility or network frailty from occuring can decrease systemic risk. This could be achieved by requiring that no exposure should represent more than a fraction γ of capital. In this case, a financial institution i is required to hold a capital larger than $\bar{c}$ given by:

$$\bar{c}(i) = \max(c(i), \frac{\max_{j \neq i}(E_{ij})}{\gamma}) \tag{14}$$

We compare the situations in which (i) these policies are applied to all financial institutions in the network (*non-targeted capital requirements*), (ii) they are applied only to the creditors of the 5% most systemic institutions (*targeted capital requirements*), by computing, in each case, the average of 5% largest Contagion Indexes (i.e. the 5% tail conditional expectation of the cross sectional distribution of Contagion Index) in the Brazilian network

Targeted capital requirements are observed to be more efficient in the sense that one can achieve the same reduction in systemic risk -in terms of the cross-sectional tail of the Contagion Index- with the same amount of capital, differently distributed across the network. Figure 22 shows that targeted capital requirements can achieve the same reduction in the size of default cascades while requiring less capital.

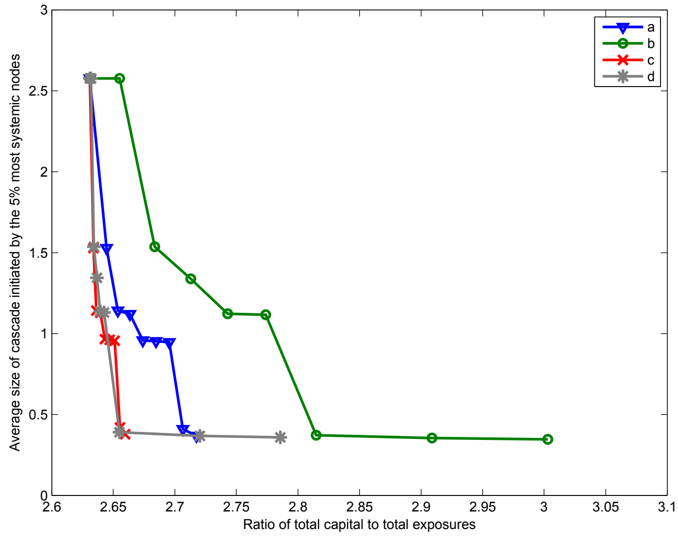


Figure 22: Comparison of various capital requirement policies: (a) imposing a floor on the capital ratio for all institutions in the network, (b) imposing a cap on the susceptibility for all institutions in the network, (c) imposing a floor on the capital ratio only for the creditors of the 5% most systemic institutions, (d) imposing a cap on the susceptibility only for the creditors of the 5% most systemic institutions.

While it is clear that raising capital requirements reduces the number of defaults by contagion, the impact on the Contagion Index is the result of two competing effects. One has to bear in mind that increasing capital requirements will mainly increase the capital of the most fragile institutions, since those already well-capitalized satisfy the requirements without any additional capital. Thus, the proportion of total capital invested in fragile institutions increases, and consequently the Contagion Index expressed in percentage of the total capital in the system may increase. In fact, we observe that the Contagion Index is decreasing when imposing these restrictions on the creditors of the 5% most systemic institutions (see figure 23), and globally decreasing when imposing these restrictions on all the institutions in the system. We also find that targeting the creditors of the most systemic nodes is a more efficient procedure to reduce the Contagion Index: for a same level of total capital the Contagion Index is smaller.

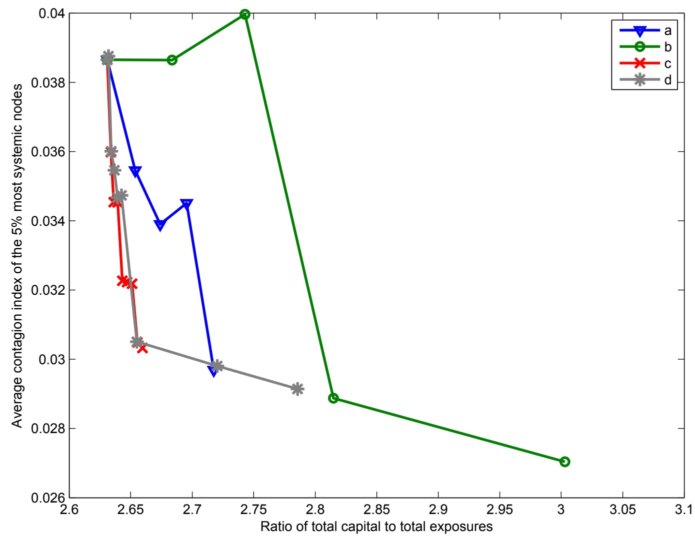


Figure 23: Comparison of various capital requirement policies: (a) imposing a floor on the capital ratio for all institutions in the network, (b) imposing a cap on the susceptibility for all institutions in the network, (c) imposing a floor on the capital ratio only for the creditors of the 5% most systemic institutions, (d) imposing a cap on the susceptibility only for the creditors of the 5% most systemic institutions.

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