MATH 1AA3 **Written Assignment #3** **March 21, 2012**

Due in the Hamilton Hall basement dropbox by noon, Wednesday April 4, 2012.

1. Find the solution to the initial value problem $y^2 = 1 - y' \cos(x)$, $y(0) = 0$ as follows:
   (a) Write $y = \sum_{n=0}^{\infty} a_n x^n$ and plug into both sides of the equation. Solve for a_0, a_1, a_2, a_3.
   (b) Guess $y(x)$. Check that your guess is correct.

2. (a) Using the fact that
   $$\arcsin(x) = \int_0^x \frac{dt}{\sqrt{1-t^2}},$$
   find a power series expansion for $\arcsin(x)$ centred at 0. State any theorems you use.
   (b) Write down the radius of convergence of the power series you derive in part (a). Find the interval of convergence of this power series.
   (c) By choosing an appropriate value of x to plug into the power series you found in part (a), find a series that converges to $\pi/2$.
   (d) Rederive your formula in (a) by determining the sequence $\{c_n\}$ of numbers for which
   $$\sum c_n \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)^n = x$$
   are equal power series.

3. Question 19., Problems Plus, p.783, Stewart 7th Ed. (Hint: Use the Maclaurin series for $\arctan(x)$).

4. (a) What is the Taylor series for e^x about $x = 0$?
   (b) For e^{-x}?
   (c) The *hyperbolic sine function* is given by the power series: $\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$. Using your answers to (a) and (b), write down a formula for $\sinh(x)$ that does not involve any infinite series.
   (d) The *hyperbolic cosine function*, $\cosh(x)$, is the derivative of $\sinh(x)$. Write down an explicit formula for $\cosh(x)$.
   (e) Determine the power series expansion for $\cosh(x)$ about $x = 0$.
   (f) What is the derivative of $\cosh(x)$?
   (g) Sketch the parametric curve $x = \cosh(t), y = \sinh(t)$. Eliminate the parameter to find a Cartesian equation of the curve. Which conic (or piece of a conic) is this?

5. Find the Taylor polynomial of smallest degree of an appropriate function about a suitable point to approximate $\sqrt{9.01}$ to within 0.00005.

6. (a) Using the Maclaurin series for e^x, $\sin(x)$, and $\cos(x)$, prove that $e^{ix} = \cos(x) + i\sin(x)$. Here, $i = \sqrt{-1}$ satisfies $i^2 = -1$. Read Stewart p. A63 if you get stuck.
   (b) Deduce that $e^{i\pi} = -1$.

7. Which of the following are hyperbolas? For the hyperbolas, determine their foci.
   (a) $x^2 - 2x - 4y^2 = 3$
   (b) $y = 1/x$
   (c) $y = 1/x^2$

8. Stewart Exercise 50, p 687.

9. Stewart Exercise 52, p 687.