MATH 1AA3 **Written Assignment #1** **January 18, 2012**

This assignment is due at the start of class on Monday January 30, 2012.

1. Stewart (7th Ed) Exercise 7.8 #64 (p. 528),

2. (a) Is the integral $\int_{-1}^{1} \frac{dx}{x}$ convergent or divergent?

   (b) Compute the limit $\lim_{t\to 0^+} \left[\int_{-1}^{-t} \frac{dx}{x} + \int_{t}^{1} \frac{dx}{x}\right]$.

   (c) Compute the limit $\lim_{t\to 0^+} \left[\int_{-1}^{-t} \frac{dx}{x} + \int_{\alpha t}^{1} \frac{dx}{x}\right]$ as a function of α for $\alpha > 0$.

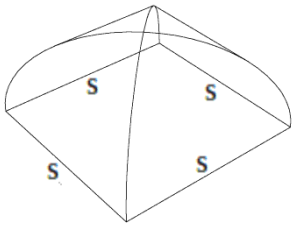
   (d) What can you say about the value of the expression $\infty - \infty$?

3. Consider the "infinite" lamina R of uniform density that lies between the lines $x = 0$ and $x = 1$, above the x-axis, and beneath the graph of the function $y = -\ln x$.

   (a) For $t \in (0, 1)$, find the centre of mass (i.e., centroid) of a lamina of uniform density that lies between the lines $x = t$ and $x = 1$, above the x-axis, and beneath the graph of the function $y = -\ln x$.

   (b) By taking the limit as $t \longrightarrow 0^+$ of your answer to part (a), compute the centre of mass of the lamina R. Is it contained within R?

4. Let C_x and C_y be two infinitely long solid cylinders of equal diameter s whose axes coincide with the x- and y-axis respectively. Consider the boundary of the intersection of these two cylinders. The *domical vault* (shown in the diagram) is the "top half" of this boundary, i.e., the boundary of the intersection of C_x and C_y that is above the xy-plane. The base of the vault is a square with side length s.

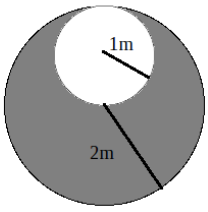


(a) Derive *and evaluate* an integral for the surface area of the vault (excluding the square base) in terms of s.

(b) Find the volume V of the vault in terms of s. How does dV/ds relate to the surface area you found in part (i)? Explain why this relationship should be expected.

Hint for (a) *and* (b): *Consider horizontal cross-sections of the vault.*

5. You are placed in an empty cylindrical tank constructed from two pipes, a small one with radius $1m$ and a larger one with radius $2m$. The end door of the tank is a vertical circular disk with a circular hole removed as pictured. The tank is then submerged at the bottom of a $7m$ deep still lake, oriented as shown. Compute the total hydrostatic force required to push open the end of the tank.



6. Stewart (7th Ed) Exercise 9.2 #20 (p. 593),

7. Stewart (7th Ed) Exercise 9.3 #40 (p. 601),

8. Stewart (7th Ed) Problems Plus p. 633 #2.