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STAT 3J04
PROBABILITY AND STATISTICS FOR ENGINEERING
In-Class Quiz #1

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October 17, 2006

1. A contractor is submitting bids to two jobs A and B. The probability that he will win job A is $P(A) = 0.25$ and that for job B is $P(B) = 0.33$.

(a) Assuming that winning job A and winning job B are independent events, what is the probability that the contractor will get at least a job?

(b) What is the probability that the contractor got job A if he has won at least one job?

(c) If he is also submitting a bid for job C with probability of winning it $P(C) = 0.25$, what is the probability that he will get at least one job? Again assume statistical independence among A, B and C. What is the probability that the contractor will not get any job at all?

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 Solution:

(a) $P(\text{contractor will win at least a job})$

$$= P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.25 + 0.33 - 0.25 \times 0.33 = 0.4975$$

$$(b) P(A | A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.25}{0.4975} = 0.503$$

(c) $P(\text{contractor will win at least a job})$

$$= P(A \cup B \cup C)$$

$$= 1 - P(\overline{A \cup B \cup C})$$

$$= 1 - P(\bar{A} \bar{B} \bar{C})$$

$$= 1 - (1 - 0.25)(1 - 0.33)(1 - 0.25) = 0.623$$

$P(\text{contractor will not get any job})$

$$= 1 - P(\text{contractor will get at least one job}) = 1 - 0.623 = 0.377$$

2. In a building construction project, the completion of the building requires the successive completion of a series of activities. Define

E = excavation completed on time; and $P(E) = 0.8$

F = foundation completed on time; and $P(F) = 0.7$

S = superstructure completed on time; and $P(S) = 0.9$

Assume statistical independence among these events.

(1) Define the event "project completed on time" in terms of E, F and S. Compute the probability of on-time completion.

(2) Define, in terms of E, F, S and their complements, the following event:

G = excavation will be on time and at least one of the other two operations will not be on time

Calculate $P(G)$

(3) Define the event:

H = only one of the three operations will be on time.

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Solution:

$$(1) \text{ project completed on Time} = E \cap F \cap S$$

$$P(\text{project completed on Time}) = P(E \cap F \cap S)$$

$$= P(E) \cdot P(F) \cdot P(S)$$

$$= 0.8 \times 0.7 \times 0.9 = 0.504$$

$$(2) G = E \cap (\bar{F} \cup \bar{S})$$

$$P(\bar{F} \cup \bar{S}) = P(\bar{F}) + P(\bar{S}) - P(\bar{F}) \cdot P(\bar{S})$$

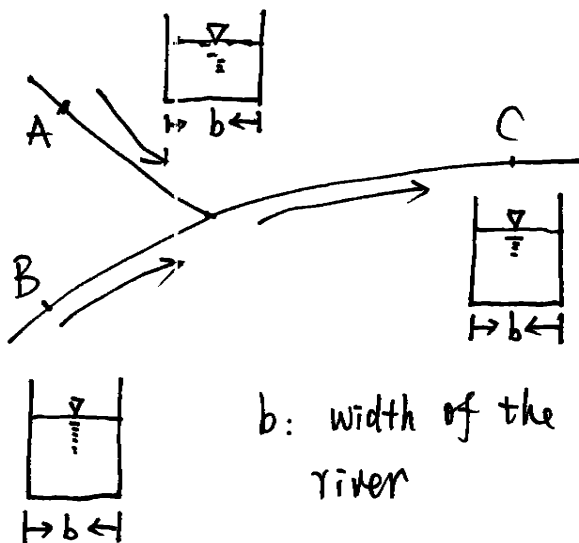
$$= 0.3 + 0.1 - 0.3 \times 0.1 = 0.37$$

$$\cdot P(G) = P(E) \cdot P(\bar{F} \cup \bar{S})$$

$$= 0.8 \times 0.37 = 0.296$$

$$(3) H = (E \bar{F} \bar{S}) \cup (\bar{E} F \bar{S}) \cup (\bar{E} \bar{F} S)$$

3. The cross-sections of the rivers at A, B, and C are shown below. The flood levels at A and B, above mean flow level, are as follows:



Flood level at A (ft)	Probability
0	0.25
2	0.25
4	0.25
6	0.25

Flood level at B (ft)	Probability
0	0.20
2	0.20
4	0.20
6	0.20
8	0.20

Assume that the flow velocities at A, B, and C are the same. What is the probability that the flood at C will be higher than 6 ft above the mean level? Assume statistical independence between flood levels at A and B.

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Solution:

Let Q_A , Q_B , and Q_C denote the flow rate in streams A, B, and C, respectively. From continuity, $Q_A + Q_B = Q_C$

$$\text{thus } b \cdot h_A \cdot v_A + b \cdot h_B \cdot v_B = b \cdot h_C \cdot v_C$$

$$\text{Since } v_A = v_B = v_C, \text{ we have } h_A + h_B = h_C$$

$P(\text{the flood at C will be higher than 6 feet})$

$$= P(h_C > 6 \text{ ft}) = P(h_A + h_B > 6 \text{ ft})$$

h_A	h_B	$h_A + h_B$	probability of occurrence
0	8	8	$P(h_A=0) \cdot P(h_B=8) = 0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$ $0.25 \times 0.20 = 0.05$
2	6	8	
2	8	10	
4	4	8	
4	6	10	
4	8	12	
6	2	8	
6	4	10	
6	6	12	
6	8	14	

$$\text{So } P(h_C > 6 \text{ ft}) = 10 \times 0.05 = 0.50$$