E4 -1

Exercise 4:

A building contractor requires a roll of roofing felt. There are three suppliers in the area and the probabilities (based on his previous experiences and the location of the suppliers) that the contractor will instruct his vanman to visit a particular supplier are:

A: the vanman goes to supplier A, P[A] = 0.6
B: the vanman goes to supplier A, P[B] = 0.2
C: the vanman goes to supplier A, P[C] = 0.2

Each supplier stocks roofing felt produced by two manufacturers, X and Y. Both types of roofing felt sell at the same price and both satisfy the current building regulations. The stock situation at each of the suppliers is:

Supplier	No. of 'X' rolls	No. of 'Y' rolls
A	10	30
B	30	20
C	30	10

The vanman will be told by his employer which supplier to visit.

(1)Which roll type is the vanman most likely to return with?
(2)If the vanman returned with an X type of roofing felt, determine the probability that he obtained it from supplier B.

Solution:

(1) Let $P(X)$ be the probability that the vanman will return with roll type X. Then, considering the stock position of each supplier:

$$P(X|A) = \frac{10}{40} = 0.25$$

$$P(X|B) = \frac{30}{50} = 0.6$$

$$P(X|C) = \frac{30}{40} = 0.75$$

From the law of total probability:

$$P(X) = P(X|A)\,P(A) + P(X|B)\cdot P(B) + P(X|C)\cdot P(C)$$

$$= 0.25 \times 0.6 + 0.6 \times 0.2 + 0.75 \times 0.2$$

$$= 0.42$$

E4-3

(2) Let events ~~A~~, B and C be B_1, B_2, and B_3, so that $P(B|X)$ is expressed as $P(B_2|X)$. By Bayes' theorem,

$$P(B_2|X) = \frac{P(X|B_2)\cdot P(B_2)}{\sum_{i=1}^{3} P(X|B_i)\,P(B_i)}$$

$$= \frac{0.6 \times 0.2}{0.25\times 0.6 + 0.6\times 0.2 + 0.75\times 0.2}$$

$$= 0.286$$

Ex-1

Exercise 5:

A foundation will be subjected to two dependent loadings, A and B, which can both vary in magnitude from 200 to 800 kN.

The design engineer has estimated values for P [Bi], the probabilities of B achieving magnitudes of 200, 400, 600 and 800 kN and for P [Ai| Bi], the conditional probabilities of A achieving the same magnitudes.

These values are set out in the table.

	$P(A_i \mid B_i)$					$P(B_i)$
	A_{200}	A_{400}	A_{600}	A_{800}	$\sum P(A_i \mid B_i)$	
B_{200}	0.1	0.5	0.3	0.1	1.0	0.3
B_{400}	0.3	0.5	0.15	0.05	1.0	0.4
B_{600}	0.6	0.25	0.1	0.05	1.0	0.2
B_{800}	0.7	0.15	0.1	0.05	1.0	0.1

(1) Determine the probabilities that A will achieve 200, 400, 600, and 800 kN.
(2) Determine the probabilities that the total load will be equal 1000 kN.

E5-2

Solution

(1) $$P(A_{400}) = \sum_{i=200}^{800} P(A_{400}|B_i)\, P(B_i)$$

$$= 0.3 \times 0.3 + 0.5 \times 0.4 + 0.25 \times 0.2 + 0.15 \times 0.1$$

$$= 0.355$$

(2) Let $(L=1000)$ be the event that the total foundation load equals 1000 kN. Then:

$$(L=1000) = (B_{200} \cap A_{800}) \cup (B_{400} \cap A_{600}) \cup (B_{600} \cap A_{400}) \cup (B_{800} \cap A_{200})$$

Hence:

$$P(L=1000) = P\left[(B_{200} \cap A_{800}) \cup (B_{400} \cap A_{600}) \cup (B_{600} \cap A_{400}) \cup (B_{800} \cap A_{200})\right]$$

$$= P(B_{200} \cap A_{800}) + P(B_{400} \cap A_{600}) + P(B_{600} \cap A_{400}) + P(B_{800} \cap A_{200})$$

$$= P(A_{800}|B_{200}) \cdot P(B_{200}) + P(A_{600}|B_{400}) \cdot P(B_{400})$$

E5-3

$$+ P(A_{400} \mid B_{600}) \cdot P(B_{600})$$

$$+ P(A_{200} \mid B_{800}) \cdot P(B_{800})$$

$$= 0.1 \times 0.3 + 0.15 \times 0.4 + 0.25 \times 0.2 + 0.7 \times 0.1$$

$$= 0.21$$

Exercise 6:

Three factories F1, F2, and F3 in an industrial area occasionally release lethal waste into a river. Probabilities of fish killing ingredients (as event A) being in the waste releases of the three factories are : P(A| F1) =0.75, P(A| F2) =0.85, and P(A| F3)=0.05. The a priori probabilities of factories releasing this particular waste are equal, P(F1) = P(F2) = P(F3) = 1/3. The factories are not related in their operations. When a waste release has occurred, what is probability of killing the fish and what are probabilities, of each factory being responsible.

Solution:

total probability

$$P(A) = P(A|F_1)\cdot P(F_1) + P(A|F_2)\cdot P(F_2) + P(A|F_3)\cdot P(F_3)$$

$$= 0.75 \times \frac{1}{3} + 0.85 \times \frac{1}{3} + 0.05 \times \frac{1}{3}$$

$$= 0.55$$

Bayes' theorem:

$$P(F_1|A) = \frac{P(A|F_1)\cdot P(F_1)}{\sum_{i=1}^{3} P(A|F_i)\cdot P(F_i)}$$

E6-2

$$= \frac{0.75 \times \frac{1}{3}}{0.55}$$

$$= 0.45$$

$$P(F_2|A) = \frac{P(A|F_2)\cdot P(F_2)}{\sum_{i=1}^{3} P(A|F_i)P(F_i)}$$

$$= \frac{0.85 \times \frac{1}{3}}{0.55}$$

$$= 0.52$$

$$P(F_3|A) = \frac{P(A|F_3)\cdot P(F_3)}{\sum_{i=1}^{3} P(A|F_i)\cdot P(F_i)}$$

$$= \frac{0.05 \times \frac{1}{3}}{0.55}$$

$$= 0.03$$

Therefore, the investigation should start at F_2, proceed to F_1 if F_2 is not found responsible and finish with F_3 if neither F_2 nor F_1 are responsible.