A05 Q3 SOLUTIONS

FIRST, ENTER ALL 12 OBS. INTO YOUR CALCULATOR TO GET GRAND SUM AND SUM OF SQUARES

$$\sum\sum\sum y_{ijk} = 193.3 \qquad \sum\sum\sum y_{ijk}^2 = 3200.71$$

NEXT, COMPUTE TOTAL AT EACH DESIGN POINT AND GET ROW & COLUMN TOTALS. GRAND TOTAL CHECKS WITH VALUE ABOVE

48.5	58.4	106.9
36.9	49.5	86.4
85.4	107.9	193.3

IT IS MORE CONVENIENT TO WORK WITH TOTALS RATHER THAN MEANS. NOTE THAT EVERY SQUARED TOTAL GETS DIVIDED BY THE NUMBER OF OBSERVATIONS IN THE TOTAL.

COMPARE ROW MEANS TO GRAND MEAN:

$$SSA = \frac{106.9^2}{6} + \frac{86.4^2}{6} - \frac{193.3^2}{12} = 35.02083$$

COMPARE COLUMN MEANS TO GRAND MEAN:

$$SSB = \frac{85.4^2}{6} + \frac{107.9^2}{6} - \frac{193.3^2}{12} = 42.1875$$

COMPARE INDIVIDUAL OBS. TO DESIGN-POINT MEANS:

$$SSE = \left(\frac{17.1^2}{1} + \frac{16.5^2}{1} + \cdots + \frac{16.7^2}{1}\right) - \left(\frac{48.5^2}{3} + \frac{36.9^2}{3} + \frac{58.4^2}{3} + \frac{49.5^2}{3}\right)$$

$$= 3200.71 - 3191.55\dot{6} = 9.15\dot{3}$$

COMPARE INDIVIDUAL OBS. TO GRAND MEAN:

$$SST = 3200.71 - \frac{193.3^2}{12} = 86.96916\dot{6}$$

* A05 Q3 *

GET THE INTERACTION SUM OF SQUARES BY SUBTRACTION:

$$SSI = SST - (SSA + SSB + SSE)$$
$$= 0.6075$$

PUT THE SS'S INTO THE ANOVA TABLE, COMPUTE MEAN SQUARES & F RATIOS, LOOK UP P-VALUES

SV	SS	DF	MS	F	P
METAL	35.0283	1	35.0283	30.608	$<<.01$
TIME	42.1875	1	42.1875	36.872	$<<.01$
METAL×TIME	0.6075	1	0.6075	.531	$>.25$
ERROR	9.1533	8	1.14417		
TOTAL	86.96916	11			

NOTES: DO NOT ROUND ANY VALUES UNTIL YOU HAVE COMPUTED THE F RATIOS; KEEP FULL CALCULATOR ACCURACY.

P-VALUES ARE BEST I CAN DO WITH TEXTBOOK TABLES.

CONCLUSIONS:

THERE IS NO EVIDENCE ($P>.25$) FROM THESE DATA OF AN INTERACTION BETWEEN METAL TYPE AND SINTERING TIME, SO WE CAN TEST THE MAIN EFFECTS. BOTH METAL TYPE AN SINTERING TIME ARE HIGHLY SIGNIFICANT.

THE ESTIMATE OF RESIDUAL VARIANCE IS $\hat{\sigma}^2 = MSE = 1.14417$ ON DFE = 8 D.F. SO THE 95% CI IS $\left(\frac{MSE}{\frac{\chi^2_{.975,DFE}}{DFE}}, \frac{MSE}{\frac{\chi^2_{.025,DFE}}{DFE}}\right)$

$= \left(\frac{1.14417}{17.535/8}, \frac{1.14417}{2.179/8}\right) = (0.522, 4.199)$

AOS Q4 SOLUTIONS

SIMPLE LINEAR REGRESSION ANALYSIS:

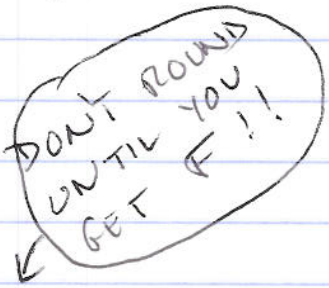
BY CALCULATOR $n = 9$ $\Delta_x = 4.76970$

$\hat{\beta}_0 = 14192.10989$ $\hat{\beta}_1 = -148.97802$ $\Delta_y = 831.05716$

$SS_\beta = \hat{\beta}_1^2 (n-1) \Delta_x^2 = 4,039,390.089$

$SST = (n-1) \Delta_y^2 = 5,525,248$

$SSE = SST - SS_\beta = 1,485,857.911$



SV	SS	DF	MS	F	P
SLOPE of AGE	4039390.089	1	4039390.089	19.03	$\ll .01$
RESIDUAL	1485857.911	7	212265.416		
TOTAL	5525248	8			

1-FACTOR ANOVA ANALYSIS:

AGE			
20	25	31	
11450	10840	9470	
10420	11170	9190	
11142	10540	9540	
3	3	3	9
33012	32550	28200	93762
363823064	353366100	265148600	982337764

$SST = 982337764 - \frac{93762^2}{9}$

$= 5525248$

* SAME AS IN TABLE ABOVE!

$SS_B = \frac{33012^2}{3} + \frac{32550^2}{3} + \frac{28200^2}{3} - \frac{93762^2}{9} = 4699032$

$SSPE = SST - SS_B = 5525248 - 4699032 = 826216$

SV	SS	DF	MS	F	P
FACTOR AGE	4699032	2	2349516	17.06	$\ll .01$
ERROR	826216	6	137702.6̇		
TOTAL	5525248	8			

* A05 Q4 *

WE CAN TAKE THE RESIDUAL FROM THE SLR ANOVA (WHICH MEASURES THE SCATTER ABOUT THE FITTED LINE) AND SUBTRACT THE PURE ERROR FROM THE 1-FACTOR ANOVA; THE DIFFERENCE IS A MEASURE OF "LACK OF FIT", THAT IS, HOW WELL THE DATA ARE FITTED BY A STRAIGHT LINE.

FROM SLR REG. ↓ FROM 1-FACTOR REG. ↓

$$SSLOF = SSE - SSPE = 1485857.911 - 826216 = 659641.911$$

SV	SS	DF	MS	F	P
SLOPE of AGE	4039390.089	1	4039390.089	29.33	$<<.01$
NON-LINEARITY	659641.911	1	659641.911	4.79	$.05<P<.1$
PURE ERROR	826216	6	137702.6̇		
TOTAL	5525248	8			

NOTES:
* DON'T ROUND UNTIL YOU GET F
* IF NON-LINEARITY IS SIGNIFICANT, THE SLOPE HAS NO MEANING SO DON'T TRY TO INTERPRET THE F-TEST FOR SLOPE.
* HERE, LINEARITY IS ACCEPTED (AT 5% LEVEL) SO WE TEST THE SLOPE AND CONCLUDE THAT AGE HAS A SIGNIFICANT LINEAR EFFECT.

THE ESTIMATE OF RESIDUAL VARIANCE IS $\hat{\sigma}^2 = MSPE = 137702.6̇$ ON $DFPE = 6$ D.F. SO THE 99% CI IS

$$\left(\frac{MSPE}{\chi^2_{.995,DFPE}/DFPE}, \frac{MSPE}{\chi^2_{.005,DFPE}/DFPE}\right)$$

$$= \left(\frac{137702.6̇}{18.548/6}, \frac{137702.6̇}{.6757/6}\right) = (44545, 1222707)$$

A05 Q6 SOLUTIONS

(a) 1-FACTOR ANOVA: DATA FROM Q2b.

RUTTED: YES	RUTTED: NO
1.48	3.06
1.90	2.58
1.88	1.70
1.29	5.76
3.53	2.44
2.43	2.03
1.00	1.76
	4.63
	2.86
	2.82
	1.04
	5.92
7	12
13.51	36.60
30.3647	138.9566

$$\frac{13.51^2}{7} + \frac{36.60^2}{12} = 137.70430$$

$$\frac{50.11^2}{19} = 132.15855316$$

$$SST = 169.3213 - 132.15855316 = 37.16277$$

$$SSB = 137.7043 - 132.15855316 = 5.54577$$

$$SSE = SST - SSB = 31.617$$

19
50.11
169.3213

SV	SS	DF	MS	F	P
RUTTING	5.54577	1	5.54577	2.9819	≈ 0.1
ERROR	31.617	17	1.85982		
TOTAL	37.16277	18			

FROM Q2b SOLUTION: $t_0 = 1.7268$ REF $t(17)$

$t_0^2 = 2.9819 = F_0$ REF $F(1, 17)$

NOTE ALSO: $MSE = 1.85982 = s_p^2$

* AOS Q6 *

(b) 2-FACTOR ANOVA: DATA FROM Q2a.

$\sum\sum y_{ij} = 286.5$ $\quad \sum\sum y_{ij}^2 = 7009.39$

27.2	18.1	27.2	19.7	24.5	22.1	138.8
24.1	19.3	26.8	20.1	27.6	29.8	147.7
51.3	37.4	54.0	39.8	52.1	51.9	286.5

COMPARE ROW MEANS TO GRAND MEAN:

$$SSA = \frac{138.8^2}{6} + \frac{147.7^2}{6} - \frac{286.5^2}{12} = 6.600833$$

COMPARE COL. MEANS TO GRAND MEAN:

$$SSB = \frac{51.3^2}{2} + \frac{37.4^2}{2} + \frac{54.0^2}{2} + \frac{39.8^2}{2} + \frac{52.1^2}{2} + \frac{51.9^2}{2} - \frac{286.5^2}{12}$$

$$= 129.0675$$

COMPARE INDIVIDUAL OBS TO DESIGN-POINT MEANS:

$$SSE = \left(\frac{27.2^2}{1} + \frac{18.1^2}{1} + \cdots + \frac{29.8^2}{1}\right) - \left(\frac{27.2^2}{1} + \frac{18.1^2}{1} + \cdots + \frac{29.8^2}{1}\right) = 0$$

COMPARE INDIVIDUAL OBS. TO GRAND MEAN:

$$SST = 7009.39 - \frac{286.5^2}{12} = 169.2025$$

GET INTERACTION BY SUBTRACTION:

$$SSI = SST - (SSA + SSB + SSE) = 33.53417$$

SV	SS	DF	MS	F	P
PROCESSOR	6.60083	1	6.00083	0.9842	$> .25$
CODE	129.0675	5	25.8135	3.849	$.1 < P < .25$
PROCESSOR × CODE	33.53417	5	6.70683		
~~ERROR~~	0	0			
TOTAL	169.2025	11			

COMPARE: $t_5^2 = -.9921^2 = 0.9842$

IF THERE IS AN INTERACTION, $E(MSI) > \sigma^2$, SO THE DENOMINATOR OF THE F-TEST WILL TEND TO BE TOO LARGE, SO THE F STATISTIC WILL BE TOO SMALL AND THE TEST WILL BE CONSERVATIVE (LESS LIKELY TO REJECT H_0).