2004-11-05

STATS 3N03/3J04 TEST 2a SOLNS

1) The Reverend Thomas Bayes, b. 1702 (London), d. 1761 (Tunbridge Wells), studied logic and theology at the Univ. of Edinburgh, ordained a Nonconformist minister, good mathematician, ideas on probability, including "Bayes Theorem" for inverting conditional probabilities were considered controversial but led to the modern "Bayesian" approach to statistical inference. (1)

2) Let X = Number of passengers who show up

Assume: Passengers show up or not, independently of each other. (1)

$$\Rightarrow X \sim Bin(125, 0.9)$$

$$\therefore P(\text{At least 1 empty seat}) = P(X < 120) \quad (5)$$

$$= 1 - \sum_{x=120}^{125} \binom{125}{x} .9^x .1^{125-x} = 0.9885678$$

Normal approx. without cc: $\Phi\left(\frac{119-112.5}{\sqrt{11.25}}\right) = \Phi(1.94) = 0.974$

with cc: $\Phi\left(\frac{119.5-112.5}{\sqrt{11.25}}\right) = \Phi(2.09) = 0.982$

3) Assume: (1) All observations independent, and (2) Central limit theorem applies. (2)

$$\Rightarrow \bar{X}_1 \sim AN\left(75, \left(\tfrac{8}{4}\right)^2\right), \quad \bar{X}_2 \sim AN\left(70, \left(\tfrac{12}{3}\right)^2\right)$$

$$\Rightarrow \bar{X}_1 - \bar{X}_2 \sim AN\left(75-70, \left(\tfrac{8}{4}\right)^2 + \left(\tfrac{12}{3}\right)^2\right) = AN(5, 20)$$

$$\therefore P(3.5 \le \bar{X}_1 - \bar{X}_2 \le 5.5) \simeq \Phi\left(\frac{5.5-5}{\sqrt{20}}\right) - \Phi\left(\frac{3.5-5}{\sqrt{20}}\right)$$

$$= \Phi(.112) - \Phi(-.335) = .54 - .37 = 0.17 \quad (5)$$

4) If the interarrival time is exponential with mean $\mu = 1$ hr, the arrivals must form a poisson process with rate 1 hr^{-1}!
(1)

Let X = Number of arrivals in 1 hour
$\therefore X \sim \text{Pois}(1)$

$$P(X > 3) = 1 - P(X \le 3) = 1 - e^{-1} - e^{-1} - \tfrac{1}{2}e^{-1} - \tfrac{1}{6}e^{-1}$$
$$= 0.0190 \quad (3)$$

Let W = Number of 1-hr. intervals out of 30 that have > 3 arrivals

$\therefore W \sim \text{Bin}(30, P(X>3))$

$$\Rightarrow P(W=0) = P(X \le 3)^{30} = 0.5626 \quad (3)$$

5) PARAMETER: A scalar or vector that indexes a family of probability distributions
(1)

STATISTIC: Any function of the observations in a sample. It may not include any unknown parameters.
(1)

SAMPLING DISTRIBUTION: The distribution of a statistic. It describes how it will vary from one sample to another.
(1)