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STATS 3N03/3J04

31-1

2-FACTOR ANOVA MODEL:

$$Y_{ijk} = \bar{Y}_{+++} + (\bar{Y}_{i++} - \bar{Y}_{+++}) + (\bar{Y}_{+j+} - \bar{Y}_{+++})$$

ROW, COL, REP

$$+ (\bar{Y}_{ij+} - \bar{Y}_{i++} - \bar{Y}_{+j+} + \bar{Y}_{+++}) + (Y_{ijk} - \bar{Y}_{ij+})$$

NOT INTERESTED IN THE OVERALL MEAN, SO WRITE:

$$(Y_{ijk} - \bar{Y}_{+++}) = (\bar{Y}_{i++} - \bar{Y}_{+++}) + (\bar{Y}_{+j+} - \bar{Y}_{+++})$$

$$+ (\bar{Y}_{ij+} - \bar{Y}_{i++} - \bar{Y}_{+j+} + \bar{Y}_{+++}) + (Y_{ijk} - \bar{Y}_{ij+})$$

SQUARE BOTH SIDES, SUM OVER ALL OBSERVATIONS; IF DESIGN IS BALANCED, CROSS-TERMS WILL SUM TO ZERO.

$$\sum\sum\sum (Y_{ijk} - \bar{Y}_{+++})^2 = \sum\sum\sum (\bar{Y}_{i++} - \bar{Y}_{+++})^2 + \sum\sum\sum (\bar{Y}_{+j+} - \bar{Y}_{+++})^2$$

$$+ \sum\sum\sum (\bar{Y}_{ij+} - \bar{Y}_{i++} - \bar{Y}_{+j+} + \bar{Y}_{+++})^2 + \sum\sum\sum (Y_{ijk} - \bar{Y}_{ij+})^2$$

$i = 1, \ldots, a$ ROWS

$j = 1, \ldots, b$ cols

$k = 1, \ldots, n$ replications

$$\sum\sum\sum (Y_{ijk} - \bar{Y}_{+++})^2 = nb \sum_i (\bar{Y}_{i++} - \bar{Y}_{+++})^2$$

$$+ na \sum_j (\bar{Y}_{+j+} - \bar{Y}_{+++})^2$$

$$+ n \sum_i \sum_j (\bar{Y}_{ij+} - \bar{Y}_{i++} - \bar{Y}_{+j+} + \bar{Y}_{+++})^2 + \sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{ij+})^2$$

COMPUTATION:

$$\sum\sum\sum (Y_{ijk} - \bar{Y}_{+++})^2 = (nab-1)s^2$$

- OVER ALL nab OBSERVATIONS

$$= \sum\sum\sum Y_{ijk}^2 - \frac{Y_{+++}^2}{nab}$$

← NUMBER OF OBS. IN TOTAL.

$$nb \sum_i (\bar{Y}_{i++} - \bar{Y}_{+++})^2 = \sum_i \frac{Y_{i++}^2}{nb} - \frac{Y_{+++}^2}{nab}$$

$$na \sum_j (\bar{Y}_{+j+} - \bar{Y}_{+++})^2 = \sum_j \frac{Y_{+j+}^2}{na} - \frac{Y_{+++}^2}{nab}$$

$$\sum\sum\sum (Y_{ijk} - \bar{Y}_{ij+})^2 = \sum\sum\sum Y_{ijk}^2 - \sum_i \sum_j \frac{Y_{ij+}^2}{n}$$

MODEL:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

WHERE $\epsilon_{ijk} \sim IN(0, \sigma^2)$

$$\hat{\sigma}^2 = MSE = \frac{\sum\sum\sum (Y_{ijk} - \bar{Y}_{ij+})^2}{ab(n-1)}$$

$$DF = ab(n-1)$$

ESTIMATE OF σ^2 IN ANY ANOVA TABLE IS ALWAYS THE BOTTOM MEAN SQUARE!!