STATS 3N03 / 3J04 2004-11-18

28-1

CALCULATING P-VALUES

- EXAMPLES (R & TABLES)

INTERPRETING P-VALUES

$P < 0.001$

"STRONG EVIDENCE FROM THESE DATA THAT THE HYPOTHESIS IS FALSE"

$0.001 < P < 0.1$

"SOME EVIDENCE FROM THESE DATA THAT THE HYPOTHESIS IS FALSE"

$P > 0.1$

"NO EVIDENCE FROM THESE DATA THAT THE HYPOTHESIS IS FALSE"

$\Rightarrow$ P-VALUE EXPRESSES THE WEIGHT OF EVIDENCE AGAINST THE HYPOTHESIS

ACCEPT - REJECT TESTING AT A GIVEN LEVEL OF SIGNIFICANCE α

$P < \alpha$

"REJECT H_0 AT THE $100\alpha\%$ LEVEL OF SIGNIFICANCE"

$P > \alpha$

"ACCEPT H_0 AT THE $100\alpha\%$ LEVEL OF SIGNIFICANCE"

[USUALLY : $\alpha = 0.05$, LESS OFTEN $\alpha = 0.01$]

$\Rightarrow$ LEADS TO A DECISION

- DOES NOT SAY WHETHER IT WAS A CLEAR DECISION OR BORDERLINE
- BUT - IF USED REPEATEDLY, WE WILL KNOW HOW OFTEN WE WILL MAKE TYPE I OR TYPE II ERRORS.

TYPE I ERROR

H_0 IS TRUE BUT WE REJECT H_0
"FALSE POSITIVE"

TYPE II ERROR

H_0 IS FALSE BUT WE ACCEPT H_0

"FALSE NEGATIVE"

$P(\text{REJECT } H_0 \mid H_0 \text{ TRUE}) = \alpha$

$\therefore \alpha$ FIXES THE TYPE I ERROR RATE

$P(\text{ACCEPT } H_0 \mid H_0 \text{ FALSE}) = \beta$

BUT β DEPENDS ON HOW FALSE H_0 IS

EX $X_1, \dots, X_n \sim IN(\mu, \sigma^2)$

σ^2 KNOWN

$H_0 : \mu = \mu_0$

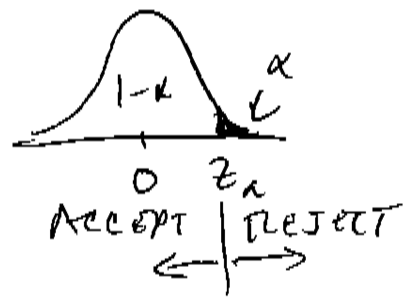
$H_1 : \mu = \mu_0 + \delta$

WHERE $\delta > 0$

TEST STATISTIC: $Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

REF DIST: $N(0,1)$

REJECTION REGION: $Z_0 > Z_\alpha$



$$P(\text{REJECT } H_0 \mid H_0) = P(Z_0 > Z_\alpha \mid Z_0 \sim N(0,1))$$
$$= 1 - \Phi(Z_\alpha) = \alpha$$

BUT IF H_1 IS TRUE

$$E(Z_0) = \frac{\mu_0 + \delta - \mu_0}{\sigma/\sqrt{n}} = \sqrt{n}\,\frac{\delta}{\sigma}$$

$$\mathrm{Var}(Z_0) = 1$$

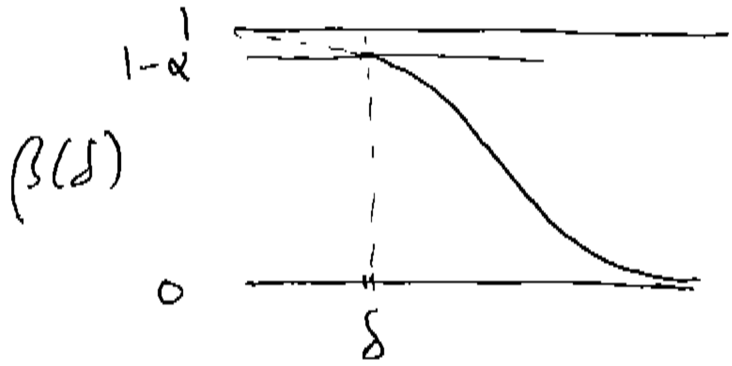
$$P(\text{REJECT } H_0 \mid H_1) = P\left(Z_0 > Z_\alpha \mid Z_0 \sim N\left(\sqrt{n}\frac{\delta}{\sigma}, 1\right)\right)$$
$$= 1 - \Phi\left(Z_\alpha - \sqrt{n}\frac{\delta}{\sigma}\right)$$

OPERATING CHARACTERISTIC CURVE:

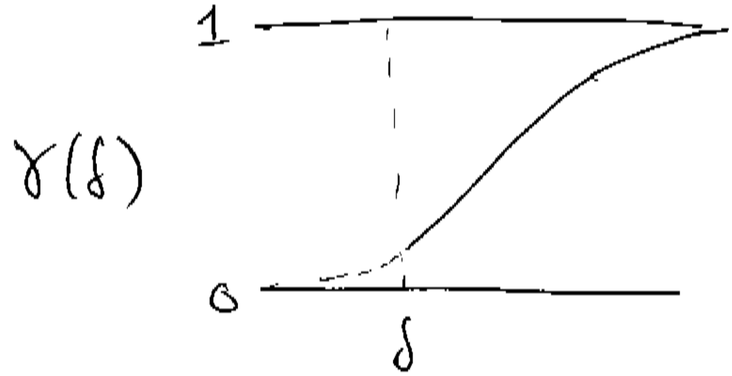
$$\beta(\delta) = \Phi\left(Z_\alpha - \sqrt{n}\frac{\delta}{\sigma}\right)$$

POWER

$$\gamma(\delta) = 1 - \beta(\delta) = 1 - \Phi\left(Z_\alpha - \sqrt{n}\frac{\delta}{\sigma}\right)$$



OC $= 1 -$ POWER



POWER $= 1 -$ OC

SAMPLE SIZE CALCULATION:

GIVEN α, β, δ, FIND n

<u>EX</u> WE KNOW THAT $\sigma = 1.2$. WE ARE CONCERNED THAT $\mu > 30$ AND WE WANT TO BE 90% CERTAIN OF DETECTING WHEN $\mu > 30.5$, HOW MANY OBSERVATIONS WILL WE NEED IF WE SET THE PROBABILITY OF A TYPE I ERROR TO BE 5%?

ii) GIVEN $\sigma = 1.2$, $\alpha = .05$

FIND n SO THAT

$$\gamma(0.5) = 0.9$$

OR $\beta(0.5) = 0.1$

SEE TEXT P. 294 FOR 2-SIDED CASE

TABLES: $z_{.05} = 1.645$

SOLVE FOR n

$$0.1 = \Phi\left(1.645 - \sqrt{n}\,\frac{0.5}{1.2}\right)$$

TABLES:

$$0.1 = \Phi(-1.282)$$

$$1.645 - \sqrt{n}\,\frac{0.5}{1.2} = -1.282$$

$$n = \left((1.645 + 1.282)\,\frac{1.2}{0.5}\right)^2$$

$$= 49.35$$

GENERALLY: $\boxed{n = \frac{(z_\alpha + z_\beta)^2\,\sigma^2}{\delta^2}}$