STATS 3N03/3J04

2004-11-17
27-1

PIVOTAL QUANTITIES:

① $$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

② $$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim AN(0,1)$$

③ $$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t(n-1)$$

④ $$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$$

⑤ $$\frac{\bar{d}-\Delta}{s_d/\sqrt{n}} \sim t(n-1)$$

⑥ ⑦ $$\frac{(\bar{X}_1-\bar{X}_2)-\Delta}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1) \text{ OR } AN(0,1)$$

(8) $$\frac{(\bar{X}_1 - \bar{X}_2) - \Delta}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

(9) $$\frac{s_1^2 / s_2^2}{\theta} \sim F(n_1 - 1,\ n_2 - 1)$$

(10) $$\frac{MSA / MSE}{\theta} \sim F(DFA,\ DFE)$$

ALTERNATIVE TO (8) WHEN YOU CAN'T ASSUME HOMOSCEDASTICITY

SEE (10-15), (10-16) ON P. 342.

- AVOID!!
- IF SLIGHTLY HETEROSCEDASTIC, BETTER TO USE (8).
- IF STRONGLY HETEROSCEDASTIC, ASK WHY YOU WANT TO COMPARE MEANS.

NOTES ON EXAMPLE 10-6

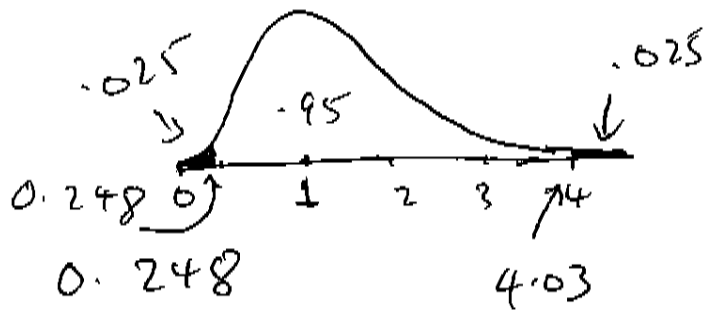
- LOOKS LIKE RURAL ARIZONA HAS MORE HIGH VALUES THAN PHOENIX, HENCE MORE VARIABLE.

BUT: $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$f_0 = \frac{s_1^2}{s_2^2} = \left(\frac{7.63}{15.35}\right)^2 = 0.247$$

$$F_{.025,9,9} = 4.03$$



$$F_{.975,9,9} = \frac{1}{4.03} = 0.248$$

⇒ JUST AT EDGE OF REJECTION REGION FOR A 5% TEST

ASSUMING $\sigma_1^2 = \sigma_2^2$ $\qquad s_p = 12.1221$

$$t_0 = \frac{12.5 - 27.5}{12.1221\sqrt{\frac{1}{10} + \frac{1}{10}}} = -2.77$$

↑ WHY IS THIS SAME AS t_0^* IN THIS EXAMPLE?

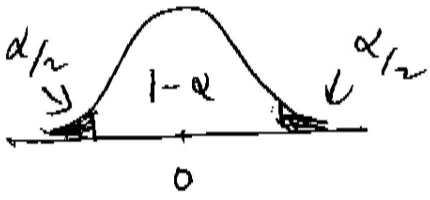
REF: $t(18)$

⇒ t_0^* WILL GIVE SLIGHTLY LESS SIGNIFICANT RESULT.

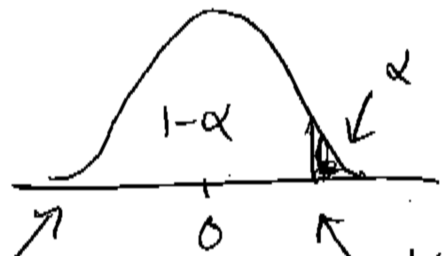
ONE-SIDED VS TWO-SIDED

$H_0: \mu = \mu_0$ ← "SIMPLE" HYPOTHESIS i.e. SINGLE VALUE

$H_1: \mu \neq \mu_0$ ← "COMPOSITE" ALTERNATIVE i.e. RANGE OF VALUES



$H_1: \mu > \mu_0$ ONE-SIDED (DON'T CARE IF $\mu < \mu_0$, OR ASSUME IT WON'T EVER HAPPEN)



NO POWER HERE

MORE POWER HERE BECAUSE CRITICAL VALUE IS LOWER.

⇒ ALWAYS USE 2-SIDED ALTERNATIVES

P-VALUE

p. 292

- PROBABILITY OF GETTING A TEST STATISTIC AS EXTREME AS, OR MORE EXTREME THAN, THE OBSERVED VALUE, IF H_0 WERE TRUE

- SMALLEST α THAT REJECTS H_0

- LARGEST α THAT ACCEPTS H_0

ONE-SIDED P-VALUE

= TAIL AREA UNDER REFERENCE DISTRIBUTION

TWO-SIDED P-VALUE

= TWICE RIGHT TAIL AREA IF IN RIGHT TAIL

OR TWICE LEFT TAIL AREA IF IN LEFT TAIL

27-6

```
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`help.start()' for a HTML browser interface to help.
Type `q()' to quit R.

> 1-pt(2.75, 5)
[1] 0.02015511
> 2*(1-pt(2.75, 5))
[1] 0.04031022
> pchisq(6, 10)
[1] 0.1847368
> 2*pchisq(6, 10)
[1] 0.3694735
> 1-pchisq(15,10)
[1] 0.1320619
> 1-pchisq(18,10)
[1] 0.05496364
> 2*(1-pchisq(18,10))
[1] 0.1099273
> pf(.356,5,12)
[1] 0.1312862
> 2*pf(.356,5,12)
[1] 0.2625723
> 1-pf(3.11, 8, 15)
[1] 0.02782143
> 2*(1-pf(3.11, 8, 15))
[1] 0.05564286
>
```