STATS 3N03/3J04

2004-11-10

24-1

* A03 IS NOW ONLINE
* CHECK YOUR TERM MARKS!!

ANALYSIS OF VARIANCE (ANOVA) IS ALL ABOUT SUMS OF SQUARES

- BREAK DOWN A TOTAL SUM OF SQUARES INTO COMPONENTS FROM DIFFERENT SOURCES

- COMPARE SUMS OF SQUARES

$\Rightarrow$ WE NEED TO KNOW THE DISTRIBUTION OF A SUM OF SQUARES TO MAKE INFERENCES ON VARIANCES

CHI-SQUARED DISTRIBUTION

χ^2

DEFINE: IF $Z_1, \dots, Z_n \sim IN(0,1)$

THEN

$$Y_n = Z_1^2 + \cdots + Z_n^2 \sim \chi^2(n)$$

"CHI-SQUARED ON n DEGREES OF FREEDOM"

DEGREES OF FREEDOM
= NUMBER OF "FREE" VARIABLES IN THE SUM

= (NUMBER OF VARIABLES)
− (NUMBER OF CONSTRAINTS)

USEFUL RESULT:

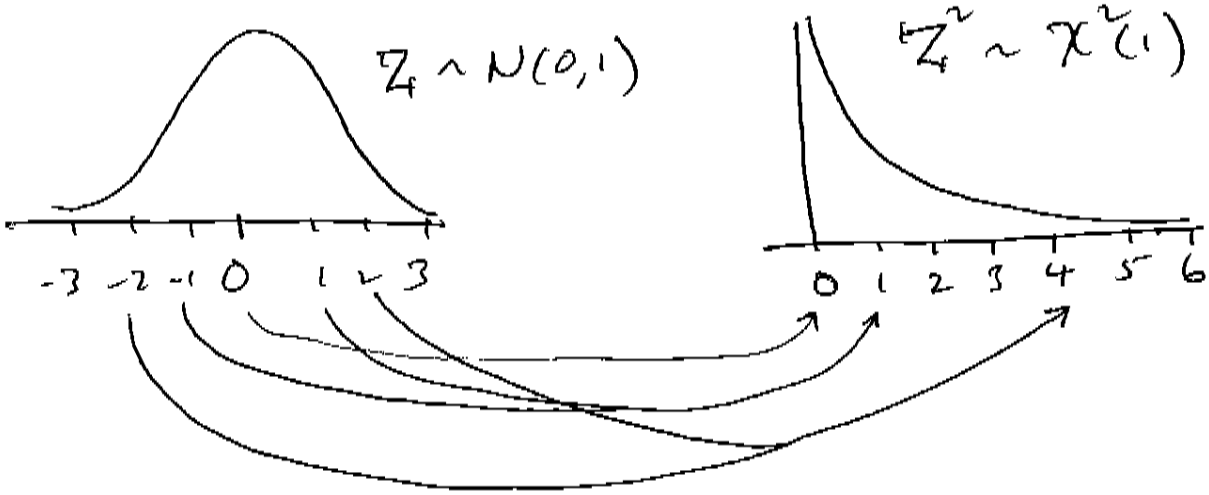
$$Var(X) = E(X^2) - E^2(X)$$

COMPARE:

$$s_x^2 = \frac{n}{n-1}\left\{ \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \right\}$$

PROPERTIES OF THE χ^2 DIST.

1 DF



$$E(Y_1) = E(Z^2) = Var(Z) = 1$$

$$Var(Y_1) = Var(Z^2) = E(Z^4) - E(Z^2)^2$$

$$= 3 - 1 = 2$$

$$\therefore E(Y_n) = n \qquad Var(Y_n) = 2n$$

AND BY CENTRAL LIMIT THEOREM, SHAPE $\rightarrow$ NORMAL AS $n \rightarrow \infty$.

TABLE III p. 655

dchisq, pchisq, qchisq, rchisq

APPLICATION

- INDEP. NORMAL DATA,
INFERENCE ABOUT σ^2
WHEN μ IS UNKNOWN

$$X_1, \dots, X_n \sim IN(\mu, \sigma^2)$$

$$\frac{X_1-\mu}{\sigma}, \dots, \frac{X_n-\mu}{\sigma} \sim IN(0,1)$$

$$\left(\frac{X_1-\mu}{\sigma}\right)^2 + \dots + \left(\frac{X_n-\mu}{\sigma}\right)^2 \sim \chi^2(n)$$

$$\left(\frac{X_1-\bar{X}}{\sigma}\right)^2 + \dots + \left(\frac{X_n-\bar{X}}{\sigma}\right)^2 \sim \chi^2(n-1)$$

NOTE: ONE CONSTRAINT

$$\left(\frac{X_1-\bar{X}}{\sigma}\right) + \dots + \left(\frac{X_n-\bar{X}}{\sigma}\right) \equiv 0$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

THIS IS A PIVOTAL QUANTITY !!

$100(1-\alpha)\%$ CI FOR σ^2

$$\left(\frac{s^2}{\frac{\chi^2_{\frac{\alpha}{2}, n-1}}{n-1}}, \frac{s^2}{\frac{\chi^2_{1-\frac{\alpha}{2}, n-1}}{n-1}} \right)$$

TEST $H_0: \sigma = \sigma_0$

TEST STATISTIC $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

REF. DIST: $\chi^2(n-1)$

EX $\underline{x} = (23.1, 27.2, 24.4, 25.0)$

$n = 4 \qquad s_x = 1.7115 \qquad \sigma_0 = 1.2$