2004-11-08

PIVOTAL QUANTITY

- FUNCTION OF THE STATISTIC AND THE PARAMETER OF INTEREST THAT FOLLOWS A STANDARD DISTRIBUTION

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

VALID IF:

$$X_1, \dots, X_n \sim IN(\mu, \sigma^2)$$

NORMALITY ONLY APPROXIMATE IF:

$$X_1, \dots, X_n \sim I*(\mu, \sigma^2)$$

HYPOTHESIS AND ALTERNATIVE:

$H_0: \mu = \mu_0$ "NULL HYPOTHESIS"

$H_1: \mu \neq \mu_0$ "ALTERNATIVE HYPOTHESIS"

CASE: σ KNOWN

IF H_0 IS TRUE, THEN

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

TEST STATISTIC ← REF. DIST.

NOW GIVEN THE DATA:

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

OBSERVED TEST STATISTIC

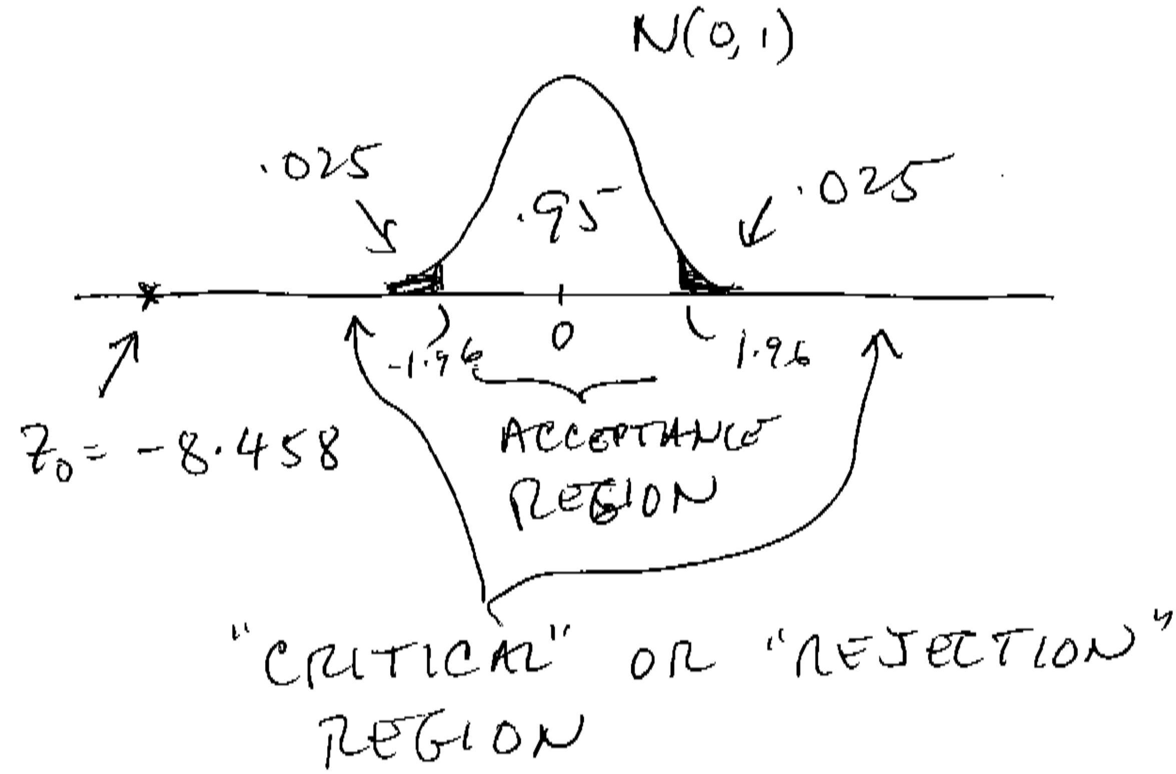
IF H_0 IS TRUE, THE TEST STATISTIC IS AN OBSERVATION FROM THE REFERENCE DISTRIBUTION.

COMPUTE THE TEST STATISTIC, LOOK AT THE REFERENCE DISTRIBUTION.

$\bar{x} = 24.925$ $\quad \mu_0 = 30$ $\quad \sigma = 1.2$ $\quad n = 4$

$$z_0 = \frac{24.925 - 30}{1.2/\sqrt{4}} = -8.458$$

FOR A TEST AT SIGNIFICANCE LEVEL $\alpha = 0.05$ ("5% TEST"), TWO-SIDED ALTERNATIVE



BUT VALUES AS EXTREME AS -8.458 "NEVER" COME FROM N(0,1), SO, EITHER:

- H_0 IS FALSE <u>OR</u>
- AN IMPOSSIBLE EVENT HAPPENED

OR

- DATA NO GOOD.

IN GENERAL.

- FIND A PIVOTAL QUANTITY INVOLVING THE PARAMETER OF INTEREST AND APPROPRIATE STATISTICS

CONFIDENCE INTERVAL:

MAKE A PROBABILITY STATEMENT ABOUT PIVOTAL QUANTITY, SOLVE FOR PARAMETER.

TEST STATISTIC:

REPLACE PARAMETER WITH HYPOTHESIZED VALUE TO GET TEST STATISTIC, COMPARE WITH REFERENCE DISTRIBUTION.

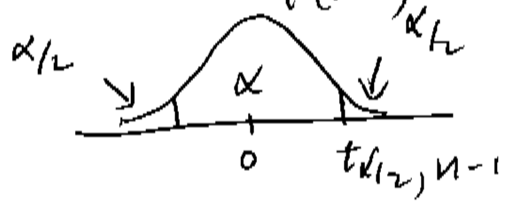
INFERENCE ON μ WHEN σ^2 UNKNOWN.

RESULT: IF $X_1, \dots, X_n \sim IN(\mu, \sigma^2)$ THEN

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$$

"STUDENT'S t DISTRIBUTION ON $n-1$ DEGREES OF FREEDOM"

(BECAUSE: s^2 IS AN ESTIMATE OF σ^2 ON $n-1$ D.F.)



SEE p. 258

$$P\left(-t_{\alpha/2, n-1} < \frac{\bar{X} - \mu}{S/\sqrt{n}} < t_{\alpha/2, n-1}\right) = 1 - \alpha$$

SOLVE FOR μ:

$$\bar{X} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$$

THIS IS A RANDOM INTERVAL COVERS μ $100(1-\alpha)\%$ OF THE TIME.

GIVEN DATA : $x_1, \ldots, x_n$

$$\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$100(1-\alpha)\%$ <u>CONFIDENCE INTERVAL</u>.

<u>EX</u> $x = (23.1, 27.2, 24.4, 25.0)$

$n = 4 \qquad \bar{x} = 24.925 \qquad s_x = 1.7115$

$t_{.025,3} = 3.182$

$$\bar{x} \pm t_{.025, n-1} \frac{s_x}{\sqrt{n}} = 24.925 \pm 3.182 \frac{1.7115}{\sqrt{4}}$$

$$= 24.925 \pm 2.723$$

$$= (22.202, 27.648)$$

23-7

t Confidence Intervals — Demonstration by Simulation

Generating a display like Fig. 8-1 on p. 251

```
> n <- 10
> normdata <- matrix(rnorm(1000*n, 10, 10), ncol=n)
> xbar <- apply(normdata, 1, mean)
> sx <- sqrt(apply(normdata, 1, var))
> llim <- xbar - qt(.975, n-1)*sx/sqrt(n)
> ulim <- xbar + qt(.975, n-1)*sx/sqrt(n)
> mean(10 > llim & 10 < ulim)
[1] 0.951
> plot(c(0,20), c(-5,25), type="n", xlab="Interval number",
ylab="x")
> for (i in 1:20)
  { points(i, xbar[i], pch=16)
  lines(c(i,i), c(llim[i],ulim[i])) }
> abline(h=10, lty=3)
```