# STATS 3N03/3J04

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22-1

MARGINAL DISTRIBUTION OF X

$$N(\mu_X, \sigma_X^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2}$$

CONDITIONAL DISTRIBUTION OF Y GIVEN X=x

$$N\left(\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X),\ \sigma_Y^2(1-\rho^2)\right)$$

$$f_{Y|x}(y|x) = \frac{1}{\sqrt{2\pi}\,\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2}\left(\frac{y-\{\mu_Y+\rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X)\}}{\sigma_Y\sqrt{1-\rho^2}}\right)^2}$$

FIND $f_{X,Y}(x, y)$

BUT $P(X=x \wedge Y=y)$

$$= P(Y=y|X=x)P(X=x)$$

$$\therefore f_{X,Y}(x,y) = f_{Y|x}(y|x)\,f_X(x)$$

$$\frac{1}{\sqrt{2\pi}\,\sigma_x}\ \frac{1}{\sqrt{2\pi}\,\sigma_y\sqrt{1-\rho^2}} = \frac{1}{2\pi\,\sigma_x\sigma_y\sqrt{1-\rho^2}}$$

$$\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\{\mu_y+\rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)\}}{\sigma_y\sqrt{1-\rho^2}}\right)^2$$

$$= \ \ldots$$

$$= \frac{1}{(1-\rho^2)}\left\{\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right\}$$

$\Rightarrow$ (5-32) page 178

NOTE:

THIS IS THE FORMULA FOR AN ELLIPSE CENTERED AT $(\mu_x, \mu_y)$, $\sigma_x$, $\sigma_y$, $\rho$ DETERMINE SLOPE AND RELATIVE DIMENSIONS. CONTOURS OF CONSANT DENSITY ARE ELLIPTICAL.

NOTE:

WHEN $\rho = 0$

$$Y \mid X = x \sim N(\mu_Y, \sigma_Y^2)$$

$\Rightarrow$ SAME DISTRIBUTION AS MARGINAL DISTRIBUTION OF $Y$

$\Rightarrow$ $Y, X$ INDEPENDENT

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REVIEW PROBLEMS:

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