# STATS 3N03/3J04

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19-1

CHECK YOUR TERM MARKS ON THE WEB!

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## VARIANCE OF A LINEAR COMBINATION - GENERAL CASE

$$Y = \underset{\sim}{A}\underset{\sim}{X} = (a_1, \cdots, a_n)\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \sum_{i=1}^{n} a_i X_i$$

$$Var(\underset{\sim}{X}) = \underset{\sim}{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{12} & \sigma_{22} & & \sigma_{2n} \\ \vdots & \vdots & & \\ \sigma_{1n} & \sigma_{2n} & \cdots & \sigma_{nn} \end{bmatrix}$$

$$Var(Y) = \underset{\sim}{A}\underset{\sim}{\Sigma}\underset{\sim}{A}' = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \sigma_{ij}$$

$$= \sum_{i=1}^{n} a_i^2 \sigma_{ii} + 2\sum\sum_{i<j} a_i a_j \sigma_{ij}$$

$$= \sum_{i=1}^{n} a_i^2 \sigma_i^2 + 2\sum\sum_{i<j} a_i a_j \sigma_{ij}$$

# NOTATION:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

CORRELATION

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$$

COVARIANCE

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## MEASUREMENT MODEL:

$$E(\underset{\sim}{X}) = \begin{pmatrix} \mu \\ \vdots \\ \mu \end{pmatrix} = \mu \underset{\sim}{1}_n$$

$$Var(\underset{\sim}{X}) = \begin{bmatrix} \sigma^2 & & 0 \\ & \ddots & \\ 0 & & \sigma^2 \end{bmatrix} = \sigma^2 \underset{\sim}{I}_n$$

## MEASUREMENT MODEL WITH CORRELATED DATA

$$Var(X) = \begin{bmatrix} \sigma^2 & \sigma^2\rho_{12} & \cdots & \sigma^2\rho_{1n} \\ \sigma^2\rho_{12} & \sigma^2 & \cdots & \sigma^2\rho_{2n} \\ \vdots & & \ddots & \vdots \\ \sigma^2\rho_{1n} & \sigma^2\rho_{2n} & \cdots & \sigma^2 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{12} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & & \vdots \\ \rho_{1n} & \rho_{2n} & \cdots & 1 \end{bmatrix}$$

← CORRELATION MATRIX

$$\text{Var}(\bar{X}) = \sigma^2 \left(\tfrac{1}{n}, \cdots, \tfrac{1}{n}\right) \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{12} & 1 & & \\ \vdots & & \ddots & \\ \rho_{1n} & & & 1 \end{bmatrix} \begin{pmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{pmatrix}$$

$$= \frac{\sigma^2}{n^2} \left[ n + 2 \sum\sum_{i<j} \rho_{ij} \right]$$

$$= \frac{\sigma^2}{n} \left[ 1 + \frac{2}{n} \sum\sum_{i<j} \rho_{ij} \right]$$

WHAT WOULD IT MEAN IF $\rho_{ij} > 0 \; \forall_{i,j}$?

WHAT WOULD IT MEAN IF $\rho_{ij} = 1 \; \forall_{i,j}$?

EX

WASHER & PIN

- WHAT DOES IT MEAN IF SOMEONE TRIES TO MATCH LARGE PIN WITH LARGE WASHER, SMALL WITH SMALL?

EX BAG OF APPLES

- WHAT ARE IMPLICATIONS IF APPLES ARE COLLECTED INTO BINS "LARGE" "MEDIUM" "SMALL" AND BAGS ARE FILLED FROM BINS?