2004-10-14

STATS 3N03/3J04

13-1

1. PAGE 11-7 WAS NOT SHOWN IN CLASS.
2. TESTS 2 & 3 MAKEUP
   ~~FRI 8:30,~~ FRI 9:30, ~~OR BOTH?~~
3. TEST SOLUTIONS NOW ON THE WEB.
4. WATCH FOR A02
5. TRY: EXERCISE 2, PAST YEAR'S A02, T02.

   TEXT PROBLEMS: CHAPT 2-3
   START WITH EASY PROBLEMS, SKIP TO HARDER ONES.

CONTINUOUS PROBABILITY DISTRIBUTIONS

SAMPLE SPACE: ALL OR PART OF THE REAL LINE eg $(-\infty,\infty)$, $(0,\infty)$, $(0,1)$, etc.

PROBABILITY DENSITY FUNCTION

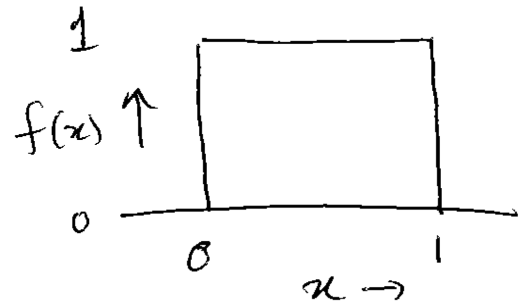
$$\underbrace{\underbrace{f(x)}_{\text{DENSITY}}\,dx}_{\text{PROBABILITY}} = P(x < X \le x+dx)$$

RANDOM VARIABLE

CUMULATIVE DISTRIBUTION FUNCTION:

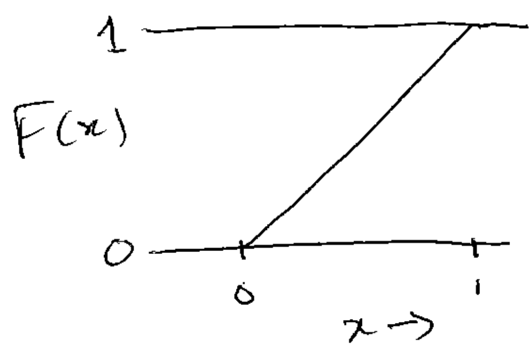
$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x')\,dx'$$

EX $U(0,1)$



$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

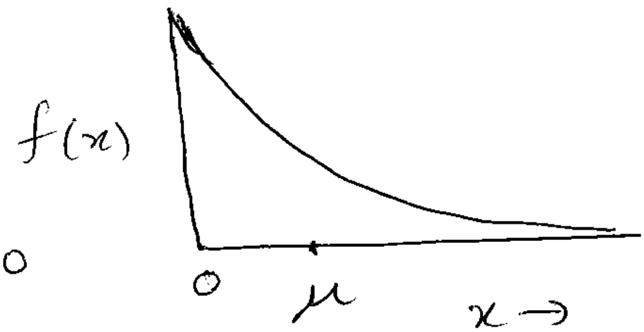


APPLICATION:

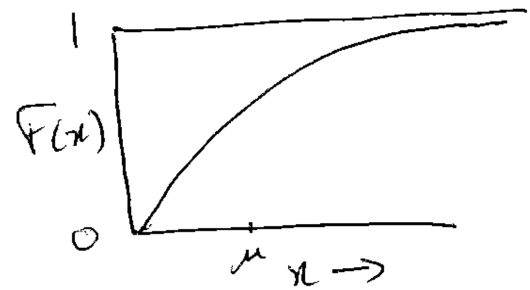
- EASY TO GENERATE BY COMPUTER OR CALCULATOR FOR RANDOM SIMULATION
- EASILY TRANSFORMED TO OTHER DISTRIBUTIONS.

EX EXPONENTIAL DISTRIBUTION

EXP(μ)



$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}} & x > 0 \\ 0 & x < 0 \end{cases}$$



$$F(x) = \int_0^x \frac{1}{\mu} e^{-\frac{x'}{\mu}} \, dx'$$

$$= \begin{cases} 1 - e^{-\frac{x}{\mu}} & x > 0 \\ 0 & x < 0 \end{cases}$$

NOTE: LIKE GEOMETRIC DISTRIBUTION BUT IN CONTINUOUS TIME.

SHOW:

$$E(X) = \int_0^\infty x \frac{1}{\mu} e^{-\frac{x}{\mu}} \, dx = \mu$$

$$\sigma^2 = Var(X) = \int_0^\infty (x-\mu)^2 \frac{1}{\mu} e^{-\frac{x}{\mu}} \, dx = \cdots = \mu^2$$

$$\therefore \quad \sigma = \mu$$

APPLICATION:

- TIME TO THE NEXT EVENT IN A POISSON PROCESS.

PROOF:

THE TIME TO THE NEXT EVENT WILL BE $\leq t \iff$ AT LEAST ONE EVENT HAPPENS IN $(0, t]$.

LET T BE TIME TO NEXT EVENT. RECALL: X_t = NO. EVENTS IN $(0, t]$.

$$P(T \leq t) = P(X_t \geq 1)$$

$$= 1 - P(X_t = 0)$$

$$= 1 - e^{-\nu t}$$

BUT THIS IS EXPONENTIAL CDF WITH MEAN $\mu = \frac{1}{\nu}$

SIMULATION OF A POISSON PROCESS WITH $\nu = 1$

13-5

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> plot(cumsum(rexp(30)), rep(1,30), pch=19, xlab="Time",ylab="", xlim=c(0,20),
col="red")
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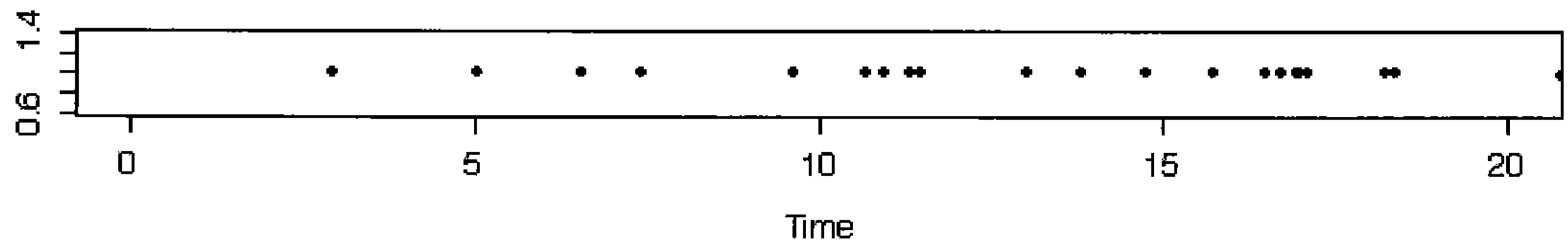
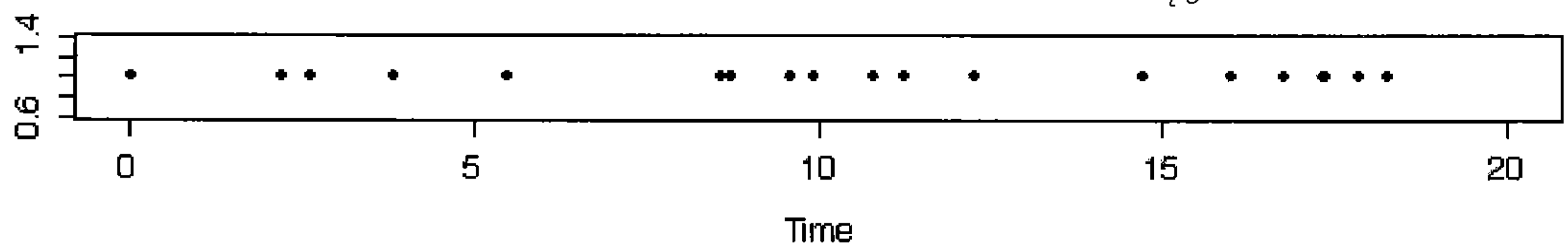
DEFAULT $\nu = 1$ $\mu = 1$

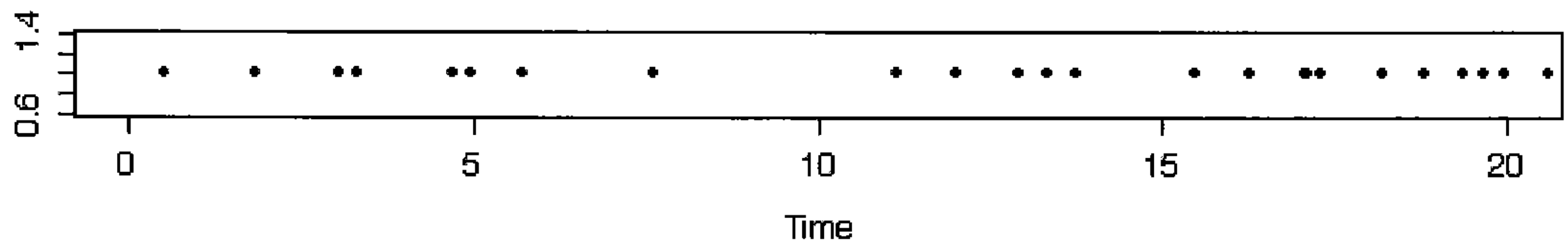
$\hat{\nu} = \frac{19}{20} = 0.95$ $\hat{\mu} = \frac{1}{.95} = 1.05$



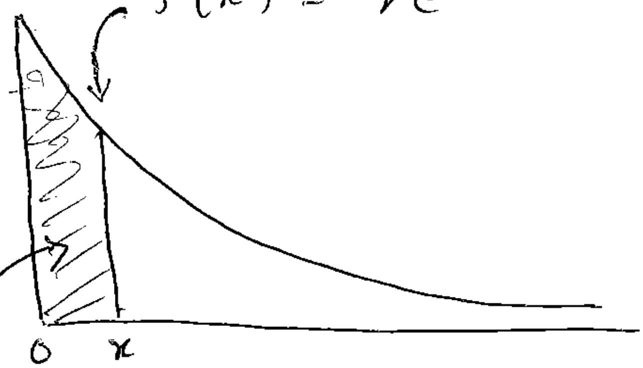
$\hat{\nu} = \frac{19}{20} = 0.95$ $\hat{\mu} = \frac{1}{.95} = 1.05$



$\hat{\nu} = \frac{22}{20} = 1.1$ $\hat{\mu} = \frac{1}{1.1} = 0.91$



$f(x) = \nu e^{-\nu t}$



$F(x) = 1 - e^{-\nu t}$

RATE $= \nu$ MEAN $= \frac{1}{\nu}$

dexp(x, rate=1) $\nu e^{-\nu t}$

pexp(q, rate=1) $\{ p = 1 - e^{-\nu q}$

qexp(p, rate=1) $\left\{ \begin{array}{l} \text{SOLVES FOR } q \\ 1 - e^{-\nu q} = p \end{array} \right.$

rexp(n, rate=1)

EX YOU ARE HITCH-HIKING ON A NORTHERN ROAD, ON AVERAGE THERE ARE 2 CARS PER HOUR.

WHAT IS THE PROBABILITY THAT THE NEXT CAR WILL ARRIVE WITHIN 15 MIN?
AFTER 4 HOURS?

EX INTERSECTION WITH AVERAGE OF 3 ACCIDENTS / MONTH (FROM PAST DATA).

WHAT IS THE PROBABILITY THAT NEXT MONTH THERE WILL BE:

NO ACCIDENTS?
6 ACCIDENTS?
6 OR MORE ACCIDENTS?