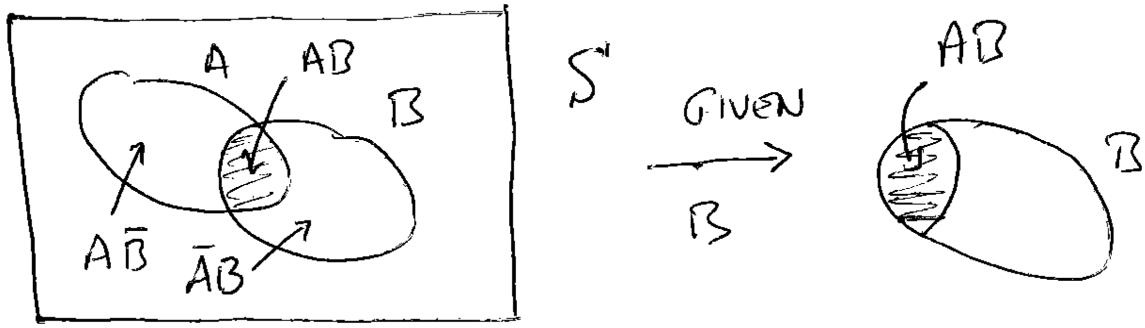
STATS 3N03/3J04



$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(AB) = P(A|B)\,P(B)$$
$$= P(B|A)\,P(A)$$

EVENTS A, B ARE "STATISTICALLY INDEPENDENT" IF:

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(AB) = P(A)\,P(B)$$

SHOW: ANY ONE CONDITION IMPLIES THE OTHER 2.

i.e. KNOWING THAT B HAS OCCURRED DOES NOT GIVE YOU ANY INFORMATION AS TO WHETHER OR NOT A HAS OCCURRED, AND

KNOWING THAT A HAS OCCURRED DOES NOT GIVE YOU ANY INFORMATION AS TO WHETHER OR NOT B HAS OCCURRED.

IN VENN DIAGRAM.

RATIO OF A TO S IS SAME AS RATIO OF AB TO B, AND

RATIO OF B TO S IS SAME AS RATIO OF AB TO A.

IN PRACTICE:

EVENTS ARE INDEPENDENT IF THERE IS NO PHYSICAL CONNECTION.

e.g. TOSS A "FAIR" COIN TWICE, MAKE SURE RESULT OF FIRST TOSS DOESN'T AFFECT RESULT OF SECOND.

$$P(H_1) = P(H_2) = \frac{1}{2}$$

$$P(H_1 H_2) = P(H_1)\,P(H_2) = \frac{1}{4}$$

$$P(2 \text{ HEADS}) = P(H_1 H_2) = \frac{1}{4}$$

$$P(1 \text{ HEAD}) = P(H_1 \bar{H}_2 \cup \bar{H}_1 H_2)$$

$$= P(H_1 \bar{H}_2) + P(\bar{H}_1 H_2)$$

$$= P(H_1)P(\bar{H}_2) + P(\bar{H}_1)P(H_2) = \frac{1}{2}$$

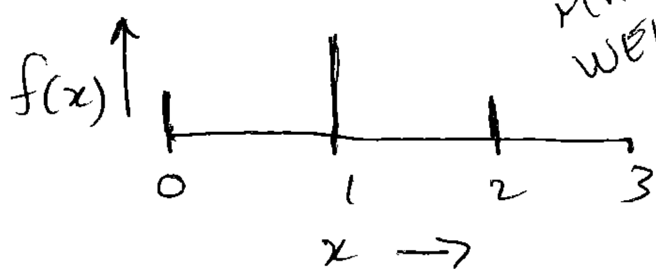
$$P(0 \text{ HEADS}) = P(\bar{H}_1 \bar{H}_2)$$
$$= P(\bar{H}_1)P(\bar{H}_2) = \frac{1}{4}$$

THIS GIVES THE "PROBABILITY MASS FUNCTION" FOR THE "RANDOM VARIABLE"

X = NUMBER OF HEADS ON 2 INDEPENDENT TOSSES OF A FAIR COIN

THINK OF POINT MASSES ON A WEIGHTLESS ROD.

x	$f(x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$



DEFINITION OF RANDOM VARIABLE:

A FUNCTION THAT ASSIGNS A REAL NUMBER TO EACH OUTCOME IN THE SAMPLE SPACE OF A RANDOM EXPERIMENT.

HERE:

RANDOM EXPT:

TWO INDEP. TOSSES OF A FAIR COIN

OUTCOMES:

$$\underbrace{H_1 H_2}_{X=2} \quad \underbrace{H_1 \bar{H}_2 \quad \bar{H}_1 H_2}_{X=1} \quad \underbrace{\bar{H}_1 \bar{H}_2}_{X=0}$$

PROBABILITY MODEL:

$$f(x) = P(X = x) \quad x = 0, 1, 2$$

$\Rightarrow$ MODEL LETS US DERIVE THE THEORETICAL PROBABILITY MASS FUNCTION. THIS MODEL HAS TWO ASSUMPTIONS, "FAIR" AND "INDEPENDENT"

TOTAL PROBABILITY AND BAYES' THEOREM

EX MACHINE A DOES 75% OF PRODUCTION, MACHINE B DOES 25%.

DEFECTIVE RATES:

10% FOR MACHINE A

3% " " B

DEFINE EVENTS: FOR A RANDOM UNIT

A : CAME FROM MACHINE A

$\bar{A}$: " " " B

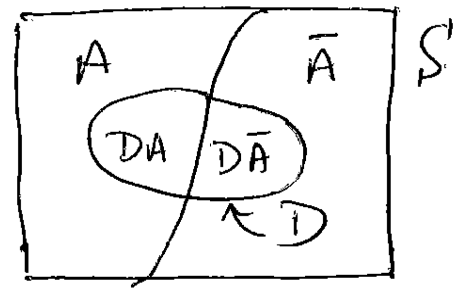
D : UNIT IS DEFECTIVE

GIVEN PROBABILITIES:

$P(A) = 0.75 \qquad P(\bar{A}) = 0.25$

$P(D|A) = 0.10 \qquad P(D|\bar{A}) = 0.03$

WHAT IS THE OVERALL DEFECTIVE RATE ?

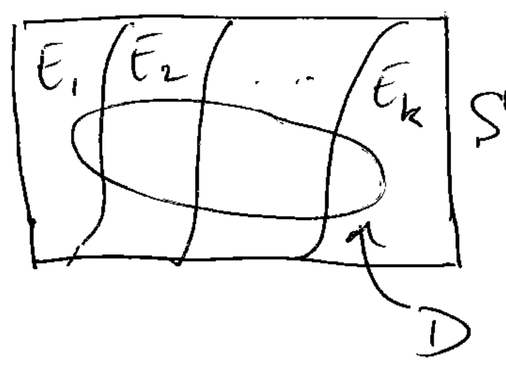


$$P(D) = P(DA \cup D\bar{A})$$
$$= P(DA) + P(D\bar{A})$$
$$= P(D|A)P(A) + P(D|\bar{A})P(\bar{A})$$
$$= (0.10)(.75) + (0.03)(.25)$$
$$= 0.075 + 0.0075 = 0.0825$$

i.e. A WEIGHTED AVERAGE OF THE TWO RATES.

"TOTAL PROBABILITY" p. 43

$$P(D) = \sum_{i=1}^{k} P(D|E_i)P(E_i)$$



APPLICATION:

YOU ARE GIVEN CONDITIONAL PROBABILITY UNDER EVERY POSSIBLE CONDITION, AND THE PROBABILITY THAT EACH CONDITION HOLDS.

BAYES' THEOREM (P.51)

EX WHAT IS THE PROBABILITY THAT A RANDOMLY CHOSEN UNIT IT CAME FROM MACHINE A?

ANSWER: $P(A) = 0.75$

IF THE RANDOMLY CHOSEN UNIT IS TESTED AND FOUND TO BE DEFECTIVE, WHAT IS THE PROBABILITY IT CAME FROM MACHINE A?

$$P(A|D) = \frac{P(AD)}{P(D)}$$

$$= \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|\bar{A})P(\bar{A})}$$

TOTAL PROBABILITY

$$= \frac{(0.10)(0.75)}{(0.10)(0.75) + (0.03)(0.25)}$$

$$= \frac{0.075}{0.0825} = 0.\dot{9}\dot{0}$$

NOTE: HERE $P(A|D) > P(A)$ WHY?

APPLICATION:

TURN AROUND CONDITIONAL PROBABILITIES.

GIVEN $P(D \mid A)$, $P(D \mid \bar{A})$

WANT $P(A \mid D)$.

PROBABILITY MASS FUNCTION

(OR DENSITY FUNCTION)

p. 53

RANDOM VARIABLE X

SET OF POSSIBLE VALUES $\mathcal{X}$

"SAMPLE SPACE"

A PARTICULAR VALUE x

p. 61

$$f(x) = P(X = x) \qquad x \in \mathcal{X}$$

"THE PROBABILITY THAT THE RANDOM VARIABLE X WILL ASSUME THE PARTICULAR VALUE x"

EX RANDOM EXPERIMENT:

ROLL A "FAIR" 6-SIDED DIE; OBSERVE SCORE ON UPTURNED FACE.

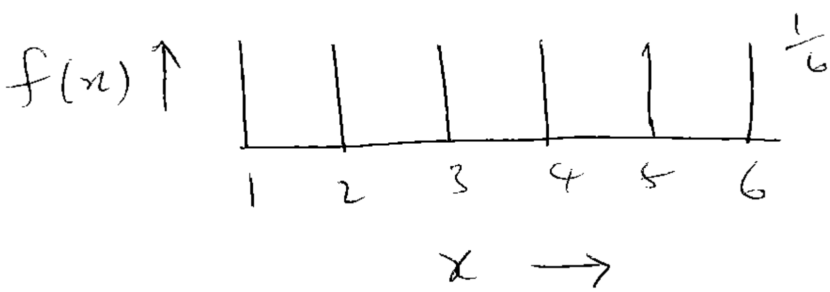
RANDOM VARIABLE:

X = SCORE ON FACE

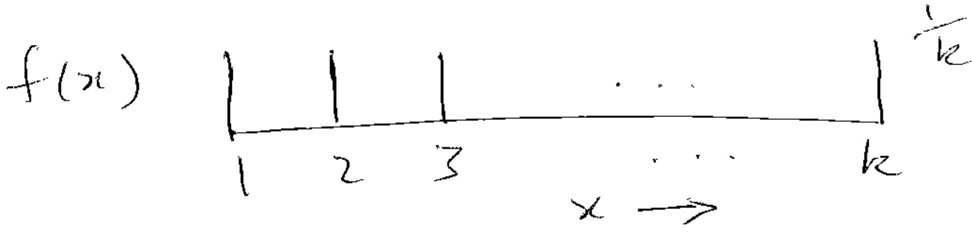
$\mathcal{X} = \{1, 2, 3, 4, 5, 6\}$

PROBABILITY MASS FUNCTION:

$$f(x) = \frac{1}{6} \qquad x = 1, 2, 3, 4, 5, 6$$



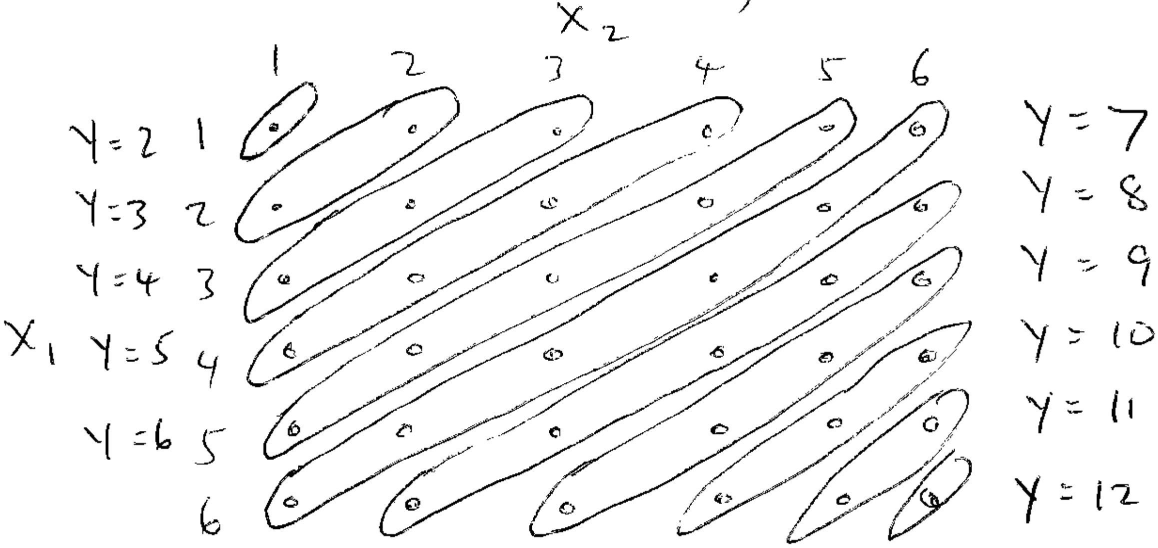
THIS IS CALLED THE "DISCRETE UNIFORM" DISTRIBUTION



EX ROLL TWO DICE INDEPENDENTLY, OBSERVE X_1 ON FIRST, X_2 ON SECOND, FIND TOTAL

$$Y = X_1 + X_2$$

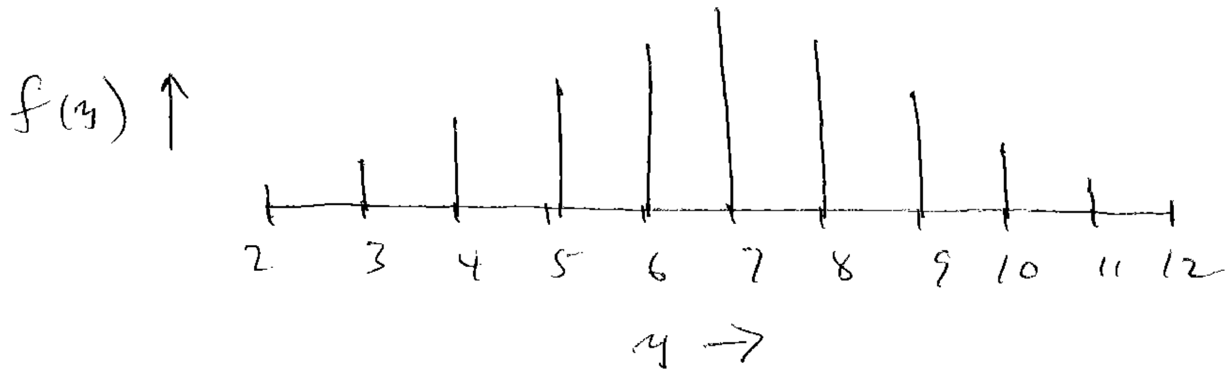
Y IS A R.V., FIND ITS PMF



EACH OF THE 36 POSSIBLE OUTCOME HAS SAME PROBABILITY

$$P(X_1 = x_1 \wedge X_2 = x_2)$$
$$= P(X_1 = x_1)\,P(X_2 = x_2) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

y	2	3	4	5	6	7	8	9	10	11	12
$f(y)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



CENTRAL LIMIT THEOREM:

THE MORE RANDOM VARIABLES YOU ADD UP, THE CLOSER THE DISTRIBUTION OF THE SUM WILL BE TO A NORMAL DISTRIBUTION.