STATS 3N03/3J04 TEST #02 SOLUTIONS
2003-10-30

Solution of Q.1 [Total 14]

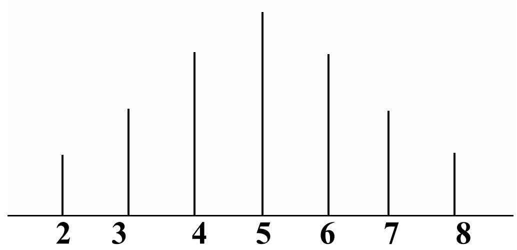
Let X denote the total score when two four-sided dice are rolled independently. The sample space will be

# on first dice	# on second dice	Total of the two dice
1	1	2
2	1	3
3	1	4
4	1	5
1	2	3
2	2	4
3	2	5
4	2	6
1	3	4
2	3	5
3	3	6
4	3	7
1	4	5
2	4	6
3	4	7
4	4	8

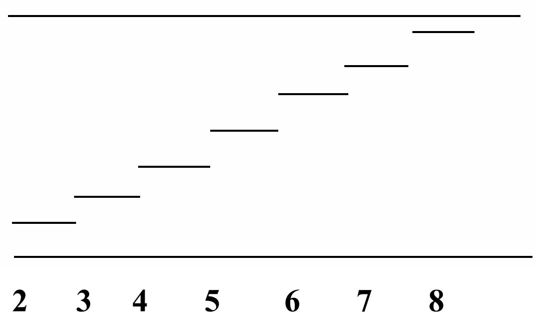
Frequency distribution [4]

Observation X	Frequency f_x	Probability	Cumulative Probability
2	1	$1/16$	$1/16$
3	2	$2/16$	$3/16$
4	3	$3/16$	$6/16$
5	4	$4/16$	$10/16$
6	3	$3/16$	$13/16$
7	2	$2/16$	$15/16$
8	1	$1/16$	$16/16=1$
Total	**16**		

Probability Mass Function: [2]



Cumulative Distribution Function: [2]



Mean, Median and Mode: [3]

Mean = $\sum X \times Probability = 5$

By symmetry, Mean=Median=Mode=5

Variance and Standard Deviation : [2+1]

Variance(X)

$$= \sum (X - mean)^2 \times Probability$$

$$= \frac{1(2-5)^2}{16} + \frac{2(3-5)^2}{16} + \frac{3(4-5)^2}{16} + \frac{4(5-5)^2}{16} + \frac{3(6-5)^2}{16} + \frac{2(7-5)^2}{16} + \frac{1(8-5)^2}{16}$$

$$= 2\left(\frac{9}{16} + \frac{8}{16} + \frac{3}{16}\right)$$

$$= \frac{40}{16} = 2.5$$

Standard deviation = $\sqrt{Variance(X)} = \sqrt{2.5} = 1.5811$

Solution of Q.2 [Total 10]

Interpretaion of the R code: [2]

The R code simulates 20 rolls of two 4-sided dice. It first generates a sample of 40 observations between 1 and 4, puts them in two columns (20 observations in each) and then adds the two columns to produce a vector of “total scores from the two columns”.

Stem and leaf plot: [2]

Cumulative Freq.	Freq.		
1	1	2	0
6	5	3	00000
7	1	4	0
12	5	5	00000
17	5	6	00000
19	2	7	00
20	1	8	0

Mean, median, mode, variance, SD: [6]

Mean = 4.9
Sample variance = 2.7263
SD= 1.6511

Median = 5
Mode = {3,5,6}

Solution of Q.3 [Total 12]

Let X be the weight of a pill.
Assume that the weight is normally distributed, i.e., $X \sim N(50,1^2)$

Then $p = P(X < 48) = \Phi\left(\frac{48-50}{1}\right) = \Phi(-2) = 1-\Phi(2) = 1-0.97725 = 0.02275$

Let Y be the number of pill in a box of 10 that weigh less than 48gm. Assuming independence, $Y \sim Binomial(10, p)$. Then the probability that at most 1 pill in a given box will weigh less than 48 gm is:

$$
\begin{aligned}
P(Y \le 1) &= \binom{10}{0} p^0 (1-p)^{10} + \binom{10}{1} p^1 (1-p)^9 \\
&= (0.02275)^0 (0.97725)^{10} + 10(0.2275)^1 (0.97725)^9 \qquad \textbf{[6]} \\
&= 0.9794
\end{aligned}
$$

Let Z be the number of pills in the bottle of 1000 that weigh less than 48 gm. Assuming independence, $Z \sim Binomial(1000, p)$

$$
\begin{aligned}
P(Z \le 20) &= \sum_{z=0}^{20} \binom{1000}{z} p^z (1-p)^{1000-z} \\
&\approx \Phi\left(\frac{20.5-1000p}{\sqrt{1000p(1-p)}}\right) \\
&= \Phi\left(\frac{20.5-22.75}{4.715}\right) \\
&= \Phi(-0.477) \qquad \textbf{[6]} \\
&= 1-\Phi(0.477) \\
&= 1-0.683 \\
&= 0.317
\end{aligned}
$$

Solution of Q.4 [Total 7]

Assume that accident happen one at a time, at a constant average rate of 3 per month. Also ignore the difference in the length of month. **[2]**

Let X denote the number of accidents in this month. Assuming that accidents happen independently, => $X \sim Poisson(3)$. So the probability of 6 or more accidents :

$$
\begin{aligned}
P(X \geq 6) &= 1 - P(X \leq 5) \\
&= 1 - \sum_{x=0}^{5} \frac{e^{-3} 3^x}{x!} \\
&= 1 - \left[e^{-3} \left(1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} \right) \right] \qquad \textbf{[4]} \\
&= 1 - e^{-3}(18.4) \\
&= 0.0839
\end{aligned}
$$

The probability is not very small; an average we would expect this to happen in one month every year, as $12(0.0839) = 1.01$

Therefore, this would not be an unusual event. **[1]**

Solution of Q.5 [Total 2]

Random variable: A function that assigns a real number to each outcome in the sample space of a random experiment. **[1]**

Parameter: A scaler or vector that includes a family of probaility distributions.
In another words, it can be defined as the unknown population characteristics. **[1]**