Math 3D03
M. Min-Oo
Assignment #2

DUE: TUESDAY, FEBRUARY 4TH, 2014 IN CLASS (AT THE BEGINNING OF THE LECTURE PERIOD)

Note: You can use symbolic software **only** *to check your answers (for the integrals for example) and to plot graphs, but you are required to show your calculations*

1. (*8 marks*) Evaluate the following definite (real-valued) integrals:

$$(i)\ \int_0^\infty \frac{(\log(x))^2}{1+x^2}\,dx \qquad (ii)\ \int_{-\infty}^\infty \frac{e^{ax}}{1+e^x}\,dx \quad \text{for } 0<a<1$$

$$(iii)\ \int_0^\infty \frac{\log(x)}{x^{\frac{3}{4}}(1+x)}\,dx \qquad (iv)\ \int_0^\infty \frac{dx}{1+x^n} \quad \text{where } n\geq 2 \text{ is an integer}$$

2. (*2 marks*) How many zeros of the polynomial $z^4 - 5z + 1$ lie in the annulus $1 \leq |z| \leq 2$?

3. (*6 marks*) Sum the following infinite series:

$$(a)\ \sum_{n=1}^\infty \frac{1}{n^2+9} \qquad (b)\ \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^4} \qquad (c)\ \sum_{n=-\infty}^\infty \frac{n^2}{n^4-\pi^4}$$

4. (*4 marks*) Do problem 25.14 on page 922 - 923 in the text book.

5. (*5 marks*) Show that the map

$$w = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

maps circles centered at the origin in the z-plane to ellipses in the w-plane. Draw some images. What happens to other circles? Find the image of the circle centered at the point $z_0 = -\frac{1}{5}(1-i)$ with radius $\frac{1}{5}\sqrt{37}$ (*Use Matlab or some other software to plot the graphs)*

6. (bonus question)

(i) Suppose that $f(z)$ is a non-constant analytic function defined for all $z \in \mathbb{C}$. Show that for every $R>0$ and for every $M>0$ there exists a z such that $|z|>R$ and $|f(z)|>M$.

(ii) Suppose that $f(z)$ is a non-constant polynomial. Show that for every $M>0$ there exists an $R>0$, such that $|f(z)|>M$ for all $|z|>R$.

(iii) Show that there exists an $M>0$, such that for every $R>0$, there exists a z satisfying $|z|>R$ and $|e^z| \leq M$.