Math 3D03
Assignment #1

DUE: TUESDAY, JANUARY 21ST, 2014 IN CLASS (AT THE BEGINNING OF THE LECTURE PERIOD)

Note: You are required to show your calculations. You can use symbolic software **only** *to check your answers.*

1. Compute the Taylor, respectively Laurent series expansion and determine the region of convergence of the following functions around the point $z = 0$:

$$(a)\ \ f(z) = \frac{1}{2i}\,\log\left(\frac{1+iz}{1-iz}\right) \qquad\qquad (b)\ \ f(z) = \frac{e^{\frac{1}{z}}}{1-z}$$

2. Classify all the singular points and compute the residues at the poles of the following functions:

$$(a)\ \ f(z) = \frac{\pi\, z}{\sin(\pi z)} \qquad (b)\ \ f(z) = \frac{z}{1-z^2}\,\sinh\frac{1}{1-z} \qquad (c)\ \ f(z) = \frac{z}{1-e^{-z}}$$

3. Evaluate the following complex contour integrals:

$$(a)\ \ \oint_C \frac{dz}{1+z^4} \qquad (b)\ \ \oint_C \frac{e^{i\,z}dz}{1-z^2} \qquad (c)\ \ \oint_C \frac{z^3\,dz}{(z+1)^2(z^2+4)}$$

where C is the ellipse defined by: $3x^2 + 4y^2 = 10^{10}$

4. Let a be a positive real number. Compute (using an appropriate contour)

$$\int_0^\infty \cos(a\,x^2)\,dx$$

.

5. Compute

$$\int_0^{2\pi} \sin^n\theta\,d\theta$$

What happens when $n \to \infty$?

6. (*bonus question*) Consider the $n-1$ diagonals connecting one fixed vertex to all the other vertices of a regular n-gon inscribed in a unit circle. Prove that the products of their lengths is equal to n.