Math 3C03
M. Min-Oo
Assignment #1

Due: Thursday, September 19th, 2013 in class at the beginning of the lecture

1. Do problem 8.42 on page 314 in the textbook.

2. Do problem 9.1 on page 329 in the textbook.

3. Show that
$$\begin{vmatrix} x & a & b \\ x^2 & a^2 & b^2 \\ a+b & x+b & x+a \end{vmatrix} = (b-a)(x-a)(x-b)(x+a+b)$$

4. The four functions $\mathbf{v}_0 = 1, \mathbf{v}_1 = t, \mathbf{v}_2 = t^2, \mathbf{v}_3 = t^3$ form a basis for the vector space of polynomials of degree ≤ 3. Apply the Gram-Schmidt procedure to find an orthonormal basis with respect to the inner product: $< \mathbf{f}, \mathbf{g} >= \int_{-1}^{1} f(t)g(t)\,dt$.

5. Consider the two almost identical linear equations:
$$\begin{pmatrix} 0.11 & 0.19 & 0.10 \\ 0.49 & -0.31 & 0.21 \\ 1.55 & -0.70 & 0.70 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
and
$$\begin{pmatrix} 0.11 & 0.19 & 0.10 \\ 0.49 & -0.31 & 0.21 \\ 1.55 & -0.70 & 0.71 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
Show by using MATLAB or MAPLE, or any other appropriate computing software, that the solutions are wildly different. Can you give an explanation?

6. (*bonus question*)
Compute the Fourier sine series of the odd function $f(x) = x^3 - 4x$; $-2 \leq x \leq 2$ (periodically extended with period = 4). Use Parseval's identity to compute $\zeta(6) = \sum_{n=1}^{\infty} n^{-6}$. Plot the first few partial sums using Matlab or some other computer software.