## 9. SOLUTIONS TO EXERCISES

### Basic Algebra

**1.** (a) $\pi$, $-1/2$, $0.33$ (b) $1$.

**2.** (a) $81$ (b) $1/64$ (c) not defined (d) $8$.

**3.** (a) $3x^3 - 7x^2 + 7x - 4$ (b) $4x^2y^2 - 4xy^3 + y^4$ (c) $x^4 - 0.4x^2 + 0.04$

(d) $x^7 - x$ (e) $8x^3 - 12x^2 + 6x - 1$.

**4.** (a) $(x-2)(x+5)$ (b) $(2x-1)(x+4)$ (c) $2(x-3)(x+4)$

(d) $(x-1)(x^2+x+1)(x+1)(x^2-x+1)$.

**5.** (a) $(2x^2+y)(y-2)$ (b) $x^2(x-1)(x+1)$ (c) $3(3a-5b)(3a+5b)$

(d) $x(x-1)(x+1)(x^2+1)$.

**6.** (a) $-5/4$ (b) $1/\sqrt{2}$ (c) $5+2\sqrt{6}$ (d) $-1$ (e) $27/8$.

**7.** (a) $\dfrac{x^2+3x+9}{x+3}$ (b) $\dfrac{x^2(x-1)}{2x+1}$.

**8.** $\dfrac{x}{x-3}$.

**9.** $\dfrac{3(\sqrt{x}+\sqrt{2})}{x-2}$.

**10.** (a) $\dfrac{x+8-6\sqrt{x}}{4-x}$ (b) $\dfrac{x}{x-2}$ (c) $\dfrac{12x^2-x-3}{x^3-x}$.

### Equations and Inequalities

**1.** $x = -1$.

**2.** $x = -47$.

**3.** (a) $x = 5/3$ (b) $x = -9/5$.

**4.** (a) $x = 9$ (b) $x = -1$ (c) $x = 5/2$ (d) $x = -3$, $x = 2$.

**5.** (a) $(2, \infty)$ (b) $[2, 3]$ (c) $[5/3, 2]$.

**6.** (a) $(-\infty, -7)$ or $(3, \infty)$ (b) $(-\infty, 5)$ or $[8, \infty)$ (c) $(-\infty, -1]$ or $[0, 1]$.

**7.** $x = -2$ and $x = 5$.

**8.** (a) $1/2 < x < 5/2$ (b) $(-\infty, -8/3]$ or $[0, \infty)$.

**9.** (a) $x = 3$, $y = -7$ (b) $x = 1$, $y = 5$ and $x = -2$, $y = 2$.

### Elements of Analytic Geometry

**1.** $\sqrt{85}$.

**2.** $y = -7x + 10$.

**3.** Line, goes through the origin, of slope $-1/2$.

**4.** (a) No. Their slopes are $-1/2$ and $3$ (b) $y = x/2 - 5/2$ (c) Yes. Their slopes are $3/2$ and $-2/3$.

**5.** (a) Ellipse with semi-axes 3 and $\sqrt{3}$; $x$-intercepts are $\pm 3$, $y$-intercepts are $\pm\sqrt{3}$ (b) circle, centered at $(1, 3)$, radius $\sqrt{10}$.

**6.** (a) $x$-intercepts at $x = -3 \pm \sqrt{6}$; $y$-intercept at $y = 3$; vertex at $(-3, -6)$; points upward

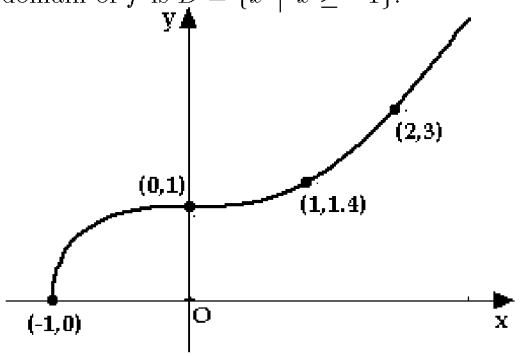
(b) $x$-intercept at $x = 2$; $y$-intercept at $y = 4$; vertex at $(2, 0)$; points upward (c) no $x$-intercepts; $y$-intercept at $y = -1$; vertex at $(0, -1)$; points downward.

**Functions**

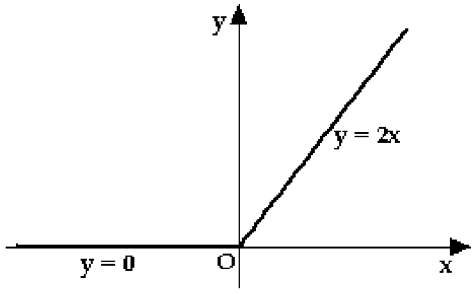
**1.** (a) $D = \mathbb{R}$, $R = \mathbb{R}$ (b) $D = \{x \mid x \neq 0\}$, $R = \{y \mid y > 0\}$ (c) $D = \mathbb{R}$, $R = \{y \mid y \geq 0\}$.

**2.** (a) $D = \{x \mid x > 2\}$ (b) $D = \{x \mid x \neq 0, 1, -1\}$ (c) $D = \{x \mid x < 0 \text{ or } x \geq 1\} = (-\infty, 0)$ or $[1, \infty)$.

**3.** Note: the domain of $f$ is $D = \{x \mid x \geq -1\}$.




**4.** (a) Horizontal line, crosses the $y$-axis at $(0, -4)$ (b) Mirror image of $y = x^2$ with respect to the $x$-axis (c) $f(x) = 0$, if $x < 0$ and $f(x) = 2x$ if $x \geq 0$; see the graph below.




**5.** (a) $3x/2 + 4$ (b) $9x + 16$ (c) $(3x + 4)/2$.

**6.** (a) $(x^2 + x + 1)^5$ (b) $x^{25}$ (c) $x^{10} + x^5 + 1$.

**7.** (a) Move the parabola $y = x^2$ 2 units to the left and 3 units up (b) Move the graph of $y = \sqrt{x}$ 4 units to the right and 2 units down (c) Move the graph of $y = 1/x^2$ 1 unit to the right and then 2 units up.

**8.** (a) Take the graph of $y = x^3$, compress it by a factor of 2 and then mirror it with respect to the $x$-axis (b) Start with the graph of $y = \sqrt{x}$, move it 3 units to the left, then stretch by the factor of 3, and then move one unit up.

**Trigonometry**

**1.** (a) $5\pi/4$ radians (b) 210 degrees.

**2.** (a) $\sin\frac{5\pi}{6} = 1/2$, $\cos\frac{5\pi}{6} = -\sqrt{3}/2$ and $\tan\frac{5\pi}{6} = -1/\sqrt{3}$ (b) $\sin\left(-\frac{\pi}{6}\right) = -1/2$, $\cos\left(-\frac{\pi}{6}\right) = \sqrt{3}/2$ and $\tan\left(-\frac{\pi}{6}\right) = -1/\sqrt{3}$.

**3.** $\sin\theta = 1/3$, $\cos\theta = \sqrt{8}/3$ and $\tan\theta = 1/\sqrt{8}$.

**4.** $\sin(-3\pi/2) = -1$, $\cos(-3\pi/2) = 0$; $\tan(-3\pi/2)$ and $\sec(-3\pi/2)$ are not defined.

**5.** $2\pi/a$.

**6.** To obtain $\cos 3x$, compress the graph of $\cos x$ along the $x$-axis by a factor if 3; to obtain $\cos 0.5x$, stretch the graph of $\cos x$ along the $x$-axis by a factor of 2.

**7.** $\pi/a$.

**8.** (a) Use the addition formula for $\sin x$ (b) Write $\tan x = \sin x/\cos x$ and compute the common denominator (c) Combine the first and the third terms and use (b).

**9.** $\sin 3x = 3\sin x - 4\sin^3 x$.

**10.** Use the addition and the subtraction formulas for $\sin x$ and multiply out the terms that you get. Then combine terms and simplify using the basic trig identity.

**11.** (a) $x = \pi + 2k\pi$ (b) $x = -\frac{\pi}{4} + k\pi$.

**12.** (a) $x = \frac{2\pi}{3} + 2k\pi$ and $x = \frac{4\pi}{3} + 2k\pi$ (b) $x = \frac{\pi}{3} + k\pi$ (c) $x = \frac{\pi}{4} + 2k\pi$ and $x = \frac{3\pi}{4} + 2k\pi$.

**13.** $x = k\pi$, $x = \frac{\pi}{3} + 2k\pi$ and $x = \frac{5\pi}{3} + 2k\pi$.

**14.** $x = \frac{\pi}{6} + k\pi$ and $x = \frac{5\pi}{6} + k\pi$.

**15.** (a) $\sin\alpha = \cos\beta = 21/29$, $\cos\alpha = \sin\beta = 20/29$ (b) $a = 12$, $b = 5$, $\sin\beta = 5/13$, $\tan\beta = 5/12$ (c) $\sin\beta = 0.8 = 4/5$, $\cos\beta = 0.6 = 3/5$, $\tan\beta = 4/3$, $\csc\beta = 5/4$, $\sec\beta = 5/3$, and $\cot\beta = 3/4$.

**16.** (a) third (b) fourth (c) third (d) second.

**17.** (a) positive (b) positive (c) negative (d) positive.

**18.** (a) $\sin 1$ (b) $\cos 2^0$ (c) $\tan 1$.

**19.** (a) $\tan^2 x$ (b) $1/\cos x$ (c) $2/\sin x$ (d) 2.

**20.** (a) $x = -\frac{\pi}{6} + k\pi$ (b) $x = \frac{3\pi}{4} + k\pi$ (c) $x = \frac{\pi}{6} + 2k\pi$ and $x = \frac{11\pi}{6} + 2k\pi$.

**21.** (a) $x = \frac{\pi}{4} + 2k\pi$ and $x = \frac{3\pi}{4} + 2k\pi$ (b) no solution (c) $(2k+1)\pi$.

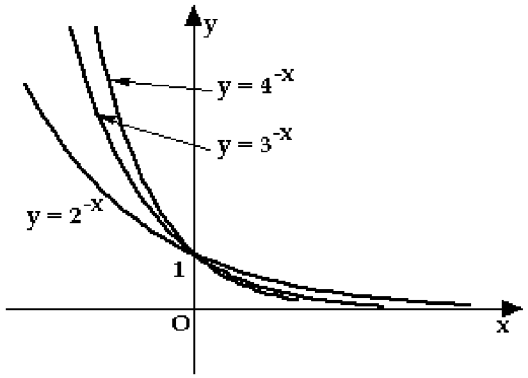
**22.** (a) graph of $\cos x$, moved $\pi/4$ units to the left (b) graph of $\sin x$, moved $\pi$ units to the right (c) graph of $\tan x$, moved 1 unit to the left.

**23.** (a) cross-multiply (b) replace $\tan x$ by $\sin x/\cos x$ and $\cot x$ by $\cos x/\sin x$, and simplify fractions.

**Exponential and Logarithmic Functions**

**1.** (a) Move the graph of $y = 2^x$ 4 units up (b) Move the graph of $y = 2^x$ 4 units to the right (c) Reflect the graph of $y = 2^x$ with respect to the $x$-axis (d) $-2^{-x} = -(1/2)^x$; reflect the graph of $y = (1/2)^x$ with respect to the $x$-axis.

**2.**




**3.** (a) $5^{-x+4}$ (b) $3^{6x-14}$ (c) $2^{-x+20}$.

**4.** (a) $x = \pm\sqrt{3}$ (b) $x = 1$ (c) $x = 0$ and $x = 2$.

**5.** (a) $x = 16$ (b) $x = 243$ (c) $x = 2$ only ($x = 0$ and $x = -2$ are not solutions since they are not in the domain of $\log_2 x$) (d) $x = 2$.

**6.** (a) 20 (b) $\ln(48e^2)$ (c) $x = (\ln 4 + 2)/3$ (d) $x = -2$ and $x = 1$.

**7.** (a) $10^{17}$ (b) $3^8$ (c) $3^{12}$ (d) 6.

**8.** (a) 5 (b) $10^4$ (c) $-2$.

**9.** (a) $\sqrt{8}$ (b) 5 (c) 5.

**10.** (a) $-2$ (b) 2 (c) 1/23.

**11.** (a) $-2$ (b) $-1/2$ (c) $-2$.

**12.** (a) $-8/3$ (b) $\ln 2$.

**Calculus: Limits and Derivatives**

**1.** (a) $-7/2$ (b) 0.

**2.** (a) 3/2 (b) 3/7 (c) 5/7.

**3.** $y = -2x + 3$.

**4.** (a) $y' = 1/(2\sqrt{x+1})$ (b) $y' = -1/(2x^{3/2})$.

**5.** (a) $f'(x) = 1/(2\sqrt{x}) + 1/(2x^{3/2})$ (b) $f'(x) = -8x^{-7/3}$ (c) $y' = (3x^2 - 1)/(2x^{3/2})$ (d) $y' = 2x + \pi x^{\pi-1}$.

**6.** $y = 1$.

**7.** (a) $y' = \cos x$ (b) $y' = 3\sin x + 3\sin x \sec^2 x$ (c) Apply the quotient rule to $\sec x = 1/\cos x$.

**8.** (a) $y' = -(3x^2+1)/(x^3+x-2)^2$ (b) $y' = (\sqrt{x}+1)/\sqrt{x}$ (c) $y' = 2x\sec^2(x^2) + 2x\sec^2(x^2+1)$ (d) $y' = e^x\sec(e^x)\tan(e^x)$ (e) $y' = -4x\sin(x^2)\cos(x^2)$ (f) $y' = 2x\sin(1/x) - \cos(1/x)$.

**9.** (a) $y' = \dfrac{1}{2x} + \dfrac{1}{2x\sqrt{\ln x}}$ (b) $y' = \dfrac{-2}{x(1+\ln x)^2}$ (c) $f'(x) = 2^{e^x}e^x\ln 2$
(d) $y' = e^x + ex^{e-1}$ (e) $2\sec^2(\ln x)\tan(\ln x)/x$.

**10.** (a) $-\dfrac{\cot x\csc^2 x}{\sqrt{\cot^2 x + 2}}$ (b) $\dfrac{e^x}{(e^x+4)\ln 2}$.