

This assignment is due 18 November.

- [4] 1. (a) In what way is $x^3 - x^2 + \epsilon = 0$ a singular perturbation problem?
 [6] (b) Use perturbation theory to approximate the roots of the polynomial in (a) for small ϵ to $O(\epsilon^2)$.

- [6] 2. (a) The function $y(x, \epsilon)$ satisfies

$$\epsilon y'' + y y' - y = 0 \quad \text{in } 0 \leq x \leq 1$$

and is subject to the boundary condition $y(0) = 0$ and $y(1) = 3$. Assuming that there is a boundary layer only near $x = 0$, find the leading order terms in the outer and inner approximations, match them and write down the uniform approximation.

- [4] (b) Plot the solution you found in (a) compared with a numerical solution for $\epsilon = 0.1, 0.01$. [Hint: use Matlab function `bvp4c` to find the numerical solution.]
- [10] 3. Consider the problem $\epsilon y'' + x^\alpha y' + y = 0$ with $y(0) = y(1) = 1$ as $\epsilon \rightarrow 0_+$. For what values of α is there a boundary layer at $x = 0$? What is the thickness of the boundary layer?

- [10] 4. Use boundary layer theory to solve $\epsilon y'' + a(x)y' + b(x)y = \delta(x)$, where $y(x) = 0$ for $x < 0$, to leading order in ϵ as $\epsilon \rightarrow 0_+$. Assume that $a(x) > 0$ for $x \geq 0$.

- [6] 5. (a) Use boundary layer theory to find a leading-order uniform approximation to the solution of

$$\epsilon y''(x) + 2xy'(x) - 4x^2y(x) = 0; \quad -1 \leq x \leq 1; \quad y(-1) = 0, \quad y(+1) = e$$

in the limit $\epsilon \rightarrow 0$.

- [4] (b) Plot the solution you found in (a) compared with a numerical solution for $\epsilon = 0.1, 0.01$. [Hint: use Matlab function `bvp4c` to find the numerical solution.]

Total: 50