

This assignment is due 28 October in class.

- [8] 1. Use the method of stationary phase to find the leading behaviour of

$$\int_0^1 \sin\left[x\left(t + \frac{1}{6}t^3 - \sinh t\right)\right] \cos t \, dt.$$

- [8] 2. Find the first two terms in the asymptotic behaviour of $\int_0^{\pi/4} \cos(xt^2) \tan^2 t \, dt$ as $x \rightarrow +\infty$.

- [10] 3. Show that

$$\int_0^{1/e} \frac{e^{-xt}}{\ln t} \, dt \sim -\frac{1}{x \ln x}, \quad x \rightarrow +\infty.$$

Note any special behaviour.

- [12] 4. Find the leading behaviour as $x \rightarrow +\infty$ of the integral

$$\int_C \frac{e^{ix(t^3/3-t)}}{t-a} \, dt,$$

where C is a contour connecting $-\infty$ to $+\infty$ in the upper half plane and a is a real constant. Investigate separately the cases $|a| < 1$, $|a| > 1$, $a = \pm 1$.

- [12] 5. Evaluate the first two terms as $r \rightarrow 0$ and the first four terms (counting $\ln r$ and 1 as different orders of magnitude) as $r \rightarrow \infty$ of

$$\int_0^\infty \frac{rx \, dx}{(r^2 + x)^{3/2}(1+x)}.$$

[Total: 50]